# **Index and glossary**

### Α

abstract: a predicate or functor [body]<sub>vartist</sub>, which associates blanks in the body with places of the predicate or functor by filling them with variables from the list varlist (if the variables are each used just once and in order, the abstract may be abbreviated by simply bracketing the body, with the blanks left open)—see 6.2.1

abstractor. brackets with a subscripted list of variables applied to a formula or term to form an abstract (or brackets alone in the abbreviated notation for abstracts)—see 6.2.1

Absurdity: the particular absurd sentence ⊥—see 1.2.5

absurdity: the property of being false in every possible world—see 1.2.5, 1.4.5

accommodation (to an utterance): adjusting one's beliefs and various aspects of the context to make the utterance count as true and appropriate—see 1.3.1

accumulated goals (of a path): a collection including every sentence that appears as a goal at any point in the development of the path—see 7.7.4

accumulated resources (of a path): a collection including every sentence that appears as an active resource at any point in the development of the path—see 7.7.4

active resource (of a gap): an available resource of the gap that has not been exploited in that gap—see 2.2.3, 2.2.8

active for a term (in a gap): an available resource of the gap that has not exploited for that term in that gap—see 7.5.4

Adams, Ernest W. (1926-2009)—see 5.1.3

Adjunction—see Adj under Rule Labels

affirmative generalization: a generalization is affirmative when it is the property expressed by the predicate of the generalization (rather than, say, the denial of this predicate) that is said to hold generally—see 7.1.4

agreement (of interpretations) for a gap: interpretations agree for a gap when they have the same referential range and give the same interpretation to all vocabulary appearing in the gap and all its ancestors—see 7.7.2

alias set: a set of terms which are all co-aliases (and which includes any co-alias of its members)—see 6.3.2

alphabetic variant: a formula that differs from a given formula only in the choice of its bound variables (not in the pattern of binding marked by the distribution of variables in the formula)—see 6.2.2

alternatives: sentences offered as a group at least one of which is true—a generalization of the idea of a conclusion, which is offered as a sole alternative—see 1.4.3

analysis: a restatement of sentence to make its logical form explicit—see 2.1.1—compare with synthesis

anaphoric pronoun: a pronoun whose function is to have the same reference as another expression—see 6.1.6

ancestor gap: a parent of a gap, or a parent of its parent, or ...—see 2.3.3

and, logical—see ∧ under Symbols

antecedent of a conditional: one of two components of a conditional; the antecedent appears at the tail of an arrow in symbolic notation and is typically expressed in English using a subordinate clause (compare consequent)—see 5.1.1

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any—see 7.3.3
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application of a functor—see functor application

appropriate assertion: an assertion that is in accord with various maxims governing language use—see 1.3.4

argument: the premises and conclusion of an inference considered in abstraction from the way the conclusion is reached from the premises—see 1.1.3

argument tree: the proximate arguments of the gaps of a derivation arranged according to parent-child relations among gaps—see 2.2.2

Aristotle (384-322 BCE)—see 7.1.1

assumption: a sentence considered (perhaps along with others) as a starting point of inference (compare premise)—see 1.1.2—see also premise, supposition

at least one-see 4.1.3

atomic formula: a formula (perhaps a sentence) that is either unanalyzed or is a predication—see 3.3.1, 6.2.2

attachment rule: a rule that adds a new but inactive resource that is entailed by resources already available—see 2.4.4

attribute of a generalization: the property that is said to fit all reference values in the generalization's domain—see 7.1.4

attributive adjective: an adjective used to modify a common noun or comparable phrase—see 2.1.3 Austin, J. L. (1911-1960)—see 1.3.3

auxiliary resource: a resource that must be available (though not necessarily active) for a rule to be applied; it is not exploited by applying the rule (compare main resource)—see 4.3.1

available resource: a resource is available in a gap when it is a resource appearing above the gap and next to a scope line that continues unbroken to the left of the gap; available resources will include the initial premises of the derivation and any resources added in ancestors of the gap—see 2.2.8

