Notes for Elementary Symbolic Logic

Preface

This is the text for PHI 270. That is, the text for the course lives on line. A link to the table of contents is in the navigation table at the top or bottom of this window, and you can find links to individual sections in the calendar and list of topics (which are among the syllabus pages, to which there is also a link in table). Most of the content of the text is also available in PDF format (there are links to that both in the table of contents and the list of topics); however, there are some animated or interactive components that can be used only with a web browser.

The text uses a number of special symbols. These must be available in fonts on your system in order for them to appear in the online HTML version of the text. Such fonts are available on most recent operating systems, but browsers (older versions of Internet Explorer in particular) will not always find their way to the correct symbols. There is a test for problems at the bottom of this page; but some problems are intermittent, so they can show up later even if they don't appear now. You will stand the best chance of avoiding them if you use a browser other than IE (e.g., Firefox, Safari, or Chrome) and also have the "STIX" fonts installed on your system. Those fonts are available at:

http://sourceforge.net/projects/stixfonts/

The following table lists the more important symbols we will use. For each, it shows how your current set-up displays the symbol, what the symbol looks like in the STIX fonts, and the standard code number for it.

browser	STIX	Unicode	browser	STIX	Unicode
∧	∧	U+2227	⊨	⊨	U+22A8
¬	¬	U+00AC	⊭	⊭	U+22AD
∨	∨	U+2228	≃	≃	U+2243
→	→	U+2192	⊤	⊤	U+22A4
←	←	U+2190	⊥	⊥	U+22A5
∀	∀	U+2200	△	△	U+25B5
∃	∃	U+2203	▽	▽	U+25BF
●	●	U+25CF	⋈	⋈	U+22C8
○	○	U+25CB			

It doesn't matter if the symbol doesn't look exactly like the image shown at its right, but you will need to be able to recognize it as the same sort of shape. Again, you may run into problems later even if things look fine now; so whenever you see a symbol you don't expect, you should compare what you are

seeing with the pdf version of the text or homework. You should be particularly wary if you see a box-like shape (e.g., □) since that is often used when a font doesn't contain the symbol called for; and, although the symbol is used in logic, it's not one of the symbols we will use in this course, so you can be sure something has gone wrong.

Glen Helman 01 Aug 2013

Table of contents

⊨ 1. Deduction

1.1. Formal deductive logic

1.1.0. Overview — 1.1.1. Logic — 1.1.2. Inference — 1.1.3. Arguments — 1.1.4. Deductive *vs.* non-deductive inference — 1.1.5. Bounds on inference — 1.1.6. Entailment and exclusion — 1.1.7. Inconsistency and exhaustiveness — 1.1.8. Formal logic — 1.1.s. Summary — 1.1.x. Exercises — 1.1.xa. Exercise answers

1.2. What is said: propositions

1.2.0. Overview — 1.2.1. Truth values and possible worlds — 1.2.2. Truth conditions and propositions — 1.2.3. Ordering by content — 1.2.4. Equivalence in content — 1.2.5. The extremes of content — 1.2.6. Logical space and the algebra of propositions — 1.2.7. Contrasting content — 1.2.8. Deductive relations in general — 1.2.s. Summary — 1.2.x. Exercises — 1.2.xa. Exercise answers

1.3. Beyond saying: pragmatics

1.3.0. Overview — 1.3.1. A model of language — 1.3.2. Some complications — 1.3.3. Speech acts — 1.3.4. Implicature — 1.3.5. Indexicality — 1.3.6. Vagueness — 1.3.7. Presupposition — 1.3.s. Summary — 1.3.x. Exercises — 1.3.xa. Exercise answers

1.4. General principles of deductive reasoning

1.4.0. Overview — 1.4.1. A closer look at entailment — 1.4.2. Separation — 1.4.3. Content and coverage — 1.4.4. Relative exhaustiveness — 1.4.5. A general framework — 1.4.6. Reduction to entailment — 1.4.7. Laws for entailment — 1.4.8. Duality — 1.4.s. Summary — 1.4.x. Exercises — 1.4.xa. Exercise answers

∧ 2. Conjunctions

2.1. And: adding content

2.1.0. Overview — 2.1.1. A connective — 2.1.2. A truth function — 2.1.3. Conjunction in English — 2.1.4. Limits on analysis — 2.1.5. Multiple conjunction — 2.1.6. Some sample analyses — 2.1.7. Logical forms — 2.1.8. Interpretations — 2.1.s. Summary — 2.1.x. Exercises — 2.1.xa. Exercise answers

2.2. Proofs: analyzing entailment

2.2.0. Overview — 2.2.1. Proofs as trees — 2.2.2. Argument trees — 2.2.3. Derivations — 2.2.4. Rules for derivations — 2.2.5. An example — 2.2.6. Two perspectives on derivations — 2.2.7. More rules — 2.2.8. Resources — 2.2.s. Summary — 2.2.x. Exercises — 2.2.xa. Exercise answers

