

Appendices

Appendix A. Reference

A.0. Overview

A.1. Definitions and notation for basic concepts

The full range of deductive properties and relations

A.2. Logical forms

Forms expressed using one or more logical constants together with symbolic and English notation or readings

A.3. Truth tables

Tables that stipulate the meaning of the constants of truth-functional logic

A.4. Derivation rules

A guide to the use of derivation rules with links to the rules themselves

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A.1. Definitions and notation for basic concepts

Most deductive properties or relation concerns a set or some specific number of *assumptions* and a set or some specific number of *alternatives*. When there is only one alternative, it is a *conclusion*. This is shown in the following table, where cells are labeled in boldface by the concept expressed as a noun, with the verbal or adjectival form shown in italics.

		alternatives			
		set Σ	two ψ_1, ψ_2	one (concl.) ψ	none
assumptions	set Γ	relative exhaustiveness Γ renders Σ exhaustive		entailment Γ entails ψ	inconsistency Γ is inconsistent
	two ϕ_1, ϕ_2				mutual exclusiveness ϕ_1 and ϕ_2 are mutually exclusive
	one ϕ			implication ϕ implies ψ	absurdity ϕ is absurd
	none	exhaustiveness Σ is exhaustive	(joint) exhaustiveness ψ_1 and ψ_2 are (jointly) exhaustive	tautologousness ψ is tautologous (or is a tautology)	

Not appearing in the table are two relations that each abbreviate conjunctions of two claims drawn from the ones above.

conjunctive relation	component relations
(logical) equivalence ϕ and ψ are (logically) equivalent	ϕ implies ψ ψ implies ϕ
contradictoriness ϕ and ψ are contradictory	ϕ and ψ are mutually exclusive ϕ and ψ are jointly exhaustive

There are also two alternative ways of applying the concept of inconsistency:

alternative statements (for assumptions Γ and ϕ)		
exclusion Γ excludes ϕ	relative inconsistency ϕ is inconsistent with Γ	inconsistency of the union Γ with ϕ added is inconsistent

Note that in this case all sentences involved count as assumptions.

All concepts appearing in the first table can be defined in the same way, as saying that their assumptions cannot be separated from their alternatives. This idea can be stated more specifically in two ways:

NEGATIVE DEFINITION: there is no possible world in which the assumptions (if any) are all true while the alternatives (if any) are all false.

POSITIVE DEFINITION: in each possible world in which the assumptions (if any) are all true, at least one alternative is true.

When there are no assumptions or no alternatives, the corresponding clause may be dropped from the negative form. The same is true for the clause regarding assumptions in the positive form; and, if there are no alternatives, that definition can be restated as: *in each possible world, the assumptions are not all true (i.e., at least one is false).*

The following table gives an explicit definition for each of these concepts and also indicates compact notation for the concept.

concept	negative definition	positive definition
ϕ is a tautology $\models \phi$	There is no possible world in which ϕ is false.	ϕ is true in every possible world.
ϕ is absurd $\phi \models$	There is no possible world in which ϕ is true.	ϕ is false in every possible world.
ϕ implies ψ $\phi \models \psi$	There is no possible world in which ϕ is true and ψ is false.	ψ is true in every possible world in which ϕ is true.
ϕ and ψ are mutually exclusive $\phi \Delta \psi$	There is no possible world in which ϕ and ψ are both true.	In each possible world, at least one of ϕ and ψ is false.
ϕ and ψ are (jointly) exhaustive $\phi \nabla \psi$	There is no possible world in which ϕ and ψ are both false.	In each possible world, at least one of ϕ and ψ is true.
ϕ and ψ are (logically) equivalent $\phi \simeq \psi$	There is no possible world in which ϕ and ψ have different truth values.	In each possible world, ϕ and ψ have the same truth value as each other.
ϕ and ψ are contradictory $\phi \boxtimes \psi$	There is no possible world in which ϕ and ψ have the same truth value.	In each possible world, ϕ and ψ have opposite truth values.
Γ is inconsistent $\Gamma \models$	There is no possible world in which all members of Γ are true.	In each possible world, at least one member of Γ is false.
Γ is exhaustive $\models \Gamma$	There is no possible world in which all members of Γ are false.	In each possible world, at least one member of Γ is true.
Γ entails ϕ $\Gamma \models \phi$	There is no possible world in which ϕ is false while all members of Γ are true.	ϕ is true in every possible world in which all members of Γ are true.
Γ excludes ϕ $\Gamma, \phi \models$	There is no possible world in which ϕ is true while all members of Γ are true.	ϕ is false in every possible world in which all members of Γ are true.
Γ renders Σ exhaustive $\Gamma \models \Sigma$	There is no possible world in which all members of Γ are true while all members of Σ are false.	In each possible world in which all members of Γ are true, at least one member of Σ is true.

