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Tractatus Logico-Philosophicus

DEDICATED
TO THE MEMORY OF MY FRIEND
DAVID H. PINSENT

Motto: . . . und alles, was man weiss, nicht bloss rauschen
und brausen gehört hat, lässt sich in drei Worten sagen.
Kürnberger.

Tractatus Logico-Philosophicus

PREFACE

This book will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it or similar thoughts. It is therefore not a text book. Its object would be attained if it afforded pleasure to one who read it with understanding.

The book deals with the problems of philosophy and shows, as I believe, that the method of formulating these problems rests on the misunderstanding of the logic of our language. Its whole meaning could be summed up somewhat as follows: What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.

The book will, therefore, draw a limit to thinking, or rather—not to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought).

The limit can, therefore, only be drawn in language and what lies on the other side of the limit will be simply nonsense.

How far my efforts agree with those of other philosophers I will not decide. Indeed what I have here written makes no claim to novelty in points of detail and therefore I give no sources, because it is indifferent to me whether what I have thought has already been thought before me by another.

I will only mention that to the great works of Frege and the writings of my friend Bertrand Russell I owe in large measure the stimulation of my thoughts.

If this work has a value it consists in two things. First that in it thoughts are expressed, and this value will be the greater the better the thoughts are expressed. The more the nail has been hit on the head.—Here I am conscious that I have fallen far short of the possible. Simply because my powers are insufficient to cope with the task.—May others come and do it better.

On the other hand the *truth* of the thoughts communicated here seems to me unassailable and definitive. I am, therefore, of the opinion that the problems have in essentials been finally solved. And if I am not mistaken in this, then the value of this work secondly consists in the fact that it shows how little has been done when these problems have been solved.

L.W.

Vienna, 1918

1 The world is everything that is the case.*

* The decimal figures as numbers of the separate propositions indicate the logical importance of the propositions, the emphasis laid upon them is my exposition. The propositions *n.1, n.2, n.3*, etc., are comments on proposition No. *n*; the propositions *n.m1, n.m2*, etc., are comments on the proposition No *n.m*; and so on.

1.1 The world is the totality of facts, not of things.

1.11 The world is determined by the facts, and by these being
all the facts.

1.12 For the totality of facts determines both what is the case,
and also all that is not the case.

1.13 The facts in logical space are the world.

1.2 The world divides into facts.

1.21 Any one can either be the case or not be the case, and
everything else remain the same.

2 What is the case, the fact, is the existence of atomic
facts.

2.01 An atomic fact is a combination of objects (entities,
things).

...

2.02 The object is simple.

...

2.03 In the atomic fact objects hang one in another, like the
links of a chain.

...

- 2.04 The totality of existent atomic facts is the world.
- 2.05 The totality of existent atomic facts also determines which atomic facts do not exist.
- 2.06 The existence and non existence of atomic facts is the reality. (The existence of atomic facts we also call a positive fact, their non existence a negative fact.)
- ...
- 2.1 We make to ourselves pictures of facts.
- 2.11 The picture presents the facts in logical space, the existence and non existence of atomic facts.
- 2.12 The picture is a model of reality.
- 2.13 To the objects correspond in the picture the elements of the picture.
- ...
- 2.14 The picture consists in the fact that its elements are combined with one another in a definite way.
- ...
- 2.15 That the elements of the picture are combined with one another in a definite way, represents that the things are so combined with one another. This connexion of the elements of the picture is called its structure, and the possibility of this structure is called the form of representation of the picture.
- ...
- 2.16 In order to be a picture a fact must have something in common with what it pictures.
- ...
- 2.17 What the picture must have in common with reality in order to be able to represent it after its manner rightly or falsely is its form of representation.
- ...
- 2.18 What every picture, of whatever form, must have in common with reality in order to be able to represent it at all rightly or falsely is the logical form, that is, the form of reality.
- ...
- 2.19 The logical picture can depict the world.
- 2.2 The picture has the logical form of representation in common with what it pictures.

...