2.3. Failed proofs and counterexamples

2.3.0. Overview — 2.3.1. When enough is enough — 2.3.2. Dead ends and counterexamples — 2.3.3. Validity through the generations — 2.3.4. Sound and safe rules — 2.3.5. Confirming counterexamples — 2.3.6. Reaching decisions — 2.3.7. Soundness and completeness — 2.3.8. Formal validity — 2.3.s. Summary — 2.3.x. Exercises — 2.3.xa. Exercise answers

2.4. Using lemmas

2.4.0. Overview — 2.4.1. Premises, assumptions, and suppositions — 2.4.2. The dangers of lemmas — 2.4.3. Lemmas for *reductio* arguments — 2.4.4. Attachment rules — 2.4.s. Summary — 2.4.x. Exercises — 2.4.xa. Exercise answers

¬ 3. Negations

3.1. Not: contradicting content

3.1.0. Overview — 3.1.1. Connectives — 3.1.2. Contradictory propositions — 3.1.3. Negation in English — 3.1.4. Negated conjunctions and conjoined negations — 3.1.5. Some

sample analyses — 3.1.s. Summary — 3.1.x. Exercises — 3.1.xa. Exercise answers

3.2. *Reductio* arguments: refuting suppositions

3.2.0. Overview — 3.2.1. The duality of premises and alternatives — 3.2.2. Drawing negative conclusions — 3.2.3. Some examples — 3.2.s. Summary — 3.2.x. Exercises — 3.2.xa. Exercise answers

3.3. Negations as premises

3.3.0. Overview — 3.3.1. Indirect proof — 3.3.2. Using lemmas to complete *reductio*s — 3.3.3. More examples — 3.3.s. Summary — 3.3.x. Exercises — 3.3.xa. Exercise answers

3.4. Counterexamples to *reductio*s

3.4.0. Overview — 3.4.1. When *reductio*s fail — 3.4.2. Some examples of consistency — 3.4.s. Summary — 3.4.x. Exercises — 3.4.xa. Exercise answers

3.5. Being guided by the rules

3.5.0. Overview — 3.5.1. Approaching derivations — 3.5.2. An example — 3.5.3. A procedure — 3.5.s. Summary — 3.5.x. Exercises — 3.5.xa. Exercise answers

∨ 4. Disjunctions

4.1. Or: taking common content

4.1.0. Overview — 4.1.1. Hedging — 4.1.2. Inclusive and exclusive disjunction — 4.1.3. Disjunction in English — 4.1.4. Further examples — 4.1.s. Summary — 4.1.x. Exercises — 4.1.xa. Exercise answers

4.2. Arguing from and for alternatives

4.2.0. Overview — 4.2.1. Proofs by cases — 4.2.2. Proving disjunctions — 4.2.3. Further examples — 4.2.4. The duality of conjunction and disjunction — 4.2.s. Summary — 4.2.x. Exercises — 4.2.xa. Exercise answers

4.3. Detachment: eliminating alternatives

4.3.0. Overview — 4.3.1. Detachment rules — 4.3.2. More attachment rules — 4.3.s. Summary — 4.3.x. Exercises — 4.3.xa. Exercise answers

→ 5. Conditionals

5.1. If: trimming content

5.1.0. Overview — 5.1.1. Conditions — 5.1.2. The conditional as a truth-functional connective — 5.1.3. Doubts about truth-functionality — 5.1.4. Examples — 5.1.s. Summary — 5.1.x. Exercises — 5.1.xa. Exercise answers

5.2. Only if and unless

5.2.0. Overview — 5.2.1. Only if — 5.2.2. Necessary and sufficient conditions — 5.2.3. Unless — 5.2.4. Three forms compared — 5.2.s. Summary — 5.2.x. Exercises — 5.2.xa. Exercise answers

5.3. Conditional proofs: bottling inference

5.3.0. Overview — 5.3.1. Conditionalization — 5.3.2. Detachment — 5.3.s. Summary — 5.3.x. Exercises — 5.3.xa. Exercise answers

5.4. Extreme measures

5.4.0. Overview — 5.4.1. Last resorts — 5.4.2. Optional extras — 5.4.s. Summary — 5.4.x. Exercises — 5.4.xa. Exercise answers

= 6. Predications

6.1. Naming and describing

6.1.0. Overview — 6.1.1. A richer grammar — 6.1.2. Logical predicates — 6.1.3. Extensionality — 6.1.4. Identity — 6.1.5. Analyzing predications — 6.1.6. Individual terms — 6.1.7. Functors — 6.1.8. Examples and problems — 6.1.s. Summary — 6.1.x. Exercises — 6.1.xa. Exercise answers

6.2. Predicates and pronouns

6.2.0. Overview — 6.2.1. Abstracts — 6.2.2. Bound variables — 6.2.3. Variables and pronouns — 6.2.4. Expanded and reduced forms — 6.2.s. Summary — 6.2.x. Exercises — 6.2.xa. Exercise answers