All these concepts can be expressed in terms of relative exhaustiveness and also in terms of entailment. To express them in terms of relative exhaustiveness, simply list the assumptions (if any) to the left of \models and list the alternatives (if any) to its right. The expression in terms of entailment for the concepts in the first table is shown below.

		alternatives			
		Σ	ψ_1, ψ_2	ψ	none
assumptions	Γ	$\Gamma, \Sigma^{\boxtimes} \models \perp$		$\Gamma \models \psi$	$\Gamma \models \perp$
	ϕ_1, ϕ_2				$\phi_1, \phi_2 \models \perp$
	ϕ			$\phi \models \psi$	$\phi \models \perp$
	none	$\Sigma^{\boxtimes} \models \perp$	$\psi_1^{\boxtimes} \models \psi_2$	$\models \psi$	

Here θ^{\boxtimes} is any sentence contradictory to θ (such as its negation); and Σ^{\boxtimes} is any result of replacing each member of Σ by a sentence that is contradictory to it. The joint exhaustiveness of ψ_1 and ψ_2 may also be expressed by $\psi_2^{\boxtimes} \models \psi_1$ and by $\psi_1^{\boxtimes}, \psi_2^{\boxtimes} \models \perp$. The general rule is that alternatives can be dropped if their contradictories are made assumptions (and vice versa) and that \perp may be used as a conclusion if there are no alternatives already.

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A.2. Logical forms

Forms for which there is symbolic notation

	Symbolic notation	English notation or English reading	
Negation	$\neg \phi$	not ϕ	
Conjunction	$\phi \wedge \psi$	both ϕ and ψ (ϕ and ψ)	
Disjunction	$\phi \vee \psi$	either ϕ or ψ (ϕ or ψ)	
The conditional	$\phi \rightarrow \psi$ $\psi \leftarrow \phi$	if ϕ then ψ (ϕ implies ψ) yes ψ if ϕ (ψ if ϕ)	
Identity	$\tau = \upsilon$	τ is υ	
Predication	$\theta \tau_1 \dots \tau_n$	θ fits τ_1, \dots, τ_n <small>A series of terms τ_1, \dots, τ_n can be read (series) τ_1, \dots, τ_n (using the expression <i>on</i> to distinguish this use of <i>and</i> from its use in conjunction and adding <i>series</i> when necessary to avoid ambiguity)</small>	
Compound term	$\gamma \tau_1 \dots \tau_n$	γ of τ_1, \dots, τ_n γ applied to τ_1, \dots, τ_n	
Predicate abstract	$[\phi]_{x_1 \dots x_n}$	what ϕ says of $x_1 \dots x_n$	
Functor abstract	$[\tau]_{x_1 \dots x_n}$	τ for $x_1 \dots x_n$	
Universal quantification	$\forall x \theta x$	forall x θx everything, x , is such that θx	
Restricted universal	$(\forall x: \rho x) \theta x$	forall x st ρx θx everything, x , such that ρx is such that θx	
Existential quantification	$\exists x \theta x$	for some x θx something, x , is such that θx	
Restricted existential	$(\exists x: \rho x) \theta x$	for some x st ρx θx something, x , such that ρx is such that θx	
Definite description	$!x \rho x$	the x st ρx the thing, x , such that ρx	

Some paraphrases of other forms

Truth-functional compounds

neither ϕ nor ψ	$\neg(\phi \vee \psi)$ $\neg\phi \wedge \neg\psi$
ψ only if ϕ	$\neg\psi \leftarrow \neg\phi$
ψ unless ϕ	$\psi \leftarrow \neg\phi$