- 2.22 The picture represents what it represents, independently of its truth or falsehood, through the form of representation.
- ...
- 3 The logical picture of the facts is the thought.
- ...
- 3.01 The totality of true thoughts is a picture of the world.
- 3.02 The thought contains the possibility of the state of affairs which it thinks. What is thinkable is also possible.
- 3.03 We cannot think anything unlogical, for otherwise we should have to think unlogically.
- ...
- 3.04 An a priori true thought would be one whose possibility guaranteed its truth.
- 3.05 Only if we could know a priori that a thought is true if its truth was to be recognized from the thought itself (without an object of comparison).
- 3.1 In the proposition the thought is expressed perceptibly through the senses.
- 3.11 We use the sensibly perceptible sign (sound or written sign, etc.) of the proposition as a projection of the possible state of affairs. The method of projection is the thinking of the sense of the proposition.
- 3.12 The sign through which we express the thought I call the propositional sign. And the proposition is the propositional sign in its projective relation to the world.
- 3.13 To the proposition belongs everything which belongs to the projection; but not what is projected.
Therefore the possibility of what is projected but not this itself.
In the proposition, therefore, its sense is not yet contained, but the possibility of expressing it.
("The content of the proposition" means the content of the significant proposition.)
In the proposition the form of its sense is contained, but not its content.
- 3.14 The propositional sign consists in the fact that its elements, the words, are combined in it in a definite way.
The propositional sign is a fact.

- ...
- 3.2 In propositions thoughts can be so expressed that to the objects of the thoughts correspond the elements of the propositional sign.
- ...
- 3.21 To the configuration of the simple signs in the propositional sign corresponds the configuration of the objects in the state of affairs.
- 3.22 In the proposition the name represents the object.
- ...
- 3.23 The postulate of the possibility of the simple signs is the postulate of the determinateness of the sense.
- 3.24 A proposition about a complex stands in internal relation to the proposition about its constituent part.
A complex can only be given by its description, and this will either be right or wrong. The proposition in which there is mention of a complex, if this does not exist, becomes not nonsense but simply false.
That a propositional element signifies a complex can be seen from an indeterminateness in the propositions in which it occurs. *We know* that everything is not yet determined by this proposition. (The notation for generality *contains* a prototype.)
The combination of the symbols of a complex in a simple symbol can be expressed by a definition.
- 3.25 There is one and only one complete analysis of the proposition.
- ...
- 3.26 The name cannot be analysed further by any definition. It is a primitive sign.
- ...
- 3.262 What does not get expressed in the sign is shown by its application. What the signs conceal, their application declares.
- ...
- 3.3 Only the proposition has sense; only in the context of a proposition has a name meaning.
- 3.31 Every part of a proposition which characterizes its sense I call an expression (a symbol). (The proposition itself is an expression.) Expressions are everything—essential for the

sense of the proposition—that propositions can have in common with one another. An expression characterizes a form and a content.

- ...
- 3.32 The sign is the part of the symbol perceptible by the senses.
- ...
- 3.33 In logical syntax the meaning of a sign ought never to play a rôle; it must admit of being established without mention being thereby made of the *meaning* of a sign; it ought to presuppose *only* the description of the expressions.
- ...
- 3.34 A proposition possesses essential and accidental features. Accidental are the features which are due to a particular way of producing the propositional sign. Essential are those which alone enable the proposition to express its sense.
- ...
- 3.4 The proposition determines a place in logical space: the existence of this logical place is guaranteed by the existence of the constituent parts alone, by the existence of the significant proposition.
- 3.41 The propositional sign and the logical co-ordinates: that is the logical place.
- ...
- 3.42 Although a proposition may only determine one place in logical space, the whole logical space must already be given by it. (Otherwise denial, the logical sum, the logical product, etc., would always introduce new elements—in co-ordination.) (The logical scaffolding round the picture determines the logical space. The proposition reaches through the whole logical space.)
- 3.5 The applied, thought, propositional sign is the thought.
- 4 The thought is the significant proposition.
- ...
- 4.002 Man possesses the capacity of constructing languages, in which every sense can be expressed, without having an idea how and what each word means—just as one speaks without knowing how the single sounds are produced.
Colloquial language is a part of the human organism and is not less complicated than it.

