# 2. Conjunctions

## 2.1. And: adding content

### 2.1.0. Overview

In this chapter, we will study the logical properties of the English word and and certain related expressions. Along the way, we will encounter some general ways of approaching the study of logical properties that will serve us in later chapters, too.

For the next several chapters, the logical forms we consider will reflect ways that sentences are formed from other sentences; the operators which form such sentences are known as *connectives*.

2.1.1. A connective
We begin with *conjunction*, a connective that enables us to combine the content of a pair of sentences.

2.1.2. A truth function
The meaning of conjunction can be given by specifying the truth value of a conjunction in terms of the truth values of the sentences that were combined.

2.1.3. Conjunction in English
Although conjunction is most closely associated with the word and, there is a variety of ways of expressing it in English.

2.1.4. Limits on analysis
On the other hand, even the appearance of and is not a sure sign that a sentence may be analyzed as a conjunction.

2.1.5. Multiple conjunction
The operation of forming a sentence from sentences can be repeated. We will look at this sort of iteration in the case of conjunction.

2.1.6. Some sample analyses
We will then apply these ideas to analyze several examples.

2.1.7. Logical forms
And we will look in more general terms at the relation of logical forms to actual sentences.

2.1.8. Interpretations
Finally, we will introduce some ways of talking about the relation between abstract logical forms and the meanings of sentences.

### 2.1.1. A connective

We are interested in logical forms as a way of stating general laws of entailment. Let us begin by looking at cases of entailment that seem to involve the word and. Here is an example:

That bear is large and edgy ⊨ That bear is large

In attempting to understand any fact, it is useful to collect related facts. One way to search for related facts about entailment is to look for cases involving sentences similar in grammatical form to those above. If we follow this route, we run into entailments like this:

That car is cheap and reliable ⊨ That car is cheap

And we will eventually hit upon a general pattern like this:

a is P and Q ⊨ a is P.

Although we will soon move on to more general patterns than this, any pattern that abstracts from particular words makes the label "formal logic" appropriate.

If we look a little farther afield in our search for related facts, we also find examples like

It was hot and there was a storm before dark ⊨ It was hot,

which follows the pattern

$\varphi$ and $\psi \vDash \varphi$.

This pattern can be seen to operate also in examples of the first group if we paraphrase them, transforming

That bear is large and edgy,

for instance, into

That bear is large and that bear is edgy.

When we apply a pattern by first paraphrasing, as we have done here, we treat a sentence as having a form that is hidden by its surface appearance. Much of our analysis of logical form will involve this sort of transition.

Both of the patterns above give us general laws of inference. But, in the second more general pattern, it is especially clear that the word and plays a key role. If we look at what this role involves, we see that and marks a particular sort of *compound* sentence formed of *component* sentences, one that we will label a *conjunction*. So the word and is a sign for an operator that forms conjunctions. We will call an operator that forms compound sentences out of component sentences a *connective*, and we will refer to the connective we are considering here as *conjunction*, marking it with the sign $\wedge$ (one of whose names is *logical and*). (The use of the term conjunction for both the operation of conjoining, the operator that performs this operation, and the compound that results from it may seem confusing, but it follows a pattern that is used fairly often in English—as when the word distribution is used both for the act of distributing and for its result.) It will often be convenient to employ a further related term and refer to the components of a conjunction as its *conjuncts*.

Using these ideas, we can express our *analysis* of That bear is large and edgy as

That bear is large $\wedge$ that bear is edgy,

and we can express our principle of entailment as

$$\varphi \wedge \psi \vDash \varphi.$$

This symbolic notation can save space, but it is often convenient to use English to mark conjunction. When we do this, we will use the construction both … and … and write it (as done here) using a special type. So the principle above could be stated as

both $\varphi$ and $\psi \vDash \varphi$.

(The reason for using the particle both in addition to and will be discussed later.)

At this point, we have reached a stage like that reached by a physicist who recognizes pressure, temperature, and volume as physical quantities and has formulated a law relating them but who does not know why the law holds. That is, we have a generalization about entailment that we can apply in special cases, but we cannot say why this generalization is true. So let's go on to ask what it is about conjunction that makes this sort of entailment work.

We can find an answer by again scaring up some more facts. Notice, for example, that the entailment that got us started is matched by a second.

That bear is large and edgy $\vDash$ That bear is edgy.

Moreover, we can see not only that the sentence That bear is large and edgy entails each of the two sentences That bear is large and That bear is edgy but also that it is entailed in turn by the two taken together.

If we abbreviate the longer sentence by B and the two shorter sentences as L and E, respectively, we have collected the following facts:

$$B \vDash L$$
$$B \vDash E$$
$$L, E \vDash B$$

And checking other cases of conjunction would show us that these are instances of three general laws.

$$\varphi \wedge \psi \vDash \varphi$$
$$\varphi \wedge \psi \vDash \psi$$
$$\varphi, \psi \vDash \varphi \wedge \psi.$$

Or, using English to express the forms,

$$\text{both } \varphi \text{ and } \psi \vDash \varphi$$
$$\text{both } \varphi \text{ and } \psi \vDash \psi$$
$$\varphi, \psi \vDash \text{both } \varphi \text{ and } \psi.$$

So far, all we have done is to accumulate more general laws, but sometimes a larger number of facts is easier to understand because a pattern can begin to emerge.

### 2.1.2. A truth function

We can begin to provide a single account of the laws outined in 2.1.1 by recalling our definition of entailment. In positive form, it says that a set Γ entails a sentence φ if and only if φ is **T** in every possible world in which each member of Γ is **T**. Restating our three laws in these terms, we have

φ is **T** in every possible world in which φ ∧ ψ is **T**
ψ is **T** in every possible world in which φ ∧ ψ is **T**
φ ∧ ψ is **T** in every possible world in which both φ and ψ are **T**

The last of these says that φ ∧ ψ is true in a possible world if both φ and ψ are true in that world. While the first two taken together tell us that φ ∧ ψ is true in a possible world *only* if both φ and ψ are true. In short, φ ∧ ψ is true in a possible world if and only if both φ and ψ are true. In other words, the coverage of a conjunction is the shared coverage of its two components.

This means that the truth value of the compound φ ∧ ψ is determined by the truth values of the components φ and ψ, a fact we can express in the *truth table* below.

| φ | ψ | φ ∧ ψ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

This table shows the contribution of conjunction to the truth conditions of compound sentences formed using it, for it tells us how to determine the truth value of a conjunction φ ∧ ψ in any possible world once we know the truth values of the conjuncts φ and ψ. And the table also shows what lies behind the general laws of entailment that led us to it: it is because conjunction makes this sort of contribution to the meaning of sentences that those laws hold. A particular way of associating an output truth value with input truth values is a *truth function*, so we can say that conjunction expresses a truth function, and we can say the laws of entailment stated above reflect the character of the truth function that conjunction expresses.