Generalizations

All Cs are such that (... they ...)	$(\forall x: x \text{ is a } C) \dots x \dots$
No Cs are such that (... they ...)	$(\forall x: x \text{ is a } C) \neg \dots x \dots$
Only Cs are such that (... they ...)	$(\forall x: \neg x \text{ is a } C) \neg \dots x \dots$
with: among Bs	add to the restriction: $x \text{ is a } B$
except Es	$\neg x \text{ is an } E$
other than τ	$\neg x = \tau$

Numerical quantifier phrases

At least 1 C is such that (... it ...)	$(\exists x: x \text{ is a } C) \dots x \dots$
At least 2 Cs are such that (... they ...)	$(\exists x: x \text{ is a } C) (\exists y: y \text{ is a } C \wedge \neg y = x) (\dots x \dots \wedge \dots y \dots)$
Exactly 1 C is such that (... it ...)	$(\exists x: x \text{ is a } C) (\dots x \dots \wedge (\forall y: y \text{ is a } C \wedge \neg y = x) \neg \dots y \dots)$ or $(\exists x: x \text{ is a } C) (\dots x \dots \wedge (\forall y: y \text{ is a } C \wedge \dots y \dots) x = y)$

Definite descriptions (on Russell's analysis)

The C is such that (... it ...)	$(\exists x: x \text{ is a } C \wedge (\forall y: \neg y = x) \neg y \text{ is a } C) \dots x \dots$ or $(\exists x: x \text{ is a } C \wedge (\forall y: y \text{ is a } C) x = y) \dots x \dots$
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A.3. Truth tables

Tautology	Absurdity	Negation
$\frac{T}{T}$	$\frac{\perp}{F}$	$\frac{\phi}{T} \frac{\neg\phi}{F}$ $\frac{\phi}{F} \frac{\neg\phi}{T}$
Conjunction	Disjunction	Conditional
$\frac{\phi \psi}{T T} \frac{\phi \wedge \psi}{T}$ $\frac{\phi \psi}{T F} \frac{\phi \wedge \psi}{F}$ $\frac{\phi \psi}{F T} \frac{\phi \wedge \psi}{F}$ $\frac{\phi \psi}{F F} \frac{\phi \wedge \psi}{F}$	$\frac{\phi \psi}{T T} \frac{\phi \vee \psi}{T}$ $\frac{\phi \psi}{T F} \frac{\phi \vee \psi}{T}$ $\frac{\phi \psi}{F T} \frac{\phi \vee \psi}{T}$ $\frac{\phi \psi}{F F} \frac{\phi \vee \psi}{F}$	$\frac{\phi \psi}{T T} \frac{\phi \rightarrow \psi}{T}$ $\frac{\phi \psi}{T F} \frac{\phi \rightarrow \psi}{F}$ $\frac{\phi \psi}{F T} \frac{\phi \rightarrow \psi}{T}$ $\frac{\phi \psi}{F F} \frac{\phi \rightarrow \psi}{T}$

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A.4. Derivation rules

Basic system

Rules for developing gaps			Rules for closing gaps		
logical form	as a resource	as a goal	when to close	resources	goal
atomic sentence		IP		ϕ	ϕ QED
negation $\neg\phi$ (if ϕ not atomic & goal is \perp)	CR	RAA	ϕ and $\neg\phi$	\perp	Nc
conjunction $\phi \wedge \psi$	Ext	Cnj		\perp	ENV
disjunction $\phi \vee \psi$	PC	PE		$\tau = \upsilon$	EC
conditional $\phi \rightarrow \psi$ (if goal is \perp)	RC	CP	$\tau = \upsilon$	$\neg\tau = \upsilon$	\perp DC
universal $\forall x \theta x$	UI	UG	$\tau_1 \dots \upsilon_1, \dots, \tau_n \dots \upsilon_n$	$P\tau_1 \dots \tau_n$ $P\upsilon_1 \dots \upsilon_n$	QED=
existential $\exists x \theta x$	PCh	NcP	$\tau_1 \dots \upsilon_1, \dots, \tau_n \dots \upsilon_n$	$P\tau_1 \dots \tau_n$ $\neg P\upsilon_1 \dots \upsilon_n$	Nc=
			Detachment rules (optional)		
			required resources	auxiliary	rule
			$\phi \rightarrow \psi$	ϕ	MPP
			$\phi \rightarrow \psi$	$\neg\psi$	MTT
			$\phi \vee \psi$	$\neg\phi$ or $\neg\psi$	MTP
			$\neg(\phi \wedge \psi)$	ϕ or ψ	MPT

