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Frank L. H. Wolfs, *Editor*

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

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Measuring the molecular polarizability of air

M. J. Madsen, D. R. Brown, S. R. Krutz, and M. J. Milliman
Department of Physics, Wabash College, Crawfordsville, Indiana 47933

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We present an update of the "refractive index of air" experiment, which is commonly used in undergraduate advanced laboratories. The refractive index of air depends on the average molecular polarizability, which can be determined from the period of the phase shift in a Michelson interferometer as a function of air pressure. The measured average molecular polarizability of air is $\gamma_{\text{mol}}=(2.118\pm0.091)\times10^{-29}$ m^3 (95% CI). The corresponding refractive index of air at atmospheric pressure is $n=1.000265(11)$, which agrees with the accepted value of $n=1.000271375(6)$.
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I. INTRODUCTION

Measuring the refractive index of air using a vacuum cell in one arm of a Michelson interferometer is a common experiment in undergraduate advanced laboratories.[1] Typically, students determine the change in the number of bright and dark fringes as air is pumped from the cell.[2–4] Based on this number, students calculate the index of refraction of the air in the cell at atmospheric pressure. The refractive index of air depends on the particular pressure, temperature, and humidity content of the air during the experiment. These measurements do not explain why the refractive index changes as a function of pressure nor do they measure an intrinsic property of the air molecules, such as the molecular polarizability.

In this paper, we present an alternative approach to this experiment, based on a measurement of the average electric polarizability of a collection of molecules in response to an electromagnetic wave. The electric polarizability is an intrinsic property of the air that depends on the properties of the electron cloud around the molecules and can be determined from the intensity of the light at the output of a Michelson interferometer as a function of the pressure in one of its arms. We made two modifications to the interferometer: (i) An apertured light sensor was added to the output of the interferometer and (ii) a pressure sensor was connected to the vacuum cell. We will show how the refractive index of air can be calculated from the polarizability for a given pressure and temperature.

II. MODEL

Atoms and molecules respond to external electric fields by creating an induced polarization. In a mixed-element gas, such as air, a calculation of the polarization of the constituent molecules can be difficult to perform. The average dipole moment of the medium $\langle\vec{p}\rangle$ is related to the effective electric field $\vec{E}'$,

$$\langle\vec{p}\rangle=\epsilon_0\gamma_{\text{mol}}\vec{E}', \tag{1}$$

where ϵ_0 is the permittivity of free space and γ_{mol} is the molecular polarizability that depends on both the molecular structure and the composition of the gas.[5,6] The macroscopic polarization vector $\vec{P}$ is equal to $\eta\langle\vec{p}\rangle$, where η is the number of molecules per unit volume.[7] Since the macroscopic polarization is proportional to the dielectric susceptibility χ_e of a substance, we can relate the dielectric susceptibility to the molecular polarizability,[6]

$$\chi_e=\frac{\eta\gamma_{\text{mol}}}{1-1/3\,\eta\gamma_{\text{mol}}}. \tag{2}$$

For optical wavelengths, the dielectric susceptibility is related to the refractive index n: $\chi_e=n^2-1$. To find the number density as a function of the dielectric susceptibility, we solve Eq. (2) for η. Since gases at *NTP* typically have a dielectric

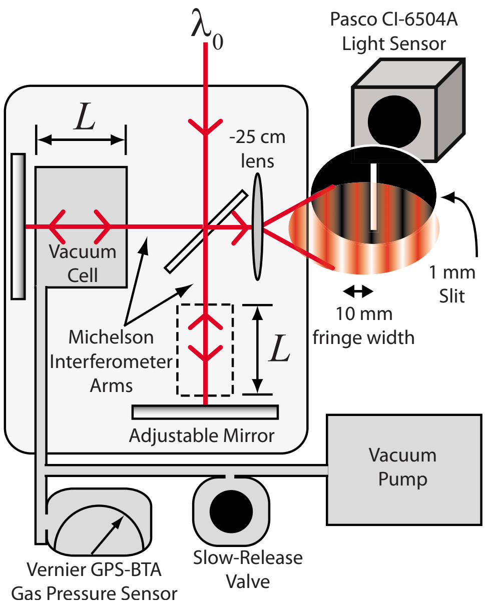

Fig. 1. A helium-neon laser was coupled to a Michelson interferometer with a vacuum cell of effective length $L=0.0256\pm0.0011$ m inserted in one arm. The alignment of the adjustable mirror was offset horizontally until the vertical output fringes were about 10 mm wide between dark stripes. A 1 mm slit was placed in front of the light sensor, resulting in a fringe contrast of about 70:1. The vacuum pump reduced the pressure in the cell to about 1 kPa, as measured with the pressure sensor. A slow-release valve released the vacuum over a time period of about 200 s.