It is worth pausing a moment to look at the way in which the proposition that is expressed by a conjunction is related to the propositions that are expressed by its components. We have already noted that its coverage is the shared coverage of its components, and that means its content is their cumulative content. That is, since a conjunction is false whenever either component is false, it rules out any possibility ruled out by either component; and, since the possibilities ruled out are the content of a sentence, this means that the effect of conjunction is to add up content.

For example, the sentence The number shown by the die is odd and less than 4 can be analyzed as the conjunction The number shown by the die is odd ∧ the number shown by the die is less than 4. The first component rules out possibilities where the die shows 2, 4, or 6 and the second rules out possibilities where it shows 4, 5, or 6. The conjunction rules out all these possibilities—that is, any possibility where the die shows 2, 4, 5, or 6. Looking at things in terms of the possibilities left open, in terms of coverage, the first component leaves open those where the die shows 1, 3, or 5 and the second leaves open those where it shows 1, 2, or 3. The conjunction leaves open a possibility when it is left open by both components, when it is part of the overlap in their coverage, so it leaves open those where the die shows 1 or 3.

This is shown pictorially in Figure 2.1.2-1 below.

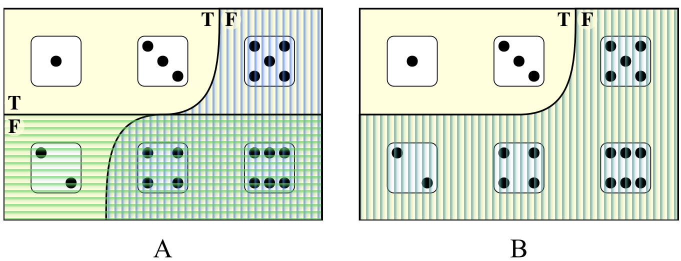


Fig. 2.1.2-1. Propositions expressed by two sentences (A) and their conjunction (B).

Here, each rectangle represents the space of all possible worlds. The die faces mark regions consisting of the possible worlds in which the die shows one or another number. In Figure 2.1.2-1A, the possibilities ruled out by the first component are at the bottom while those ruled out by the second component occupy the region at the right. The possibilities left open by the first component then form the region in the top half while those left open by the second are in the region at the left. Figure 2.1.2-1B shows the proposition expressed by the conjunction of these two sentences. The possibilities ruled out add up to form the shaded region; those left open are in the unhatched region at the top left where the ranges of possibilities left open by the original components overlap. These diagrams can be compared to the truth table for conjunction. The sort of worlds covered by first row of the table, worlds where both components are true, appear at the top left of the 2.1.2-1A; the other rows of the table correspond to the remaining three regions of the this diagram—those at the top right, the bottom left, and the bottom right, respectively.

Although the most fundamental approach to the deductive properties of the logical form will come through laws governing its role as the conclusion of an entailment or as one among its possibly many premises, specific characteristics of a logical form can often be highlighted most clear by its significance for relations between pairs of sentences, especially the positive relations of implication and equivalence. The following principles are some of the more important examples of this in the case of conjunction:

COMMUTATIVITY. *The order of conjuncts in a conjunction does not affect the content.* That is, $\varphi \land \psi \simeq \psi \land \varphi$.

ASSOCIATIVITY. *When a conjunction is a conjunct of a larger conjunction, the way components are grouped does not affect the content.* That is, $\varphi \land (\psi \land \chi) \simeq (\varphi \land \psi) \land \chi$.

IDEMPOTENCE. *Conjoining a sentence to itself does not change the content.* That is, $\varphi \land \varphi \simeq \varphi$.

COVARIANCE. *A conjunction implies the result of replacing a component with anything that component implies.* That is, if $\psi \vDash \chi$, then $\varphi \land \psi \vDash \varphi \land \chi$ and $\psi \land \varphi \vDash \chi \land \varphi$.

The names of these principles are terms used for analogous principles in other contexts. For example, you may have encountered the first two as names of principles for addition and multiplication since order and grouping do not matter for these operators. Conjunction shares the third property with numerical operators that produce the maximum or minimum of a pair of numbers, and this is not surprising since, if we think of truth values as being ordered so that falsity comes below truth, the truth value of a conjunction is just the minimum of the truth values of its components.

The last property, covariance, says roughly that the content of a conjunction varies in the same direction as the content of its components. An analogous property holds for addition and the maximum and minimum operators (e.g., if $y \leq z$ then $\min(x, y) \leq \min(x, z)$) but it doesn't hold for multiplication when negative numbers are considered (e.g., $-2 \times 3 > -2 \times 4$ even though $3 \leq 4$). We cannot say that an increase or decrease in the content of one component will produce an actual increase or decrease, respectively, in the conjunction since information added or lost in a change to one component may be provided in any case by the other component. For example, although The sign had red letters on a blue background says more than does The sign had red letters, the conjunction The sign had red letters on a blue background, and the background was light blue is equivalent to The sign had red letters, and the background was light blue. (This is analogous to the fact that, $\min(2, 3) = \min(2, 4)$ even though $3 < 4$.) What can be said is that, if the content of one component of conjunction increases, the content of the conjunction must increase if it changes at all.

One consequence of covariance is the following principle:

COMPOSITIONALITY. *Conjunctions are equivalent if their corresponding components are equivalent.* That is, if $\varphi \simeq \varphi'$ and $\psi \simeq \psi'$, then $\varphi \wedge \psi \simeq \varphi' \wedge \psi'$.

Although this follows from covariance (since equivalent components imply each other), it can hold when covariance does not. And compositionality is so fundamental that, if conjunction did not satisfy it, we might hesitate even to count it as a logical form. Since sentences are logically equivalent when they express the same proposition, this principle says that conjunctions cannot express different propositions unless there is some difference in the propositions expressed by their components. Understanding the meanings of sentences to be the propositions expressed, the principle of compositionality tell us that the meaning of a conjunction is composed out of the meanings of its components in the particular way we label "conjunction."

### 2.1.3. Conjunction in English

Conjunction is most often marked by the word and, but there are English sentences without this word that also may be analyzed as conjunctions. First of all, there are quite a number of expressions—such as also, in addition, and moreover—that serve as stylistic variants of and. But conjunctions also may employ another group of words that are not simple stylistic variants of and. The principal example is the word but.