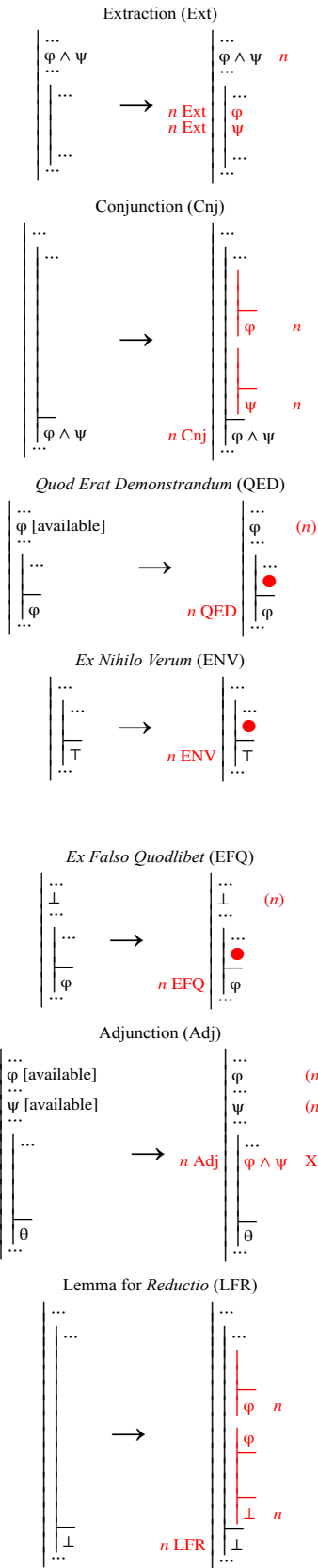
In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

Additional rules (not guaranteed to be progressive)

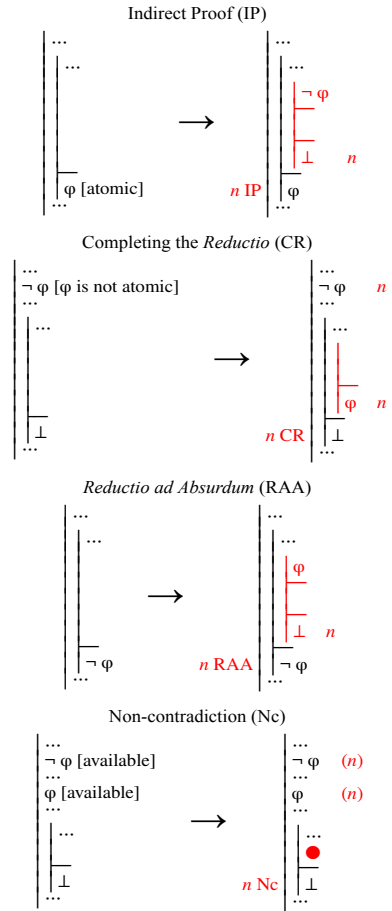
Attachment rules	Rule for lemmas
added resource rule $\frac{\phi \wedge \psi}{\phi} \text{ Adj}$ $\frac{\phi \rightarrow \psi}{\psi} \text{ Wk}$ $\frac{\phi \vee \psi}{\phi} \text{ Wk}$ $\frac{\neg(\phi \wedge \psi)}{\neg\phi} \text{ Wk}$ $\frac{\tau = \upsilon}{\tau} \text{ CE}$ $\frac{\theta \upsilon_1 \dots \upsilon_n}{\exists x \theta x} \text{ Cng}$	prerequisite rule the goal is \perp LFR