susceptibility on the order of 10^{-3}, we keep only the first order term in χ_e and find that the number density is approximately proportional to the dielectric susceptibility,

$$\eta \approx \frac{\chi_e}{\gamma_{\mathrm{mol}}}. \tag{3}$$

Treating air as an ideal gas at pressure P and temperature T, the number density η will be proportional to the pressure P: $\eta=P/kT$, where k is the Boltzmann constant. Equation (3) shows that the dielectric susceptibility is proportional to the pressure: $P\approx(kT/\gamma_{\mathrm{mol}})\chi_e$. The predicted value of the molecular polarizability, based on the empirical data from NIST, is $\gamma_{\mathrm{mol}}=2.1865(22)\times10^{-29}$ m^3.[8–10] We find that the refractive index of air is a function of pressure and, since the susceptibility for gases is small, is approximately equal to

$$n \approx 1+\frac{\gamma_{\mathrm{mol}}P}{2kT}. \tag{4}$$

In our experiment, we measure the change in the light intensity as a function of air pressure in a Michelson interferometer with two paths of approximately equal physical lengths, as shown in Fig. 1. The critical difference between the two arms of the interferometer can be characterized by two lengths: The optical path length of the vacuum cell $2n_{\mathrm{cell}}L$ in one arm and the reference optical path length $2n_{\mathrm{ref}}L$ in the other arm, noting that the light passes twice through both the vacuum cell and the reference arm. When the vacuum cell is filled with air, $n_{\mathrm{cell}}=n_{\mathrm{ref}}=n_{\mathrm{air}}$, and the path length difference (ΔOPL) between the two arms is zero. Since the index of refraction is a function of pressure [see Eq. (4)], the optical path length difference becomes

$$\Delta\mathrm{OPL}=2L\left(1+\frac{\gamma_{\mathrm{mol}}P}{2kT}-n_{\mathrm{ref}}\right). \tag{5}$$

We neglect any motion of the vacuum windows, and thus the physical length of the cell, due to changes in pressure. We are only interested in changes in phase due to changes in pressure; the constant terms in Eq. (5) give rise to a constant phase offset in the output of the interferometer. The intensity I of the light at the output of the interferometer depends on this phase difference $\Delta\phi$ between the two arms. As the phase difference increases with pressure, the intensity oscillates between a maximum I_0 and zero intensity: $I=I_0\cos^2(\Delta\phi/2)$. The phase difference between the two arms depends on the optical path length difference and is equal to $\Delta\phi=2\pi\Delta\mathrm{OPL}/\lambda_0$. Using Eq. (5), we conclude that $\Delta\phi$ depends on pressure as

$$\Delta\phi=2\pi\frac{L\gamma_{\mathrm{mol}}}{\lambda_0 kT}P+\Delta\phi_0, \tag{6}$$

where the phase offset $\Delta\phi_0$ depends on the reference arm described above and is constant and stable over long time scales. The output intensity of the interferometer is thus

$$I=I_0\cos^2\left(2\pi\frac{P}{\Pi}+\frac{\Delta\phi_0}{2}\right), \tag{7}$$

where the period of the oscillations is

$$\Pi=\frac{2\lambda_0 kT}{L\gamma_{\mathrm{mol}}}. \tag{8}$$

Using this relationship, we can determine the molecular polarizability of air from the period of the intensity-versus-pressure data. Once we have the molecular polarizability, we can use Eq. (4) to find the refractive index of air at *NTP*.

III. METHODS

For our measurements, we coupled a helium-neon laser with a vacuum wavelength $\lambda_0=632.991$ nm to a Michelson interferometer.[11] Both arms were aligned so that the path length difference was approximately zero, as shown in Fig. 1. The beams were slightly misaligned by adjusting one mirror slightly off-center in the horizontal direction in order to produce a wide vertical fringe pattern at the output of the interferometer. Any change in the phase difference between the two light beams makes the vertical fringe pattern move horizontally. We magnified the fringes using a -25 cm focal length lens and detected the output intensity I using a 1-mm-wide vertical slit in front of a photodetector.[12] A single fringe was approximately 10 mm wide, producing a fringe contrast of $\sim$70:1 at the detector.

The vacuum cell in the second interferometer arm, as shown in Fig. 1, has a total length of 3.2 ± 0.1 cm (95% CI) and two windows with a thickness of 0.318 ± 0.025 cm (95% CI), giving an effective length of the vacuum portion of the cell of $L=0.0256\pm0.0011$ m (95% CI). The vacuum cell was aligned perpendicular to the beam in the second arm of the interferometer by using the back reflections from the

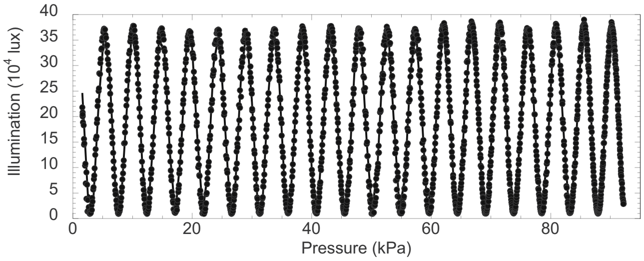

Fig. 2. The measured fringe intensity of our interferometer as a function of the pressure in the vacuum cell. The sinusoidal dependence is due to the change of the refractive index with pressure. The period of oscillation is $\Pi = 9.430 \pm 0.015$ kPa (95% CI), based on the statistical average of four trials.

glass faces of the cell. A slight vertical offset in the exiting beam was maintained to ensure that the main output beam was not disrupted.