This may be a surprise. Although a sharp ear might detect a slight difference in meaning between and and moreover, the difference between and and but is unmistakable. Consider, for example, the following two sentences, which differ only in the use of these two words:

> Adams spoke forcefully to the committee, and they agreed to the expenditure
>
> Adams spoke forcefully to the committee, but they agreed to the expenditure.

These sentences would be used under different circumstances, and it may seem odd to count them as logically equivalent, which is what we must do if we are to analyze both as conjunctions of the same two components.

This is the first of several points at which we must recall the distinctions between truth and appropriateness and between implication and implicature. As was noted in 1.3.4, our concern is with only the first concept in each pair and thus with only certain aspects of meaning. Specifically, we count two sentences as equivalent if they have the same truth conditions. Any differences between their meanings that have no effect on their truth and falsity are irrelevant for our purposes.

So we must look more closely at the nature of the difference in meaning between and and but. It is clear that the second sentence above carries a suggestion of contrast between the two components—perhaps Adams spoke against the expenditure or the committee usually rejected Adams's advice—and it is also clear that the suggestion of contrast is absent in the first sentence. Now, suppose that the second sentence was used in a context where the suggested contrast is not present—perhaps the expenditure was approved because Adams spoke for it. The assertion of the second sentence would then be inappropriate, but would it be false?

Let us use the test of a yes-no question. Imagine that you attended a meeting were Adams persuaded a committee to agree to a certain expenditure and that later someone who had heard rumors of the proceedings asked you the question Is it true that Adams spoke forcefully to the committee, but they agreed to the expenditure?. How would you reply? This is something you must decide for yourself; but, for my own part, I would say something like, “Yes, but he spoke for the expenditure, not against it.” That is, I would give a yes-but answer, reacting to the sentence whose truth was asked about as one whose assertion would be true but would be misleading. This suggests that the difference between but and and is a difference in conditions of appropriateness rather than conditions of truth, and it is for this reason that I will suggest we analyze sentences formed using but and other similar words—such as however, though, and nonetheless—as conjunctions. These words are not *merely* signs of conjunction; but their differences from and lie outside their effect on truth conditions.

There are cases of other sorts where analysis by conjunction is legitimate though not obvious. Sometimes, for example, there is no word at all marking the conjunction. The operation of conjoining produces a compound sentence that commits us to the truth of both its components, and there are linguistic devices other than the use of particular words that enable us to roll two claims up into one in this way. For example, the sentence It was a hot, windy day is equivalent to It was a hot and windy day and can be analyzed as the conjunction

It was a hot day ∧ it was a windy day.

An analysis of a sentence might even break a modifier off from the expression it modifies. One common case of this is provided by adjectives used *attributively*—i.e., applied directly to the noun they modify. For example, we may treat Sam's car is a green Chevy as if it were Sam's car is a Chevy, and it's green. But it is important to note that, for reasons discussed in the next section, these analyses work only because the adjectives appear in a predicate nominative employing the indefinite article—i.e., in the form represented by

X is a … Y

or represented by a similar form with a different tense. However, this is a very common pattern so there will be many occasions to apply this sort of analysis.

Another rather specific but important case of breaking off modifiers concerns relative clauses. There are really two cases here. The first is non-restrictive relative clauses—that is, ones marked off by commas. These can usually be analyzed as conjunctions. For example, Ann, who you met yesterday, called this morning can be understood as a conjunction of You met Ann yesterday and Ann called this morning.

The second sort of case is a restrictive relative clause—one not marked off by commas—appearing as part of a predicate nominative using the indefinite article. The grammatical pattern in this case is

X is a Y that …

or a similar pattern using a different tense or another relative pronoun (such as who or which). A sentence like this can be analyzed as a conjunction whose conjuncts are, first, X is a Y and, second, the result of putting X in the expression marked by … at the point governed by the relative pronoun. For example, Sam's car is a Chevy that's green could be analyzed as the conjunction of Sam's car is a Chevy and Sam's car is green; that is, it can be analyzed in the same way as Sam's car is a green Chevy. But relative clauses of this sort can be used to express many sorts of modification other than the simple application of adjectives. One example is The speaker was a writer who Sam admired, which can be analyzed as the conjunction of The speaker was a writer and Sam admired the speaker; here the second conjunct has the subject of the original sentence as its direct object rather than its subject.

### 2.1.4. Limits on analysis

Although the presence of and, or another word used to mark conjunction, is a good sign that conjunction will be involved in a full analysis of a sentence, it does not mean that the sentence as a whole can be analyzed as a conjunction. For that to be possible, we must be able to identify components that are independent sentences, and there are a number of things that can keep us from doing that.

One thing that can interfere is the occurrence of indefinite articles and similar expressions. Consider the sentence A friend of Ann lives in Singapore and works in London. The claim it makes may well be true, but its truth would be at least mildly surprising. However, there would be much less surprise at the truth of a sentence analyzed as A friend of Ann lives in Singapore ∧ a friend of Ann works in London since there is no longer any implication that the same person does both. Of course, we could paraphrase the original sentence (a bit awkwardly) as A friend of Ann lives in Singapore and that person works in London, but that is of no help in analyzing it since the second clause relies on the first clause for the reference of the phrase that person and thus does not function as an independent sentence. Indeed, in spite of the occurrence of the word and, there is no way to analyze this sentence as a conjunction in which the references to Singapore and London appear in different components; its analysis must await our treatment of expressions involving the indefinite article in chapters 7 and 8.

The indefinite article is one of a group of expressions also including some, every, and no that we will later study as *quantifier words*. Their presence will often preclude analysis of a sentence as a compound formed by a connective even though a word that ordinarily indicates that compound is present. Analysis as a compound formed by the connective is sometimes possible in such cases, but you should be wary if you find yourself being led to repeat a quantifier word when dividing the sentence into two components (as we would do by repeating a friend of Ann in the example above).

Similar problems can arise in other cases where we might expect to find a conjunction, as with attributive adjectives and relative clauses. For example, Tom forecast a hot and windy day next week is not equivalent to Tom forecast a hot day next week ∧ Tom forecast a windy day next week since the latter does not imply that the two forecasts are for the same day. This is the reason that 2.1.3 recommended such analyses only for predicate nominatives. In cases of that sort, the implication that two adjectives are being applied to the same thing is insured by other aspects of the sentence, but you still need to be wary of duplicating other quantifier phrases—in, for example, the subject of the sentence—when you make the analysis. And this is true even for compound predicate adjectives: Sam's car was cheap and reliable is equivalent to Sam's car was cheap ∧ Sam's car was reliable but One model is cheap and reliable is not equivalent to One model is cheap ∧ one model is reliable.