- From it it is humanly impossible to gather immediately the logic of language.
- Language disguises the thought; so that from the external form of the clothes one cannot infer the form of the thought they clothe, because the external form of the clothes is constructed with quite another object than to let the form of the body be recognized.
- The silent adjustments to understand colloquial language are enormously complicated.
- ...
- 4.01 The proposition is a picture of reality. The proposition is a model of the reality as we think it is.
- ...
- 4.016 In order to understand the essence of the proposition, consider hieroglyphic writing, which pictures the facts it describes. And from it came the alphabet without the essence of the representation being lost.
- 4.02 This we see from the fact that we understand the sense of the propositional sign, without having had it explained to us.
- ...
- 4.03 A proposition must communicate a new sense with old words. The proposition communicates to us a state of affairs, therefore it must be *essentially* connected with the state of affairs. And the connexion is, in fact, that it is its logical picture. The proposition only asserts something, in so far as it is a picture.
- ...
- 4.04 In the proposition there must be exactly as many things distinguishable as there are in the state of affairs, which it represents. They must both possess the same logical (mathematical) multiplicity (cf. Hertz's *Mechanics*, on Dynamic Models).
- ...
- 4.05 Reality is compared with the proposition.
- 4.06 Propositions can be true or false only by being pictures of the reality.
- ...
- 4.1 A proposition presents the existence and non-existence of atomic facts.
- 4.11 The totality of true propositions is the total natural science (or the totality of the natural sciences).
- ...
- 4.12 Propositions can represent the whole reality, but they cannot represent what they must have in common with reality in order to be able to represent it—the logical form. To be able to represent the logical form, we should have to be able to put ourselves with the propositions outside logic, that is outside the world.
- ...
- 4.2 The sense of a proposition is its agreement and disagreement with the possibilities of the existence and non-existence of the atomic facts.
- 4.21 The simplest proposition, the elementary proposition, asserts the existence of an atomic fact.
- ...
- 4.22 The elementary proposition consists of names. It is a connexion, a concatenation, of names.
- ...
- 4.23 The name occurs in the proposition only in the context of the elementary proposition.
- ...
- 4.25 If the elementary proposition is true, the atomic fact exists; if it is false the atomic fact does not exist.
- 4.26 The specification of all true elementary propositions describes the world completely. The world is completely described by the specification of all elementary propositions plus the specification, which of them are true and which false.
- ...
- 4.28 To these combinations correspond the same number of possibilities of the truth—and falsehood—of n elementary propositions.
- 4.3 The truth-possibilities of the elementary propositions mean the possibilities of the existence and non-existence of the atomic facts.
- 4.31 The truth-possibilities can be presented by schemata of the following kind ("T" means "true", "F" "false". The rows of T's and F's under the row of the elementary propositions mean their truth-possibilities in an

easily intelligible symbolism).

p	q	r	p	q	p
T	T	T	T	T	T
F	T	T	F	T	F
T	F	T	T	F	F
T	T	F	F	F	F
F	F	T	F	F	F
F	T	F	F	F	F
T	F	F	F	F	F
F	F	F	F	F	F

4.4 A proposition is the expression of agreement and disagreement with the truth-possibilities of the elementary propositions.

4.41 The truth-possibilities of the elementary propositions are the conditions of the truth and falsehood of the propositions.

...

4.43 Agreement with the truth-possibilities can be expressed by co-ordinating with them in the schema the mark "T" (true).

Absence of this mark means disagreement.

...

4.44 The sign which arises from the co-ordination of that mark "T" with the truth-possibilities is a propositional sign.

...

4.46 Among the possible groups of truth-conditions there are two extreme cases.

In the one case the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are *tautological*.

In the second case the proposition is false for all the truth-possibilities. The truth-conditions are *self-contradictory*.

In the first case we call the proposition a tautology, in the second case a contradiction.

...

4.5 Now it appears to be possible to give the most general

form of proposition; *i.e.* to give a description of the propositions of some one sign language, so that every possible sense can be expressed by a symbol, which falls under the description, and so that every symbol which falls under the description can express a sense, if the meanings of the names are chosen accordingly.