# В

Barbara: a name for a particular syllogistic pattern of argument, whose vowels reflects the pattern's place in a traditional classification of syllogisms (in the narrow sense) by "mood"—see 7.5.6, 8 5 4

basic rule: part of the basic system of derivation rules—see 3.5.1, App. A.4

basic system of derivations: any of the rules for closing gaps, or one of the rules for exploiting resources (including detachment rules) or planning for goals; the basic system does not include attachment rules or the rule LFR—see 3.5.1

Beth, Evert W. (1908-1964)—see 2.2.3

biconditional: a truth-functional compound that amounts to a conjunction of if and only if conditionals with the same main and subordinate clauses and is often expressed by the phrase if and only if—see 5.2.2

body (of an abstract): an expression to which an abstractor has been applied—see 6.2.1

Boole, George (1815-1864)—see 1.1.8

bound variable: a variable falling the scope of an abstractor for that variable (or in the scope of another operator, like of quantifier, that incorporates abstraction; bound variables are the symbolic analogue of anaphoric pronouns that have antecedents—see 6.2.2

bounding class (of a generalization): a class (beyond that given by the class indicator) that limits the domain of the generalization—see 7.1.5

bounds indicator: an expression (such as among Bs) that indicates a bounding class—see 7.1.5

branching conditional: a truth-functional compound that amounts to a conjunction of conditionals whose antecedents are contadictory—see 5.1.4

Brouwer, L. E. J. (1881-1966)—see 3.1.3, 8.5.2

but-not form: a truth-functional compound that asserts one component and denies another; it can be expressed by a conjunction whose second component is negative—see 3.1.4

- case argument: an argument that establishes a conclusion by showing that it holds for each of a group of alternatives that is exhaustive given current assumptions—see 4.2.1
- categorical sentence: a sentence that does not incorporate the sort of qualification or hedge found in a disjunctive or conditional claim—see 4.2.2
- category mistake: the application of predicate to something that is not of the right sort for the predicate to be either true or false—see 1.3.7
- causal condition (for a state or event): a condition that can bring about the state or event or whose absence can prevent it (compare epistemic condition)—see 5.2.2
- Celarent: a name for a particular syllogistic pattern of argument, whose vowels reflects the pattern's place in a traditional classification of syllogisms (in the narrow sense) by "mood"—see 8.5.4
- *chain law*: a law governing entailment that asserts the transitivity of the relation that holds between sets when the first entails each member of the second—*see* 1.4.7
- character. a feature of the meaning of a sentence—in particular, the way the proposition expressed by the sentence depends on its context of use—see 1.3.2
- child gap: an heir of a given gap at the next stage in a derivation, either a gap that results from applying a rule to the gap or the gap itself if a rule has been applied elsewhere—see 2.3.3
- Chrysippus (280-207 BCE)—see 2.2.3
- Church, Alonzo (1903-1995)—see 6.2.1, 7.7.1
- claim of exemplification: a claim that something provides an example of a given property; it is the sort of claim expressed by an existential sentence—see 8.1.1
- class indicator. a part of a quantifier phrase, consisting of the common noun together with any modifiers that restrict the class referred to (e.g., large city that Al has visited); the class referred to by a class indicator is the indicated class—see 7.1.4
- closed expression: a formula or term all of whose variables are bound within that expression—i.e., a formula or term with no free variables—see 6.2.2
- co-alias (of a term): a term that is an alias of the same thing as is the given term—see 6.3.1
- co-alias series: a pair of term series that have the same length and whose corresponding terms are co-aliases—see 6.3.3
- co-alias (of a term) with respect to equations: a term that has the same reference as a given term on the assumption that the given equations hold—see 6.3.2
- complement (of a set X): when relative to a set Y, it is the set consisting of the members of Y that are not members of X; when full, it is the set of all reference values that are not members of X—see 7.1.4—see also subtraction
- complementary generalization: a generalization whose domain is the full complement of its indicated class; the term only is often used to express a complementary generalization—see 7.1.4
- *complete expression*: an expression of the normal sort that is not looking for input, as distinct from an "expression-with-blanks" or operator—*see* 6.1.1—*compare with* operator
- complete system of derivations: a system of derivations that provides a proof for every valid argument—see 2.3.7, 7.7.1—compare with sound system of derivations
- *component* of an expression: the expression itself and any further expression uncovered by analyzing the given expression as the result of applying connectives or other syntactic operators—*see* 2.1.1, 2.1.7, 3.1.1
- compositionality: a law for a truth-functional compound that asserts its equivalence to any result of replacing its immediate components by sentences equivalent to them—see 2.1.2, 3.1.2, 4.1.1, 5.1.2
- compound expression: an expression that has at least one component other than itself; compound formulas and sentences can take many forms, but compound terms are alway functor applications