Even when quantifier words are not involved, analyses by conjunction cannot always be used to break modifiers off from the words they modify. For example, it would be wrong to analyze Tristram is a large flea as Tristram is a flea ∧ Tristram is large because a sentence with this analysis entails that at least one flea is to be found among the large things of the world. The problem in this case is that an adjective modifying a noun has its meaning determined in part by the noun it is applied to; large indicates a different range of sizes when it is applied to fleas than when it is applied to elephants. This is an example of a phenomenon discussed in 1.3.6: vague terms have their meaning determined in part by their context of use. A noun can contribute to the context in which an adjective is used when the adjective is applied to the noun directly and also when the adjective follows the noun in a stream of discourse. This means that it also would be wrong to analyze Tristram is a flea and Tristram is large as Tristram is a flea ∧ Tristram is large, for the adjective large acquires part of its meaning from the noun flea in the English sentence. (But the way a noun affects the meaning of a vague adjective is not simple. Although the sentence No fleas are large speaks about fleas, the range of sizes indicated by large in this sentence is different from the range indicated by its use in Tristram is a flea and Tristram is large.)

But why does the same thing not happen with the conjunction Tristram is a flea ∧ Tristram is large? Although the symbol ∧ is closely related to the English conjunction and, it is not a simple abbreviation; and we do not assume that their contribution to the meaning of a sentence is exactly the same. The symbol ∧ (and the construction both … and … that we use as an alternative notation for it) are signs for the operator conjunction. The conjunction of two sentences is a sentence that, in any context, has truth conditions that are related to those of its two components in the way shown by the table we considered earlier. And the stipulation that this is so *in any context* is a crucial one here; in particular, it need not be part of that context that either component has been asserted. So in the conjunction, we cannot assume that the meaning of the second component Tristram is large will be influenced by the meaning of the first component. In certain sorts of context, Tristram is large will have the same meaning as Tristram is large for a flea. But it is only in such contexts that Tristram is a flea ∧ Tristram is large has the same truth conditions as Tristram is a flea and Tristram is large, and our analyses should not depend on equivalences that hold only for certain contexts.

This indicates a further difference between our model of the operation of language and the way things work in English. Everything that is said in English has the potential of affecting the context of what follows it and, to a more limited extent, what precedes it. But when we analyze sentences, we treat their components as independent and as each understood in the same context. Our excuse for this limitation of our model is the same as that for many others: a model that was more accurate in this respect would require significant complications—and complications that no one yet understands very well.

Of course, we can analyze Tristram is a large flea as a conjunction after all if we modify the second component to remove its dependence on the context established by the assertion of the first. One way of doing that was suggested in passing above: we may use the conjunction Tristram is a flea ∧ Tristram is large for a flea. Here we have modified the second component to replace the implicit effect of the context with a more explicit indication of the range of sizes in question. Though generalizations about such matters are risky, something like this device can be applied in many cases where adjectives acquire part of their meaning from the surrounding context.

There are still other factors that can prevent breaking attributive adjectives off from the nouns they modify. We could be guilty of slander if we were to analyze Alfred is an alleged murderer as Alfred is a murderer ∧ Alfred is alleged to be a murderer. The difference between this and the example above is that the attributive adjective alleged modifies the meaning of a noun in a different way from an adjective like large. Adjectives like large narrow down the class of things marked out by the noun by adding a further property; in contrast, alleged shifts the membership of this class by adding as well as dropping members. The class of alleged murderers is not included in the class of murderers in the way the class of large fleas is included in the class of fleas. As a result, no analysis as a conjunction is possible.

While the issues of contextual dependence can also affect our ability to break relative clauses off from the nouns they modify, this latter problem does not occur for them. If we say Alfred is a murderer who is alleged to be one we already imply that Alfred is a murderer, and analysis as a conjunction is possible. This means that one initial test for cases where we may break an attributive adjective off from the noun it modifies is to see if restatement using a relative clause changes the meaning. While That's an unknown Rembrandt is equivalent to That's a Rembrandt that is unknown and can be analyzed as a conjunction, That's a fake Rembrandt is not equivalent to That's a Rembrandt that is fake and cannot be analyzed as a conjunction.

But, in the end, the test that an analysis must pass is that the conjunction we use to represent a sentence really has the same truth conditions. Since the truth table for conjunction is directly tied to the laws of entailment discussed in 2.1.1, one way to apply this test is to check whether the original sentence really entails both components of the analysis (when these are considered as independent sentences) and whether they, taken together, entail it. And we have used this test in the discussion of examples above; for example, because Alfred is an alleged murderer does not entail Alfred is a murderer, we cannot analyze the premise as conjunction with the conclusion as one of its conjuncts. Due to the problems associated with the contextual dependence of meaning, we must be careful, when applying this test, not to fill out the meanings of terms in one of the sentences we are considering by a surreptitious reference to another sentence.

### 2.1.5. Multiple conjunction

Although conjunction can compound sentences only two at a time, the word and in English can be used with any series of more than two items. To analyze the *serial conjunction* He went to Gary, South Bend, and Fort Wayne, we need to regard the sentence as the result of two uses of conjunction, first to join two of the components and then to tack on a third. There are two ways of doing this and, although the associativity of conjunction noted in 2.1.2 tells that they have the same content, they arrive at their common meaning in different ways.

We can represent this difference in our symbolic notation by using parentheses:

He went to Gary ∧ (he went to South Bend ∧ he went to Fort Wayne)
(He went to Gary ∧ he went to South Bend ) ∧ he went to Fort Wayne.

There are a number of ways of describing the difference displayed here. We can say, first, that in each case a different one of the two uses of conjunction is the *main* connective or the one at the *top level*. The main or top-level connective is the operator that would be used last in forming the sentence, and it marks the place the sentence would be broken first when it is decomposed. In the first sentence above, it is the first use of conjunction that is the main connective or the one at top level while, in the second sentence, it is the second use.

Another way of describing the difference between the two analyses is to speak of the *scope* of a connective, the part of the whole sentence that is made up of the connective and the components it applies to. Thus the scope of the first ∧ in the first of the sentences above is the whole sentence while the scope of the second ∧ is the portion in parentheses. This situation is reversed in the second sentence; there, the scope of the first ∧ is limited by the parentheses and is included in the scope of the second ∧.

He went to Gary ∧ (he went to South Bend ∧ he went to Fort Wayne)

(He went to Gary ∧ he went to South Bend) ∧ he went to Fort Wayne

So we say that the two examples differ in the *relative scope* of the two uses of conjunction. In one, the first use has *wider* scope; in the other, the second has wider scope.