Diagrams

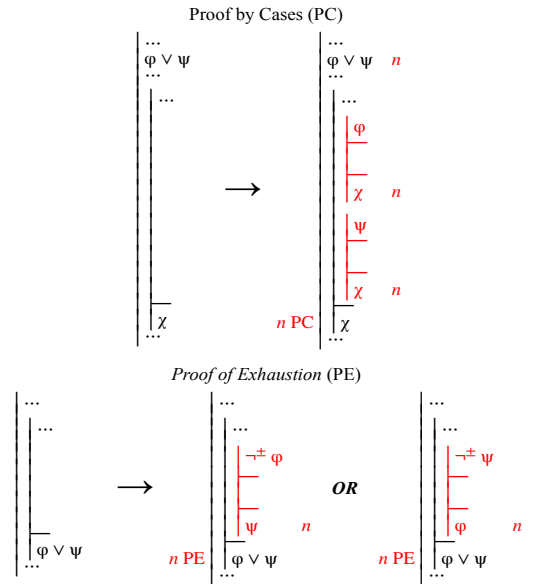
Rules from chapter 2



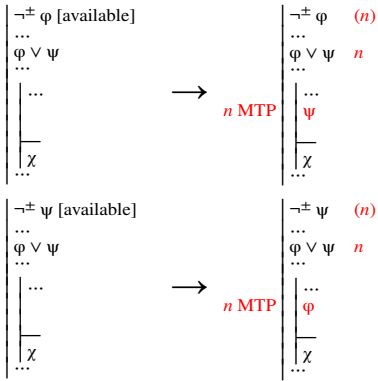
Rules from chapter 3



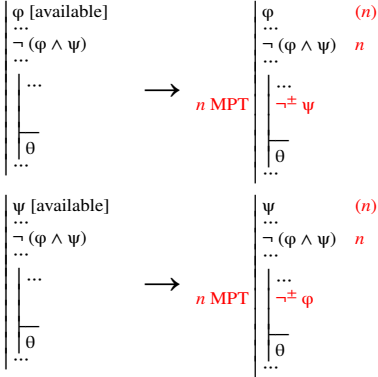
Rules from chapter 4



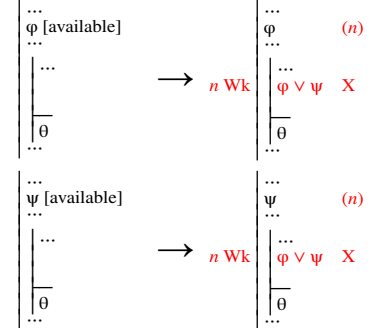
Modus Tollendo Ponens (MTP)



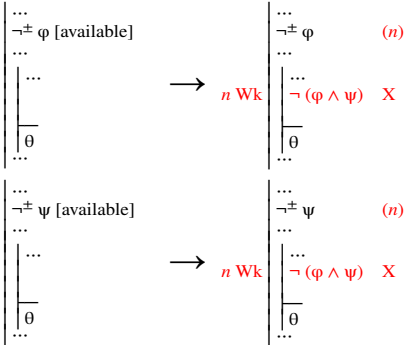
Modus Ponendo Tollens (MPT)



Weakening (Wk)

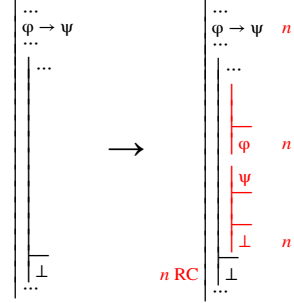


Weakening (Wk)

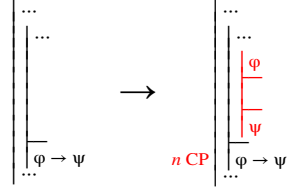


Rules from chapter 5

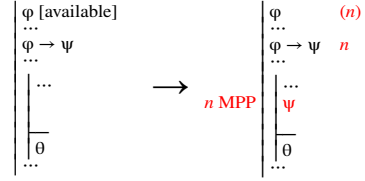
Rejecting a Conditional (RC)



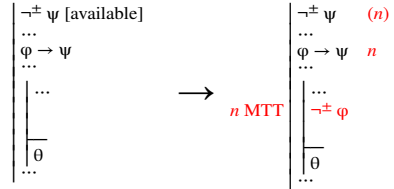
Conditional Proof (CP)



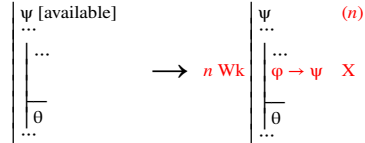
Modus Ponendo Ponens (MPP)