- and compound predicates and functors are always abstracts—see 2.1.1, 6.1.7
- *conclusion*: most generally, the output of an inference; in the context of relative exhuastiveness, a single-membered list of alternatives—see 1.1.2, 1.4.4
- conclusion tree: a proof in the form of a tree consisting of conclusions drawn from premises that are themselves conclusions from premises further up its branches; the tree is a proof for an argument whose conclusion is at its root and whose premises include all conclusions at the tips of its branches—see 2.2.1
- conditional: in a narrow sense (in which it may be referred to as "the conditional"), the if-conditional; more broadly, any of a group of related logical forms also including conditionals expressed by only if and unless—see 5.1.1—see also if-conditional, only-if-conditional, unless-conditional
- conditionalization: an operation for modifying an argument by dropping a premise and making the conclusion conditional on it—i.e., making the dropped premise the antecedent of a conditional whose consequent is the original conclusion—see 5.3.1
- confinement: a restatement of a quantificational sentence that confines the scope of the quantifier to a part of its original scope—see 8.1.4
- confirming a counterexample: a calculation of truth values for the premises and conclusion of an argument to show that a counterexample really does make the premises true and conclusion false—see 2.3.5
- conjunct: an immediate component of a conjunction—see 2.1.1
- conjunction: a truth-functional compound that is true only when its two components are both true and whose content thus includes the content of both components; also the connective that forms such a compound—see 2.1.1
- Conjunction (pattern of argument): an argument in which a conjunction is concluded from its two conjuncts—see 2.2.1—see also Cnj under Rule labels
- connective: an operator that forms a compound sentence from one or more sentences (its components); more generally, that forms a formula from one or more formulas—see 2.1.1. 3.1.1
- consequent of a conditional: one of two components of a conditional; the consequent appears at the head of an arrow in symbolic notation and is typically expressed in English using a main clause to which another clause is subordinate (compare antecedent)—see 5.1.1
- consequentia mirabilis: an argument in which a sentence is concluded from its own denial—see 4.3.1
- conservative system: a system of derivations all of whose rules are safe and sound (or, equivalently, safe and strict); a conservative system will neither add nor drop lurking counterexamples as a derivation develops—see 2.3.4
- constructive proof. a proof of an existential that produces (and thus "constructs") the sort of example whose existence is claimed by the existential—see 8.5.2—see also EG under Rule labels
- content of a sentence or proposition: the set of possible worlds in which it is false—see 1.2.2
- content of a set  $\Gamma$  as a set of alternatives: the set of possible worlds in which every member of  $\Gamma$  is false (same as the shared content of  $\Gamma$ )—see 1.4.3
- content of a set  $\Gamma$  as a set of assumptions: the set of possible worlds in which at least one member of  $\Gamma$  is false (same as the cumulative content of  $\Gamma$ )—see 1.4.3
- contingency, logical: the property of being neither tautologousness nor absurd—i.e., of being true in at least one possible world and false in at least one—see 1.2.5
- contradictory sentences: sentences that both mutually exclusive and jointly exhaustive—i.e., which have opposite truth values in every possible world—see 1.2.7, 1.4.5
- contraposition: principle of equivalence for conditionals and for generalizations; for conditionals, the equivalence between a conditional and another whose antecedent and consequent are contradictory to the consequent and antecedent, respectively, of the first; for generalizations, the equivalence between a generalization and another whose restricting and quantified formulas are contra-

dictory to the quantified and restricting formulas, respectively, of the first—see 8.3.3

conversational implicature: an implicature that derives from the content of an assertion and the conversational context rather than features of the language used—see 4.1.2

conversion: a principle of equivalence for existentials according to which an existential is equivalent to the result of interchanging its restricting and quantified formulas—see 8.1.3