These two ideas are depicted together in Figure 2.1.5-1. The main connective of each analysis appears quite literally at the top level, and the scope of each connective is the portion of the analysis that branches out from under it.

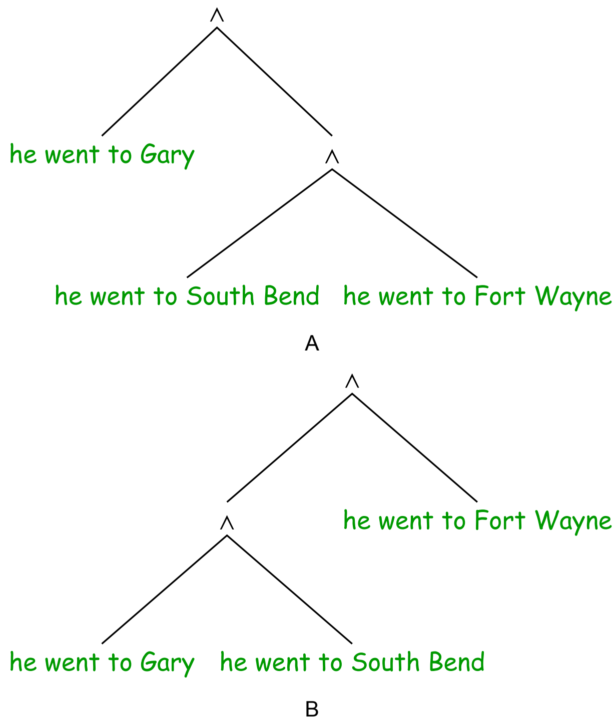


Fig. 2.1.5-1. Two analyses of a serial conjunction.

The parentheses of our symbolic notation for these analyses can be seen as a way of representing this sort of structure without resorting to two dimensions.

Although such scope distinctions make no difference in the truth conditions of English sentences, they can be marked syntactically—as in

He went to Gary and also South Bend and Fort Wayne
He went to Gary and South Bend and also Fort Wayne.

Here, also is used to emphasize the break made by one of the occurrences of and over the other. Use of punctuation is another way to emphasize one of the two ands and raise it to the top level—as in

He went to Gary—and South Bend and Fort Wayne
He went to Gary and South Bend—and Fort Wayne.

In the absence of devices like the use of also, syntactic grouping, or punctuation, the normal order of reading probably would lead us to interpret the second and as the last one used in forming the sentence—that is, as the main operator.

Still another common way of making such distinctions is to exploit the power of and to conjoin words or phrases as well as complete clauses. For example, compare

He went to Gary and to South Bend and Fort Wayne

He went to Gary and South Bend and to Fort Wayne

(in which the conjoined words and phrases are underlined). In each case, one and conjoins prepositional phrases and the other conjoins nouns to form the object of one of these phrases.

A final way of representing scope distinctions in English is one we have adapted to represent conjunction using the expression both … and …. All things being equal, we will interpret the second of two ands as having the wider scope, but this presumption can be defeated by adding the word both to get, for example

He went to Gary and both South Bend and Fort Wayne.

This sentence has the form $\varphi \wedge (\psi \wedge \chi)$; and, in general, the word both has roughly the same effect as a left parenthesis in our symbolic notation. Indeed, when we represent the forms we have been considering using English notation we get this:

both he went to Gary and both he went to South Bend and he went to Fort Wayne
both both he went to Gary and he went to South Bend and he went to Fort Wayne.

The word both appears here just where a left parenthesis would in a symbolic analysis if we were to add parentheses surrounding the whole sentence.

(He went to Gary ∧ (he went to South Bend ∧ he went to Fort Wayne))
((He went to Gary ∧ he went to South Bend ) ∧ he went to Fort Wayne)

Of course, our English notation uses both in many cases where both would not appear in ordinary English and even where a left parenthesis would not ordinarily appear in our symbolic notation. This is because the English notation is designed to make the scope of connectives unambiguous in cases where ordinary English is ambiguous. And without anything to serve as a corresponding right parenthesis, the word both may be needed in some places where a left parenthesis is not.

For example, suppose we attempt to express the form $(\varphi \wedge \psi) \wedge \chi$, using and in place of $\wedge$ and both in place of the left parenthesis. We would get

both $\varphi$ and $\psi$ and $\chi$.

If we take both to mark the left end of the scope of the first conjunction (as the left parenthesis does in the symbolic expression), we are still left with no indication of the right end of its scope: is the second component only $\psi$, or is it the whole of $\psi$ and $\chi$? If we supply a both for every and, we can write $(\varphi \wedge \psi) \wedge \chi$ as

both both $\varphi$ and $\psi$ and $\chi$.

This is hardly elegant prose, but it does make the grouping definite; finding a second both immediately following the first, we know the first component of the main conjunction is itself a conjunction. Of course, we could also mark scope using parentheses. It may seem odd to do this if we are using English notation, too; but it is possible to mix the two forms, and it can sometimes be helpful to indicate a logical form by combining the word and with grouping marked by parentheses.

Although scope distinctions can be made in English in these ways, the English and is often applied to a series of items that are all on the same level. It would be possible to treat conjunction as an operator that was similar to the English and in this respect, but it would cost us the trouble of more complex accounts of the properties of conjunction without yielding much greater insight. Still, we can (and often will) mimic the way addition and multiplication are usually treated in algebra and drop parentheses when they make no difference in the value of an expression. This introduces no real complications but it has limitations. Since our principles concerning conjunction will be stated only for 2-component conjunctions, we can apply them to a *run-on conjunction* like $\varphi \wedge \psi \wedge \chi$—or, in English notation, $\varphi$ and $\psi$ and $\chi$—only after we have chosen one of the two conjunction symbols as marking the main operator. And, although we could regard either the first or last of the three components as a component of the top-level conjunction, the middle one, $\psi$, always ends up as a component of the lower level conjunction, so we really have not put the three components on the same level.

### 2.1.6. Some sample analyses

Here are a few example analyses written out in full as models for the exercises to this section. In each case a few comments follow the actual analysis.

Roses are red and violets are blue
Roses are red ∧ violets are blue

R ∧ B
both R and B

R: roses are red; B: violets are blue

As a last step here, unanalyzed components have been abbreviated with capital letters in order to highlight logical forms. The final form is stated both symbolically and using English notation, something that will be done also in the examples to follow.

The next example is worked out in two steps, first analyzing the whole sentence as a conjunction and then analyzing one of its components.