6.3. Arguments involving equations

6.3.0. Overview — 6.3.1. Logical properties of identity — 6.3.2. A law for aliases — 6.3.3. Derivations for identity — 6.3.s. Summary — 6.3.x. Exercises — 6.3.xa. Exercise answers

6.4. Describing models

6.4.0. Overview — 6.4.1. Extensions and ranges — 6.4.2. Building structures — 6.4.3. Structures as counterexamples — 6.4.s. Summary — 6.4.x. Exercises — 6.4.xa. Exercise answers

∀ 7. Generalizations

7.1. Generalizations in English

7.1.0. Overview — 7.1.1. Theories of quantifier phrases — 7.1.2. Pronouns and quantifier phrases — 7.1.3. Finding quantifier phrases — 7.1.4. Kinds of generalizations — 7.1.5. Bounds and exceptions — 7.1.s. Summary — 7.1.x. Exercises — 7.1.xa. Exercise answers

7.2. Generalizations and quantifiers

7.2.0. Overview — 7.2.1. The universal quantifier — 7.2.2. Analyzing generalizations — 7.2.3. Compound restrictions — 7.2.s. Summary — 7.2.x. Exercises — 7.2.xa. Exercise answers

7.3. Quantifiers and connectives

7.3.0. Overview — 7.3.1. Generalizations and counterexamples — 7.3.2. Generalizations as components — 7.3.3. Any and every — 7.3.s. Summary — 7.3.x. Exercises — 7.3.xa. Exercise answers

7.4. Multiple generality

7.4.0. Overview — 7.4.1. Multiple generality — 7.4.2. Judging the scope of quantifier phrases — 7.4.s. Summary — 7.4.x. Exercises — 7.4.xa. Exercise answers

7.5. General arguments

7.5.0. Overview — 7.5.1. Conjunction and universal quantification — 7.5.2. Instantiation — 7.5.3. Generalization — 7.5.4. Adding instances — 7.5.5. General arguments in derivations — 7.5.6. Syllogisms — 7.5.s. Summary — 7.5.x. Exercises — 7.5.xa. Exercise answers

7.6. Insuring generality

7.6.0. Overview — 7.6.1. How generality can fail — 7.6.2. Multiply general arguments — 7.6.s. Summary — 7.6.x. Exercises — 7.6.xa. Exercise answers

7.7. Soundness & completeness

7.7.0. Overview — 7.7.1. Aspects of adequacy — 7.7.2. Soundness — 7.7.3. Thoroughness — 7.7.4. Effectuality — 7.7.s. Summary — 7.7.x. Exercises — 7.7.xa. Exercise answers

7.8. Finite & infinite structures

7.8.0. Overview — 7.8.1. Finding finite structures — 7.8.2. The failure of decisiveness — 7.8.s. Summary — 7.8.x. Exercises — 7.8.xa. Exercise answers

∃ 8. Numerations

8.1. Some

8.1.0. Overview — 8.1.1. Exemplification — 8.1.2. Obversion — 8.1.3. Conversion — 8.1.4. Existentials exemplified — 8.1.5. Existential commitment — 8.1.s. Summary — 8.1.x. Exercises — 8.1.xa. Exercise answers

8.2. Uniform generality

8.2.0. Overview — 8.2.1. General and uniformly general exemplification — 8.2.2. Quantifier scope ambiguities — 8.2.3. Controlling ambiguity — 8.2.s. Summary — 8.2.x. Exercises — 8.2.xa. Exercise answers

8.3. Numerical quantification

8.3.0. Overview — 8.3.1. Else — 8.3.2. Numerical quantifier phrases — 8.3.3. Exactly *n* — 8.3.s. Summary — 8.3.x. Exercises — 8.3.xa. Exercise answers

8.4. Definite descriptions

8.4.0. Overview — 8.4.1. The problem of definite descriptions — 8.4.2. Definite descriptions as quantifier phrases — 8.4.3. Definite descriptions as individual terms — 8.4.4. Examples: restrictive *vs.* non-restrictive relative clauses — 8.4.s. Summary — 8.4.x. Exercises — 8.4.xa. Exercise answers

8.5. Proofs by choice & proofs of existence

8.5.0. Overview — 8.5.1. Proof by choice — 8.5.2. Constructive and non-constructive proof — 8.5.3. Derivations for existentials — 8.5.4. First-order logic — 8.5.s. Summary — 8.5.x. Exercises — 8.5.xa. Exercise answers

8.6. Arguments involving descriptive reference

8.6.0. Overview — 8.6.1. The role of definite descriptions in entailment — 8.6.2. Derivations for the description operator — 8.6.3. Consequences for adequacy — 8.6.s. Summary — 8.6.x. Exercises — 8.6.xa. Exercise answers

Appendices

A. Reference

A.0. Overview — A.1. Definitions and notation for basic concepts — A.2. Logical forms — A.3. Truth tables — A.4. Derivation rules

B. Laws for relative exhaustiveness

Index and glossary

Glen Helman 01 Aug 2013