The pressure P in the vacuum cell was measured using a gas pressure sensor.[13] The cell was pumped down to ~ 1 kPa and the vacuum was released slowly with a manual valve. The total time for the cell to return to atmospheric pressure was about 200 s. We found that a slow pressure release prevented a systematic effect due to the vacuum cell windows moving. Data from both the pressure sensor and the light sensor were collected simultaneously using a Vernier LabPro computer interface (with an adapter for the light sensor)[14] at a sampling rate of about 20 samples/s. The data collected, as shown in Fig. 2, have a clear sinusoidal trend, while the residuals are random.

We fit the intensity I versus pressure P data using a four-parameter sinusoidal fit based on Eq. (7),

$$I = I_0 \cos^2\left(\frac{2\pi P}{\Pi_{\text{fit}}} + \phi_0\right) + I_{\text{offset}}. \tag{9}$$

The only fit parameter of interest is the period of the oscillation Π_{fit}. The experiment was carried out four times with similar conditions and we found a period of $\Pi_{\text{fit}} = 9.430 \pm 0.015$ kPa (95% CI). During all four experiments, the room was maintained at room temperature, $T = 293 \pm 1$ K (95% CI). Using Eq. (8), the molecular polarizability of air was determined to be $\gamma_{\text{mol}} = 2.118 \pm 0.091 \times 10^{-29}$ m^3 (95% CI). This result is in agreement with the value from NIST, $\gamma_{\text{mol,NIST}} = (2.1865 \pm 0.0022) \times 10^{-29}$ m^3. We found the refractive index of air at atmospheric pressure to be $n = 1.000265(11)$, which is also in agreement with the accepted value of $n = 1.000271375(6)$.[10]

IV. SUMMARY AND CONCLUSIONS

We have shown that by measuring the output intensity of the Michelson interferometer as a function of the pressure in a vacuum cell, the molecular polarizability of air can be accurately measured. We have also shown explicitly how the observed intensity depends on the pressure in the vacuum cell. This experiment highlights the techniques of making a differential measurement. A few simple improvements to the experiment can be made by fabricating a custom vacuum cell with thicker external windows in order to reduce the systematic uncertainty in the measurement of the molecular polarizability and to reduce the systematic effect of the windows flexing with a change in pressure. Additionally, various pure gases, such as nitrogen and argon, can be used and the results are compared to known values of the dielectric susceptibility for those gases. Finally, we determined the refractive index of air at atmospheric pressure, as in the traditional interferometer experiments, but our result is based on a more fundamental measurement.

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[1] Instruction Manual and Experiment Guide for PASCO Scientific Michelson Interferometer, Model OS-8501, pp. 12–14.

[2] M. A. Jeppesen, "Measurement of dispersion of gases with a Michelson interferometer," Am. J. Phys. **35**, 435–436 (1967).

[3] F. Y. Yap, "Laser measurement of refractive index of a gas," Am. J. Phys. **39**, 224–224 (1971).

[4] J. D. Hey, H. S. T. Driver, and D. B. Fish, "The refractivity of helium and its measurement by laser interferometer," Am. J. Phys. **56**, 646–652 (1988).

[5] Max Born and Emil Wolf, *Principles of Optics*, 7th ed. (Cambridge U. P., Cambridge, UK, 1999), pp. 92–95.

[6] John David Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, NY, 1999), pp. 159–164.

[7] It is common to denote the number density as n. However, since we use n for the refractive index, we will use η to represent the number density.

[8] Bengt Edlén, "The refractive index of air," Metrologia **2** (2), 71–80 (1966).

[9] Philip E. Ciddor, "Refractive index of air: New equations for the visible and near infrared," Appl. Opt. **35** (9), 1566–1573 (1996).

[10] NIST Electromagnetic Toolbox, Modified Edlén equation calculator, ⟨http://emtoolbox.nist.gov/Wavelength/Edlen.asp⟩.

[11] PASCO precision interferometer OS-9255A.

[12] PASCO light detector CI-6504A.

[13] Vernier gas pressure sensor GPS-BTA.

[14] For example, Vernier software, "LOGGER PRO," information available at ⟨http://www.vernier.com/soft/lp.html⟩.