It's cool even though it's bright and sunny
It's cool ∧ it's bright and sunny
It's cool ∧ (it's bright ∧ it's sunny)

C ∧ (B ∧ S)
both C and both B and S

C: it's cool; B: it's bright; S: it's sunny

The parentheses in the final result correspond to the grouping of bright and sunny together in the predicate of the second clause of the original sentence.

In the following example, it would not be wrong to use parentheses (or grouping with both), but that would be an artifact of our analysis and correspond to nothing in the English.

He was cool, calm, and collected
He was cool ∧ he was calm ∧ he was collected

C ∧ M ∧ T
C and M and T

C: he was cool; M: he was calm; T: he was collected

Accordingly, the analysis uses a run-on conjunction in the symbolic version, and use of both is similarly suppressed in the English statement of the form. If grouping were used here, either conjunction might be assigned widest scope.

Finally, there can be cases where some grouping reflects the structure of the

English, but other grouping does not.

It is a two-story brick building with a slate roof
It is a two-story brick building ∧ it has a slate roof
(it is a building ∧ it is made of brick ∧ it has two stories) ∧ it has a slate roof

$$(B \land R \land T) \land S$$
(B and R and T) and S

B: it is a building; R: it is made of brick; S: it has a slate roof; T: it has two stories

No grouping is used within the first three components because it is not obvious that any is imposed by the phrase two-story brick building. The English notation employs parentheses because there is no good way of indicating the combination of run-on conjunction with ordinary conjunction using both.

As in the last example, there would be nothing wrong with imposing a grouping here. If we were to group the first three components to the left, we would end up with the following in symbols and English:

$$((B \land R) \land T) \land S$$
both both both B and R and T and S

In the English notation, each of the boths tells us that a certain component is a conjunction—first the whole sentence, then its first component, and finally the first component of this component—and this settles the scope of the ands that follow.

The value of English notation does not lie in the possibility of making such a calculation but rather in our ability to understand the significance both automatically; however, that ability is limited to fairly simple forms, and a row of three boths is hard to follow without reflection. (To cite a standard example of a similar limitation in the case of a different sort of grouping, it is just possible to understand Bears bears fight fight to say what is said by Bears that bears fight themselves fight—i.e., so that the first bears is modified by a relative clause bears fight and is the subject of the second fight; but it is virtually impossible to understand Bears bears bears fight fight fight as anything other than a cheer, even though it is grammatically possible for it to say something that might be expressed by Bears which are fought by bears that bears fight themselves fight.)

### 2.1.7. Logical forms

We will conclude this first look at analysis by considering its results in more general terms. The aim of analysis is to uncover logical form. While it is natural to speak of the result of an analysis as *the* logical form of the sentence that was analyzed, a sentence will usually have many logical forms of differing complexity. Many of these may be displayed as we carry out an analysis step by step. Consider, for example, the following analysis of a fairly complex sentence:

He went to Gary, South Bend, and Fort Wayne, leaving at dawn and returning after dark

He went to Gary, South Bend, and Fort Wayne ∧ he left at dawn and returned after dark

(he went to Gary and South Bend ∧ he went to Fort Wayne) ∧ he left at dawn and returned after dark

((he went to Gary ∧ he went to South Bend) ∧ he went to Fort Wayne) ∧ he left at dawn and returned after dark

((he went to Gary ∧ he went to South Bend) ∧ he went to Fort Wayne) ∧ (he left at dawn ∧ he returned after dark)

$$((G \land S) \land F) \land (L \land R)$$

both both both G and S and F and both L and R

F: he went to Fort Wayne; G: he went to Gary; L: he left at dawn;
R: he returned after dark; S: he went to South Bend

The first line exhibits the sentence without further analysis, the second shows it as a conjunction, the third as a conjunction whose first component is a conjunction, and so on. (The first component might have been analyzed as a run-on conjunction; but, for the purposes of this example, we need a fully specified structure.)

Each line ascribes a form to the sentence, and if we ignore the identity of unanalyzed components, this is a form that the sentence shares with many other sentences. These abstract forms are indicated below (in the order in which they appear in the analysis) with symbolic notation on the left and a description of the form on the right:

| | |
|---|---|
| $\varphi$ | sentence |
| $\psi \wedge \chi$ | conjunction |
| $(\zeta \wedge \xi) \wedge \chi$ | conjunction of (i) a conjunction and (ii) a sentence |
| $((\mu \wedge \nu) \wedge \xi) \wedge \chi$ | conjunction of (i) a conjunction whose first component is a conjunction and (ii) a sentence |
| $((\mu \wedge \nu) \wedge \xi) \wedge (\theta \wedge \upsilon)$ | conjunction of (i) a conjunction whose first component is a conjunction and (ii) a conjunction |

The sentence has still further forms that might have appeared in the course of our analysis if we had reached the final result in a different way. One example is $\psi \wedge (\theta \wedge \upsilon)$, a conjunction of (i) a sentence and (ii) a conjunction.

It is important to recognize all the different forms a sentence has, even those that correspond to very partial analyses of it. Each represents a class of sentences that may share important logical properties with the sentence we are focusing on. For example, the sentence above will share some of its logical properties with all sentences, others with all conjunctions, still others with conjunctions whose first components are conjunctions, and so on.

We will apply the term *component* to any sentence that appears on any level of analysis of a given sentence. In particular, a sentence is a component of itself. We will distinguish those components of a compound to which the main connective applies as the *immediate* components of the compound, and we will refer to those that appear unanalyzed at the last stage of an analysis as the *ultimate* components (on that analysis). We will often refer to the ultimate components of a sentence also as *unanalyzed*. In the example above, the immediate components of the initial sentence are the two sentences divided at the second line of the analysis; and the ultimate components are those abbreviated with capitals at the end. Although, in principle, both roman capital letters and the lower case Greek letters may stand for any sentences, in practice, we will reserve capital letters for sentences we do not analyze further. Such sentences are ultimate components of themselves and of any larger compounds in which they appear.

### 2.1.8. Interpretations

In passing from a sentence to any of its logical forms, we abstract from the specific components that we replace by variables. In general, we also abstract from the proposition expressed by the sentence and from its truth value. Except in special cases, such as forms that are shared only by tautologies, a logical form does not express a proposition or have a truth value, but we may introduce such semantic features by *interpreting* the form.

We will consider two sorts of interpretation, an *extensional interpretation*, that provides a truth value only, and an *intensional interpretation*, which provides the proposition expressed and thus a truth value not only for the actual world but for every possible world. These two sorts of interpretation will be used for different purposes, so it will usually be clear from the context which sort is relevant; and, when this is clear, we will use the term *interpretation* without qualification.