It is clear that in the description of the most general form of proposition *only* what is essential to it may be described—otherwise it would not be the most general form.

That there is a general form is proved by the fact that there cannot be a proposition whose form could not have been foreseen (*i.e.* constructed). The general form of proposition is: Such and such is the case.

4.51 Suppose *all* elementary propositions were given me: then we can simply ask: what propositions I can build out of them. And these are *all* propositions and *so* are they limited.

4.52 The propositions are everything which follows from the totality of all elementary propositions (of course also from the fact that it is the *totality of them all*). (So, in some sense, one could say, that *all* propositions are generalizations of the elementary propositions.)

4.53 The general propositional form is a variable.

5 Propositions are truth-functions of elementary propositions. (An elementary proposition is a truth-function of itself.)

5.01 The elementary propositions are the truth-arguments of propositions.

...

5.1 The truth-functions can be ordered in series.
That is the foundation of the theory of probability.

...

5.11 If the truth-grounds which are common to a number of propositions are all also truth-grounds of some one proposition, we say that the truth of this proposition follows from the truth of those propositions.

5.12 In particular the truth of a proposition *p* follows from that of a proposition *q*, if all the truth-grounds of the second are truth-grounds of the first.

...

5.13 That the truth of one proposition follows from the truth of other propositions, we perceive from the structure of the propositions.

...

5.14 If a proposition follows from another, then the latter says more than the former, the former less than the latter.

...

5.2 The structures of propositions stand to one another in internal relations.

5.21 We can bring out these internal relations in our manner of expression, by presenting a proposition as the result of an operation which produces it from other propositions (the bases of the operation).

5.22 The operation is the expression of a relation between the structures of its result and its bases.

5.23 The operation is that which must happen to a proposition in order to make another out of it.

...

5.24 An operation shows itself in a variable; it shows how we can proceed from one form of proposition to another. It gives expression to the difference between the forms.
(And that which is common to the bases, and the result of an operation, is the bases themselves.)

...

5.25 The occurrence of an operation does not characterize the sense of a proposition.

For an operation does not assert anything; only its result does, and this depends on the bases of the operation.

(Operation and function must not be confused with one another.)

...

5.3 All propositions are results of truth-operations on the elementary propositions.

The truth-operation is the way in which a truth-function arises from elementary propositions.

According to the nature of truth-operations, in the same way as out of elementary propositions arise their truth-functions, from truth-functions arises a new one. Every truth-operation creates from truth-functions of elementary

propositions another truth-function of elementary propositions, *i.e.* a proposition. The result of every truth-operation on the results of truth-operations on elementary propositions is also the result of *one* truth-operation on elementary propositions.

Every proposition is the result of truth-operations on elementary propositions.

...

5.32 All truth-functions are results of the successive application of a finite number of truth-operations to elementary propositions.

5.4 Here it becomes clear that there are no such things as “logical objects” or “logical constants” (in the sense of Frege and Russell).

5.41 For all those results of truth-operations on truth-functions are identical, which are one and the same truth-function of elementary propositions.

...

5.43 That from a fact *p* an infinite number of *others* should follow, namely $\sim \sim p$, $\sim \sim \sim p$, etc., is indeed hardly to be believed, and it is no less wonderful that the infinite number of propositions of logic (of mathematics) should follow from half a dozen “primitive propositions”.

But all propositions of logic say the same thing. That is, nothing.

5.44 Truth-functions are not material functions. If *e.g.* an affirmation can be produced by repeated denial, is the denial—in any sense—contained in the affirmation? Does “ $\sim \sim p$ ” deny $\sim p$, or does it affirm *p*; or both? The proposition “ $\sim \sim p$ ” does not treat of denial as an object, but the possibility of denial is already prejudged in affirmation. And if there was an object called “ \sim ”, then “ $\sim \sim p$ ” would have to say something other than “*p*”. For the one proposition would then treat of \sim , the other would not.

...

5.45 If there are logical primitive signs a correct logic must make clear their position relative to one another and justify their existence. The construction of logic *out of* its primitive signs must become clear.

...

5.53 Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs.

...