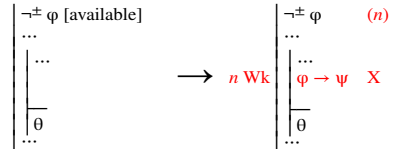
Modus Tollendo Tollens (MTT)



Weakening (Wk)

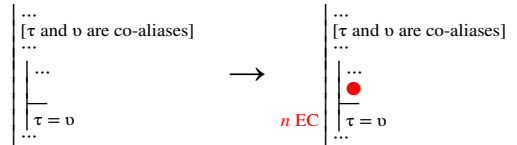


Weakening (Wk)

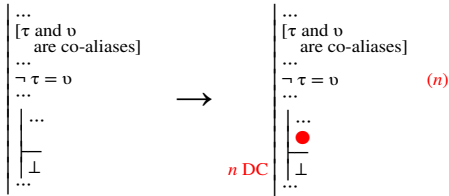


Rules from chapter 6

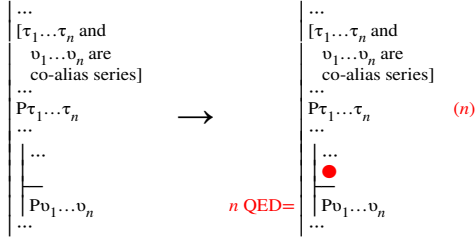
Equated Co-aliases (EC)



Distinguished Co-aliases (DC)

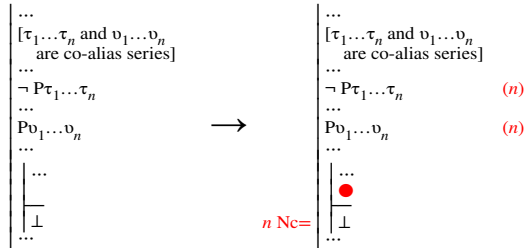


QED given equations (QED=)



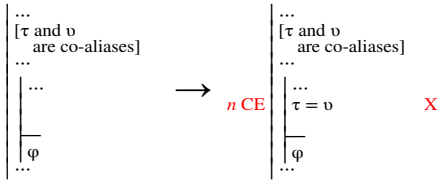
Note: Two series of terms are co-alias series when their corresponding members are co-aliases.

Non-contradiction given equations (Nc=)

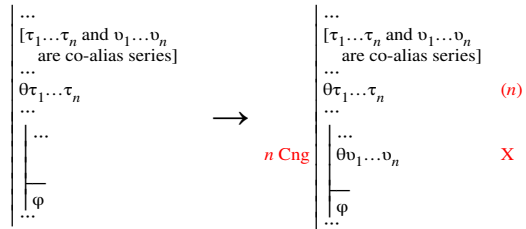


Note: Two series of terms are co-alias series when their corresponding members are co-aliases.

Co-alias Equation (CE)



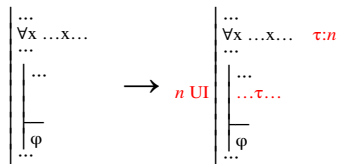
Congruence (Cng)



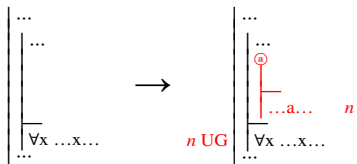
Note: θ can be an abstract, so θτ₁...τₙ and θv₁...vₙ are any formulas that differ only in the occurrence of terms and in which the corresponding terms are co-aliases.

Rules from chapter 7

Universal Instantiation (UI)

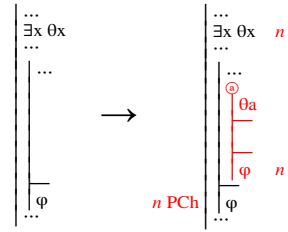


Universal Generalization (UG)

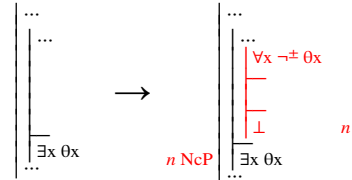


Rules from chapter 8

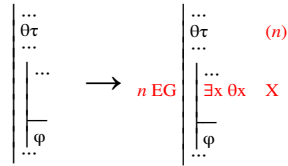
Proof by Choice (PCh)



Non-constructive Proof (NcP)



Existential Generalization (EG)



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