Problem of the Fortnight #10

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\[ f(x) + f(4x + y) + xy = f(2x - y) + x^2 + 2 \]

for all real numbers \( x \) and \( y \)

Let \( y = -x \)

\[ \Rightarrow f(x) + f(3x) - x^2 = f(3x) + x^2 + 2 \]

\[ \Rightarrow f(x) = 2x^2 - 2 \]

Let \( f(x) = 0 \) \( \Rightarrow 2x^2 - 2 = 0 \) \( \Rightarrow x = \pm 1 \)

\[ \Rightarrow x = \pm 1 \] are the roots and the only roots for \( f \)