*counterexample*: an example for which a generalization fails; a generalization is true when it has no counterexamples—*see* 1.4.1, 7.1.4

counterexample, confirmation of—see confirming a counterexample

counterexample, presentation of—see presenting a counterexample

cover a possible world: a sentence or proposition covers a possibility when it is true in that possibility—see 1.2.2

coverage of a sentence or proposition: the set of possible worlds in which it is true—see 1.2.2

coverage of a set  $\Gamma$  as a set of alternatives: the set of possible worlds in which at least one member of  $\Gamma$  is true (same as the cumulative coverage of  $\Gamma$ )—see 1.4.3

coverage of a set  $\Gamma$  as a set of assumptions: the set of possible worlds in which every member of  $\Gamma$  is true (same as the shared coverage of  $\Gamma$ )—see 1.4.3

counterfactual conditional: a non-truth-functional compound, often stated in subjunctive mood, that makes a claim which can be false, and thus has substantive content, in cases where its antecedent is false (or counter to fact); also the connective that forms such a compound—see 5.1.3

cumulative content of a set  $\Gamma$ : the set of possible worlds in which at least one member of  $\Gamma$  is false (same as the content of  $\Gamma$  as a set of assumptions)—see 1.4.3

cumulative coverage of a set  $\Gamma$ : the set of possible worlds in which at least one member of  $\Gamma$  is true (same as the coverage of  $\Gamma$  as a set of alternatives)—see 1.4.3

Curry, Haskell (1900-1982)—see 5.1.2

*cycle*: a round of development of a derivation to the extent this is possible using only terms already and not introducing new ones—*see* 7.7.3

# D

Darii: a name for a particular syllogistic pattern of argument, whose vowels reflects the pattern's place in a traditional classification of syllogisms (in the narrow sense) by "mood"—see 8.5.4

dashes: a way of reading the use of dashes around an expression to indicate its context—i.e., the plural noun dashes referring to the symbols is read as if it was a verb to indicate that the context is sentential (compare dots)—see 7.2.1

De Morgan, Augustus (1806-1871)—see 4.2.4

dead-end gap: a gap to which all available rules have been applied, so nothing more can be done to develop or close it—see 2.3.1

*decisive* system of derivations: a system of derivation is decisive when any derivation will reach a point where either there is a dead end gap or all gaps have closed—see 2.3.6

decision procedure: a decision procedure for a question is a procedure that will always produce, after a finite number of steps, a correct answer—see 2.3.6

deductive logic: the study of the norms of deductive reasoning—see 1.1.4

deductive reasoning: reasoning by way of deductive properties and relations—in particular, inference to conclusions that are entailed by the premises the inference begins with—see 1.1.4

definite description: an individual term in which the definite article the is used to refer to something as the sole example fitting some description—see 1.3.7, 6.1.6—see also 8.4

deixis: literally, pointing; an alternative term for indexicality—see indexicality

delineation: a precise specification of the range of things a term is true of—see 1.3.6

de-negation: the removal of negation; more precisely, the transition from a negation to its immediate component—see 4.2.2 descendant gap: a gap that is a child of the given gap, or the child of a child, and so on—see 2.3.3

description operator: an operator that forms an individual term when it is applied to the one-place predicate; it provides one approach to analyzing definite descriptions—see 8.4.3

detachment rule: a rule that permits the exploitation of a two-component compound given the availability of another resource (that is not exploited); a resource is added to the gap that is either a component of the exploited resource or contradictory to a component, and is in this sense "detached" from the other component—see 4.3.1

development of a gap: further analysis of the question of validity posed by a gap, involving the creation of one or more child gaps with different resources or goals—see 2.2.3

