

# Notes for Elementary Symbolic Logic

## Preface

This is the text for PHI 270. That is, the text for the course lives on line. A link to the table of contents is in the navigation table at the top or bottom of this window, and you can find links to individual sections in the calendar and list of topics (which are among the syllabus pages, to which there is also a link in table). Most of the content of the text is also available in PDF format (there are links to that both in the table of contents and the list of topics); however, there are some animated or interactive components that can be used only with a web browser.

The text uses a number of special symbols. These must be available in fonts on your system in order for them to appear in the online HTML version of the text. Such fonts are available on most recent operating systems, but browsers (older versions of Internet Explorer in particular) will not always find their way to the correct symbols. There is a test for problems at the bottom of this page; but some problems are intermittent, so they can show up later even if they don't appear now. You will stand the best chance of avoiding them if you use a browser other than IE (e.g., Firefox, Safari, or Chrome) and also have the "STIX" fonts installed on your system. Those fonts are available at:

<http://sourceforge.net/projects/stixfonts/>

The following table lists the more important symbols we will use. For each, it shows how your current set-up displays the symbol, what the symbol looks like in the STIX fonts, and the standard code number for it.

browser	STIX	Unicode	browser	STIX	Unicode
^	^	U+2227	⊨	⊨	U+22A8
¬	¬	U+00AC	⊭	⊭	U+22AD
∨	∨	U+2228	≈	≈	U+2243
→	→	U+2192	⊤	⊤	U+22A4
←	←	U+2190	⊥	⊥	U+22A5
∀	∀	U+2200	Δ	Δ	U+25B5
∃	∃	U+2203	∇	∇	U+25BF
●	●	U+25CF	⊠	⊠	U+22C8
○	○	U+25CB			

It doesn't matter if the symbol doesn't look exactly like the image shown at its right, but you will need to be able to recognize it as the same sort of shape. Again, you may run into problems later even if things look fine now; so whenever you see a symbol you don't expect, you should compare what you are seeing with the pdf version of the text or homework. You should be particularly wary if you see a box-like shape (e.g., □) since that is often used when a font doesn't contain the symbol called for; and, although the symbol is used in logic, it's not one of the symbols we will use in this course, so you can be sure something has gone wrong.

Glen Helman 01 Aug 2013

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