5.54 In the general propositional form, propositions occur in a proposition only as bases of the truth-operations.

5.541 At first sight it appears as if there were also a different way in which one proposition could occur in another.

Especially in certain propositional forms of psychology, like "A thinks, that p is the case" or "A thinks p ", etc.

Here it appears superficially as if the proposition p stood to the object A in a kind of relation.

(And in modern epistemology (Russell, Moore, etc.) those propositions have been conceived in this way.)

5.542 But it is clear that "A believes that p ", "A thinks p ", "A says p ", are of the form " p says p ": and here we have no co-ordination of a fact and an object, but a co-ordination of facts by means of a co-ordination of their objects.

...

5.5422 The correct explanation of the form of the proposition "A judges p " must show that it is impossible to judge a nonsense. (Russell's theory does not satisfy this condition.)

...

5.55 We must now answer a priori the question as to all possible forms of the elementary propositions.

The elementary proposition consists of names. Since we cannot give the number of names with different meanings, we cannot give the composition of the elementary proposition.

5.551 Our fundamental principle is that every question which can be decided at all by logic can be decided off-hand.

(And if we get into a situation where we need to answer such a problem by looking at the world, this shows that we are on a fundamentally wrong track.)

5.552 The "experience" which we need to understand logic is not that such and such is the case, but that something *is*; but that is *no* experience.

Logic *precedes* every experience—that something is *so*. It is before the How, not before the What.

5.5521 And if this were not the case, how could we apply logic? We could say: if there were a logic, even if there were no world, how then could there be a logic, since there is a world?

...

5.6 *The limits of my language* mean the limits of my world.

5.61 Logic fills the world: the limits of the world are also its limits.

We cannot therefore say in logic: This and this there is in the world, that there is not.

For that would apparently presuppose that we exclude certain possibilities, and this cannot be the case since otherwise logic must get outside the limits of the world: that is, if it could consider these limits from the other side also.

What we cannot think, that we cannot think: we cannot therefore *say* what we cannot think.

5.62 This remark provides a key to the question, to what extent solipsism is a truth.

In fact what solipsism *means*, is quite correct, only it cannot be *said*, but it shows itself.

That the world is *my* world, shows itself in the fact that the limits of the language (*the* language which I understand) mean the limits of *my* world.

...

5.63 I am my world. (The microcosm.)

...

5.64 Here we see that solipsism strictly carried out coincides with pure realism. The I in solipsism shrinks to an extensionless point and there remains the reality co-ordinated with it.

...

6 The general form of truth-function is: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$. This is the general form of proposition.

...

6.03 The general form of the cardinal number is: $[0, \xi, \xi + 1]$.

...

6.1 The propositions of logic are tautologies.

6.11 The propositions of logic therefore say nothing. (They are the analytical propositions.)

...

6.12 The fact that the propositions of logic are tautologies *shows* the formal—logical—properties of language, of the world.

That its constituent parts connected together *in this way* give a tautology characterizes the logic of its constituent parts.

In order that propositions connected together in a definite way may give a tautology they must have definite properties of structure. That they give a tautology when *so* connected shows therefore that they possess these properties of structure.

...

6.13 Logic is not a theory but a reflexion of the world.
Logic is transcendental.

6.2 Mathematics is a logical method.

The propositions of mathematics are equations, and therefore pseudo-propositions.

6.21 Mathematical propositions express no thoughts.

...

6.22 The logic of the world which the propositions of logic show in tautologies, mathematics shows in equations.

6.23 If two expressions are connected by the sign of equality, this means that they can be substituted for one another. But whether this is the case must show itself in the two expressions themselves. It characterizes the logical form of two expressions, that they can be substituted for one another.

...

6.24 The method by which mathematics arrives at its equations is the method of substitution.

For equations express the substitutability of two expressions, and we proceed from a number of equations to new equations, replacing expressions by others in accordance with the equations.

...

6.3 Logical research means the investigation of *all regularity*. And outside logic all is accident.

6.31 The so-called law of induction cannot in any case be a logical law, for it is obviously a significant proposition.—

And therefore it cannot be a law a priori either.