Diodorus Cronus (-c.284 BCE)—see 5.1.3

direct generalization: a generalization whose domain is given directly by its class indicator (compare complementary generalization)—see 7.1.4

discharged assumption: an assumption whose scope (marked by the scope line to its immediate left) has ended—see 2.4.2

disjunct: an immediate component of a disjunction—see 4.1.1

disjunction: a truth-functional compound that is false only when its two components are both false and whose coverage thus includes that of both its components (with its content then limited to the overlap in the content of its components); also the connective that forms such a compound; sometimes labeled "inclusive" disjunction to distinguish it from exclusive disjunction—see 4.1.1—compare exclusive disjunction

domain (of a generalization): the collection of reference values about which the generalization generalizes—i.e., the values that the generalization claims its attribute to fit—see 7.1.4

dots: a way of reading the use of ellipsis dots around an expression to indicate its context—i.e., the plural noun dots referring to the symbols is read as if it was a verb to indicate that the context is sentential (compare dashes)—see 7.2.1

double conditional: a compound that limits its content to cases where each of two sentences is true either by adding each successively as antecedents of a conditional or by adding them together as a conjoined antecedent—see 5.1.2

double negation: a truth functional compound that is the negation of a negation; also the operator that forms such a compound—see 3.1.3

down tack—see ⊤ under Symbols

duality: a pairing of terms (e.g., premise-alternative, absurdity-tautology, negation-negation, conjunction-disjunction) with the property that replacing such terms by their pair-mates in a deductive law or principle will yield another valid law or principle—see 1.4.8, 4.2.4

dummy restriction: a restricting predicate (such as,  $[x = x]_x$  or  $[T]_x$ ) that is bound to be true of every reference value and thus does nothing to restrict the domain of a generalization; a dummy restriction makes its possible to state the content of an unrestricted universal using the form of a restricted universal—see 7.2.1

#### F

effectual system of derivations: a system of derivations is effectual when every fully developing gap has a counterexample lurking in it; this is an adaptation of the idea of sufficiency for systems in which development cannot always be brought to an end—see 7.7.1

entailment: the relation holding between assumptions and a conclusion when there is no possible world in which the conclusion is false while the assumptions are all true; an argument is deductively valid when its premises entail its conclusion—see 1.1.6

*entity*: anything that might be referred to—see 6.1.6

epistemic condition (for a state or event): a condition that supports reasonable belief in something or whose failure undermines such belief (compare causal condition)—see 5.2.2

- equation: a predication of the identity predicate, often represented by the equals sign =—see 6.1.4
- equivalence (logical): the relation holding between a pair of sentences when there is no possible in which they have different truth values; equivalent sentences have the same content (and coverage) and thus express the same proposition—see 1.2.3
- equivalence class: a (non-empty) set consisting of all and only things equivalent (in a given sense) to any of it members; the set all things equivalent (in the relevant sense) to a given thing will form an equivalence class—see 6.3.2
- *equivalence relation*: a relation that is reflexive, symmetric, and transitive; any equivalence relation expresses some form of equivalence—*see* 1.2.3
- ex falso quodlibet: a phrase (meaning, roughly, 'from the false, whatever') used to name the principle that Absurdity (and thus any absurd sentence) entails every sentence—see 2.2.7—see also EFO under Rule labels
- ex nihilo verum: a phrase (meaning, roughly, 'from nothing, the true') used to name the principle that Tautology (and thus any tautologous sentence) is entailed by the empty set of assumptions—see 2.2.7—see also ENV under Rule labels
- exception class: a class of reference values that must be subtracted in determining the domain of a generalization—see 7.1.5
- exception indicator: a phrase that serves to indicate an exception class—see 7.1.5
- *exclusion*: a set of assumptions excludes a sentence when there is no possible world in which the sentence is true along with all of the assumptions—that is, the sentence must be false if the assumptions are all true—see 1.1.6, 1.4.5—see also inconsistency, relative
- exclusion, mutual—see mutually exclusive
- *exclusive disjunction*: a truth-functional compound of two sentences that is true only when its the sentences have opposite truth values; it adds to disjunction the claim that the disjuncts are not both true—see 4.1.2—compare disjunction
- exclusive disjunction form: a logical form expressing an exclusive disjunction, the result of conjoining a not-both form to an ordinary (i.e., inclusive) disjunction—see 4.1.2
- exemplify: to be an example of; something exemplifies a property (or a predicate expressing the property) when it has that property—see 8.1.1
- *exhaustiveness* (absolute) of a set: a set is exhaustive when its members cannot be all false; the members themselves are said to be *jointly exhaustive—see* 1.1.7, 1.2.7, 1.4.4, 1.4.5
- exhaustiveness of a set of alternatives, relative to a set of assumptions: a set of alternatives is exhaustive relative to a set of assumptions—or is rendered exhaustive by it—when the alternatives are not all false in any possible world where the assumptions are all true—i.e., when the assumptions cannot be separated from the alternatives—see 1.4.4
- existence (as a requirement for a definite description to refer): one requirement for a definite description to have a non-nil reference value is that the description be exemplified—see 8.4.1
- existential: a logical form whose main operator is an existential quantifier—see 8.1.1
- existential claim: the claim that something of a certain sort exists—i.e., that a certain property is exemplified—see 8.1.1
- existential commitment: the existential commitments of a sentence (or set of assumptions) are expressed by the existential sentences it entails—see 8.1.5—see also substantive existential commitment
- existential generalization: a pattern of argument in which an existential sentence is concluded from one of its instances—see 8.5.2—see also EG under Rule labels
- expanded form: for English sentences, the result of restating the sentence with some noun phrase (either an individual term or a quantifier phrase) as the subject followed by the phrase is such that; for symbolic analyses, a restatement as the predication of an abstract to one or more individual terms—see 6.2.3, 6.2.4, 7.1.1