The term *intensional* (spelled with an *s*) and the term *extensional* derive from a traditional distinction between, on the one hand, the means by which a term picks out a class of objects and, on the other, the class of objects it picks out. Terms that pick out the same class of objects in different ways have the same *extension* but different *intensions*. For example, if the population of Crawfordsville is 14287, the terms city with a population greater than 14287 and city more populous than Crawfordsville have the same extension but different intensions. One way to see that the two terms have different intensions is to notice that they would pick out different classes of cities if the population of Crawfordsville were not 14287.

During the past century, the concepts of intension and extension have been extended to terms that pick out single objects rather than classes of objects, so we can say that the definite descriptions the author of *Poor Richard's Almanack* and the inventor of the lightning rod both have Benjamin Franklin as their extension though they differ in their intensions.

The distinction between the object a term refers to and the way the term refers to this object is sufficiently analogous to the distinction between the truth value of a sentence and the proposition it expresses that the concepts of intension and extension are now also applied to sentences. So Indianapolis is the capital of Indiana and Sringfield is the capital of Illinois could be said to have the same extension (i.e., the value **T**) but to have different intensions. In general, the extension of sentence is the sentence's truth value while the intension is the proposition that the sentence expresses.

Since the only general way we have to specify propositions is by using sentences that express them, intensional interpretations will be specified by assigning sentences to variables. (This assumes were are working with a fixed context of use, so sentences express propositions.) This assignment is the exact inverse of the process of abbreviating ultimate components by capital letters, and we will use the same notation for the association of letters and sentences in both. For example, we can give an intensional interpretation of the form (A ∧ B) ∧ C by making the following assignment of sentences to the variables that mark its ultimate components.

A: I got it apart;
B: I don't know how I got it apart;
C: I couldn't get it together again

Since the sentences assigned to variables serve only to specify propositions, we will not be concerned about their logical forms; they may be as simple or complex as we wish.

Especially in later chapters, the proposition assigned to a compound sentence by an intensional interpretation may not be apparent until we find an idiomatic English sentence that expresses the same proposition. This can be done by a step-by-step process of *synthesizing* English that reverses the process of analysis. For the example above, this might proceed as follows:

(A ∧ B) ∧ C

(I got it apart ∧ I don't know how I got it apart) ∧ I couldn't get it together again

I got it apart but I don't know how ∧ I couldn't get it together again

I got it apart but I don't know how, and I couldn't get it together again

Of course, other wording is possible here, and the process of synthesizing English will rarely have a unique correct result.

Extensional interpretations are easier to manage and will often provide all the information we need. We will adapt the tabular notation used for truth tables.

| A | B | C | (A ∧ C) ∧ (B ∧ C) |
|---|---|---|---|
| T | F | T | T Ⓕ F |

Variables are listed at the left with the assigned value under each of them. The whole form we are interested in is displayed to their right. The values of the larger components may be calculated by using the truth table for conjunction just as a multiplication table may be used to calculate the numerical value of a product: we find the values of the smallest components first and use these to calculate the values of larger components. The truth value calculated for each compound component is displayed below the main connective of that component. The value for the sentence as a whole is shown circled. Our interest will generally be only in this final value, but examples in this text will usually also display the intermediate values in order to show how the final value was reached.

### 2.1.s. Summary

1 The prime role of the logical word and is to mark the use of a connective, called conjunction, that serves to form a compound sentence (also called a conjunction) from component sentences that may be referred to as its conjuncts. The process of interpreting a sentence as a conjunction is analysis. We use the sign $\wedge$ (logical and) as symbolic notation for the operator conjunction, marking the scope of a conjunction by parentheses. Alternatively, we can write a conjunction $\varphi \wedge \psi$ as both $\varphi$ and $\psi$, where both plays the role of a left parenthesis. The two forms can be mixed using and to mark conjunction and parentheses to mark scope. We will use capital letters to stand for unanalyzed components as we use lower case Greek to stand for any sentences, analyzed or not.

2 The effect of conjunction on the truth conditions of the compounds formed using it may be described in a truth table showing the compound to be true if and only if both components are true. The truth table specifies a truth function, so conjunction can be said to have a truth function as its meaning. Some of the properties conjunction has in virtue of its meaning have standard names. It is commutative, associative, and idempotent (i.e., the order, grouping, and number of conjuncts does not affect the content of a sentence formed using conjunction, perhaps repeatedly); and it is covariant (adding or reducing the content of a component makes the content of the conjunction vary in an analogous way).

| $\varphi$ | $\psi$ | $\varphi \wedge \psi$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

3 Conjunction is marked in English by stylistic variants of and as well as by but and similar words. Conjunction also can appear without explicit indication, particularly through the use of modifiers like attributive adjectives and relative clauses.

4 Care is needed to be sure that such modifications can be captured by conjunction and to identify components that make independent contributions to the compound. The presence of quantifier words can preclude analysis as a conjunction even when the word and is present.

5 Since conjunction is used to combine only two components, uses of conjunction to combine more than two in a multiple conjunction will involve two or more connectives of differing scope, the one with widest scope counting as the main connective of the sentence. Such differences in scope can be marked in several ways in English but such markings may be absent in a serial conjunction. Some of the effect of serial conjunction without

scope distinctions can be achieved by run-on conjunctions, such as $\varphi \wedge \psi \wedge \chi$, which suppress parentheses.

6 In all but the simplest cases, the analysis of conjunctions will find components that are themselves conjunctions. The result of an analysis will exhibit this structure using symbolic and English notation. Although it is never wrong to mark the scope of conjunction within serial conjunctions, the resulting differences in the scopes of connectives are more significant in some cases than in others.

7 The analysis of the logical form of a sentence can occur in stages in which we identify the immediate components of a compound, any immediate components of these, and so on. The last components arrived at are the ultimate components of the analysis; the full class of components includes them as well as all other sentences that could appear in the course of analysis (including the analyzed sentence itself). A sentence will usually have many logical forms representing different partial analyses of it.

8 We can specify a proposition or a truth value for a logical form by means of an intensional or extensional interpretation, assigning truth values or sentences, respectively, to its ultimate components. A sentence expressing the proposition provided by an intensional interpretation can be found by carrying out a process of synthesis that reverses the process of analysis. The truth value provided by an extensional interpretation can be found by calculation using the truth table for conjunction. The tabular notation used to write the truth table of conjunction may be used also to describe extensional interpretations and the values that they give to compound forms.