6.32 The law of causality is not a law but the form of a law.

...

6.33 We do not *believe* a priori in a law of conservation, but we *know* a priori the possibility of a logical form.

6.34 All propositions, such as the law of causation, the law of continuity in nature, the law of least expenditure in nature, etc. etc., all these are a priori intuitions of possible forms of the propositions of science.

6.341 Newtonian mechanics, for example, brings the description of the universe to a unified form. Let us imagine a white surface with irregular black spots. We now say: Whatever kind of picture these make I can always get as near as I like to its description, if I cover the surface with a sufficiently fine square network and now say of every square that it is white or black. In this way I shall have brought the description of the surface to a unified form. This form is arbitrary, because I could have applied with equal success a net with a triangular or hexagonal mesh. It can happen that the description would have been simpler with the aid of a triangular mesh; that is to say we might have described the surface more accurately with a triangular, and coarser, than with the finer square mesh, or vice versa, and so on. To the different networks correspond different systems of describing the world. Mechanics determine a form of description by saying: All propositions in the description of the world must be obtained in a given way from a number of given propositions—the mechanical axioms. It thus provides the bricks for building the edifice of science, and says: Whatever building thou wouldst erect, thou shalt construct it in some manner with these bricks and these alone.

(As with the system of numbers one must be able to write down any arbitrary number, so with the system of mechanics one must be able to write down any arbitrary physical proposition.)

...

6.35 Although the spots in our picture are geometrical figures, geometry can obviously say nothing about their actual form and position. But the network is *purely* geometrical, and all

- its properties can be given a priori.
- Laws, like the law of causation, etc., treat of the network and not of what the network describes.
- 6.36 If there were a law of causality, it might run: “There are natural laws”.
- But that can clearly not be said: it shows itself.
- ...
- 6.37 A necessity for one thing to happen because another has happened does not exist. There is only *logical* necessity.
- ...
- 6.4 All propositions are of equal value.
- 6.41 The sense of the world must lie outside the world. In the world everything is as it is and happens as it does happen. *In it* there is no value—and if there were, it would be of no value.
- If there is a value which is of value, it must lie outside all happening and being-so. For all happening and being-so is accidental.
- What makes it non-accidental cannot lie *in* the world, for otherwise this would again be accidental. It must lie outside the world.
- 6.42 Hence also there can be no ethical propositions.
- Propositions cannot express anything higher.
- ...
- 6.43 If good or bad willing changes the world, it can only change the limits of the world, not the facts; not the things that can be expressed in language.
- In brief, the world must thereby become quite another. It must so to speak wax or wane as a whole.
- The world of the happy is quite another than that of the unhappy.
- ...
- 6.44 Not *how* the world is, is the mystical, but *that* it is.
- 6.45 The contemplation of the world sub specie aeterni is its contemplation as a limited whole.
- The feeling of the world as a limited whole is the mystical feeling.
- 6.5 For an answer which cannot be expressed the question too cannot be expressed.
- The riddle* does not exist.

- If a question can be put at all, then it *can* also be answered.
- 6.51 Scepticism is *not* irrefutable, but palpably senseless, if it would doubt where a question cannot be asked.
- For doubt can only exist where there is a question; a question only where there is an answer, and this only where something *can* be said.
- 6.52 We feel that even if *all possible* scientific questions be answered, the problems of life have still not been touched at all. Of course there is then no question left, and just this is the answer.
- 6.521 The solution of the problem of life is seen in the vanishing of this problem. (Is not this the reason why men to whom after long doubting the sense of life became clear, could not then say wherein this sense consisted?)
- ...
- 6.53 The right method of philosophy would be this. To say nothing except what can be said, *i.e.* the propositions of natural science, *i.e.* something that has nothing to do with philosophy: and then always, when someone else wished to say something metaphysical, to demonstrate to him that he had given no meaning to certain signs in his propositions. This method would be unsatisfying to the other—he would not have the feeling that we were teaching him philosophy—but it would be the only strictly correct method.
- 6.54 My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.)
- He must surmount these propositions; then he sees the world rightly.
- 7 Whereof one cannot speak, thereof one must be silent.