

Problem of the Fortnight #10

Jackson Ding March 20

$$f(x) + f(x+y) + xy = f(2x-y) + x^2 - 2$$

for all real numbers x and y

$$\text{Let } y = -x$$

$$\Rightarrow f(x) + f(3x) - x^2 = f(3x) + x^2 - 2$$

$$\Rightarrow f(x) = 2x^2 - 2$$

$$\text{Let } f(x) = 0 \Rightarrow 2x^2 - 2 = 0 \Rightarrow x = \pm 1$$

$\Rightarrow x = \pm 1$ are the roots and the only roots for f .