## 2.1.x. Exercises

**1.** Analyze each of the following sentences in as much detail as possible.

**a.** Mike visited both London and Paris.

**b.** Ann wanted white wine but Bill and Carol wanted red.

**c.** It will rain and clear off, but it will rain.

**d.** That is a new but growing market.

**e.** Confucius is affable but dignified, austere but not harsh, polite but completely at ease. (*Analects* 7:37)

**f.** Although Tim lost his glasses and his wallet, each was returned.

**g.** Tim lost his glasses and his wallet, and one person found both.

**2.** Restate each of the following forms, putting English notation into symbols and vice versa (e.g., both A and B becomes $A \wedge B$, and $A \wedge B$ becomes both A and B). Indicate the scope of connectives in the result by underlining.

**a.** both A and both B and C

**b.** both both A and B and C

**c.** $(A \wedge B) \wedge (C \wedge D)$

**d.** $A \wedge ((B \wedge C) \wedge D)$

**e.** $(A \wedge (B \wedge C)) \wedge D$

**f.** both both both A and B and C and D

**3.** The logical forms below are followed by intensional interpretations of their unanalyzed components. In each case, synthesize an idiomatic English sentence that expresses the corresponding interpretation of whole form. Remember that there may be more than one correct answer.

**a.** $(V \wedge F) \wedge R$

[F: Fred visited Florence; R: Fred spent a week in Rome; V: Fred visited Venice]

**b.** $(J \wedge (S \wedge F)) \wedge K$

[F: he was fair; J: he was a judge; K: he had an excellent knowledge of the law; S: he was stern]

**c.** $(C \wedge T \wedge H) \wedge (W \wedge F \wedge S)$

[C: we arrived cold; F: we left stuffed; H: we arrived hungry; S: we left sleepy; T: we arrived tired; W: we left warm]

**d.** $O \wedge O$

[O: Old King Cole was a merry old soul]

**4.** Calculate truth values for all compound components of the forms below using the extensional interpretation provided in each case.

**a.**

| A | B | C | A ∧ (B ∧ C) |
|---|---|---|---|
| T | T | F | |

**b.**

| A | B | C | D | ((A ∧ D) ∧ C) ∧ (B ∧ A) |
|---|---|---|---|---|
| T | T | F | T | |

For more exercises, use the exercise machine.

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### 2.1.xa. Exercise answers

**1.** **a.** Mike visited London ∧ Mike visited Paris

$$L \land P$$

both L and P

L: Mike visited London; P: Mike visited Paris

**b.** Ann wanted white wine ∧ Bill and Carol wanted red wine

Ann wanted white wine ∧ (Bill wanted red wine ∧ Carol wanted red wine)

$$A \land (B \land C)$$

both A and both B and C

A: Ann wanted white wine; B: Bill wanted red wine; C: Carol wanted red wine

**c.** It will rain and clear off ∧ it will rain

(it will rain ∧ it will clear off) ∧ it will rain

$$(R \land C) \land R$$

both both R and C and R

C: it will clear off; R: it will rain

**d.** That is a market ∧ that is new relative to other markets but growing

That is a market ∧ (that is new relative to other markets ∧ that is growing)

$$M \land (N \land G)$$

both M and both N and G

G: that is growing; M: that is a market; N: that is new relative to other markets

**e.** Confucius is affable but dignified ∧ Confucius is austere but not harsh ∧ Confucius is polite but completely at ease

(Confucius is affable ∧ Confucius is dignified) ∧ (Confucius is austere ∧ Confucius is not harsh) ∧ (Confucius is polite ∧ Confucius is completely at ease)

$$(A \land D) \land (S \land H) \land (P \land E)$$

(both A and D) and (both S and H) and (both P and E)

A: Confucius is affable; D: Confucius is dignified; E: Confucius is completely at ease; H: Confucius is not harsh; P: Confucius is polite; S: Confucius is austere

**f.** Tim lost his glasses and his wallet ∧ Tim's glasses and wallet were each returned

(Tim lost his glasses ∧ Tim lost his wallet) ∧ (Tim's glasses were returned ∧ Tim's wallet was returned)

$$(G \wedge W) \wedge (R \wedge T)$$

both both G and W and both R and T

G: Tim lost his glasses; R: Tim's glasses were returned;
T: Tim's wallet was returned; W: Tim lost his wallet

**g.** Tim lost his glasses and his wallet ∧ one person found both Tim's glasses and his wallet

(Tim lost his glasses ∧ Tim lost his wallet) ∧ one person found both Tim's glasses and his wallet

$$(G \wedge W) \wedge O$$

both both G and W and O

G: Tim lost his glasses; O: one person found both Tim's glasses and his wallet; W: Tim lost his wallet

*Note*: One person found both Tim's glasses and his wallet cannot be analyzed further because One person found Tim's glasses ∧ one person found Tim's wallet does not imply that the same person found both.

**2.** **a.** A ∧ (B ∧ C)

**b.** (A ∧ B) ∧ C

**c.** both both A and B and both C and D

**d.** both A and both both B and C and D

**e.** both both A and both B and C and D

**f.** ((A ∧ B) ∧ C) ∧ D

**3.** **a.** (Fred visited Venice ∧ Fred visited Florence) ∧ Fred spent a week in Rome

Fred visited Venice and Florence ∧ Fred spent a week in Rome

Fred visited Venice and Florence, and he spent a week in Rome

**b.** (he was a judge ∧ (he was stern ∧ he was fair)) ∧ he had an excellent knowledge of the law

(he was a judge ∧ he was stern but fair) ∧ he had an excellent knowledge of the law

He was a stern but fair judge who had an excellent knowledge of the law

**c.** (we arrived cold ∧ we arrived tired ∧ we arrived hungry) ∧ (we left warm ∧ we left stuffed ∧ we left sleepy)

We arrived cold, tired, and hungry ∧ we left warm, stuffed, and sleepy

We arrived cold, tired, and hungry; and we left warm, stuffed, and sleepy

**d.** Old King Cole was a merry old soul ∧ Old King Cole was a merry old soul

Old King Cole was a merry old soul, and a merry old soul was he

**4.** Numbers below the tables indicate the order in which values were computed

**a.**

| A | B | C | A | ∧ | (B | ∧ | C) |
|---|---|---|---|---|---|---|---|
| T | T | F | | Ⓕ | | F | |
| | | | | 2 | | 1 | |

**b.**

| A | B | C | D | ((A | ∧ | D) | ∧ | C) | ∧ | (B | ∧ | A) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | F | T | | T | | F | | Ⓕ | | T | |
| | | | | | 1 | | 2 | | 3 | | 1 | |

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