- *expansion* (on a term or terms or, for English sentences, on a noun phrase): a restatement in expanded form that focuses in the terms or noun phrase in question—*see* 6.2.3, 6.2.4, 7.1.1
- expansion (of a language by a range of reference values): the addition to the language, for each of the reference values, of a term with that value—i.e., roughly, an enrichment of the language so that each thing has a name—see 7.5.1
- exploitable (in a gap): an active resource for which there is an exploitation rule that may be applied in the gap—see 7.7.3
- *exploitation*: developing a gap by analyzing the significance of an active resource; in most cases, a resource will become inactive when exploited, but this is not true for generalizations—*see* 2.2.4, 7.5.4
- extension: traditionally, the class of things a predicate is in fact true of; more generally, an aspect of meaning that, like the extension of a predicate, concerns only a single possible world—e.g., the extension of a sentence is its truth value—see 2.1.8, 6.4.1—compare with intension
- extension (of a rule) for equations: a broadening of a rule to apply in cases that do not fit the unextended rule but differ from cases fitting it only by co-aliases (i.e., could be made to fit the unextended rule if certain terms were replaced by co-aliases); the extended form is labeled by adding the equal sign = to the label of the unextended form—see 6.3.3
- extensional interpretation: an interpretation of a sentence (or set of sentences) that assigns extensions (e.g., truth values) to unanalyzed components—see 2.1.8, 6.4.1
- extensional operator: a generalization of the idea of a truth-functional connective to include any operator that forms compounds whose extensions are determined solely by the extensions of their components (i.e., without requiring information about the intensions of those components)—see 6.1.3
- extraction: a pattern of argument in which a conjunct is concluded from a conjunction—i.e., the most explicit case of the extraction of content—see 2.2.1—see also Ext under Rule labels

#### F

- Ferio:a name for a particular syllogistic pattern of argument, whose vowels reflects the pattern's place in a traditional classification of syllogisms (in the narrow sense) by "mood"—see 8.5.4
- figure: part of a traditional classification of syllogisms (in the narrow sense); only 3 predicates appear as restricting or quantified predicates in syllogism, and the figure concerns the distribution of the restricting and quantified predicates of the conclusion in the two premises (each premise must contain one of the two for the argument to be a syllogism in this sense)—see 7.5.6, 8.5.4
- first-order logic: logic for logical forms built using connectives and quantifiers for individuals (not, for example, quantifiers for properties); the logic of the identity relation is also sometimes seen as part of first-order logic—see 1.1.8, 8.5.4—compare with higher-order logic
- Fitch, Frederic (1908-1987)—see 2.2.3
- *flag*: a label on the scope line of a general argument indicating the term for which it is general—*see* 7.5.5
- *force*: the aspect of a sentence's that concerns the speech act constituted by its utterance (e.g., a question could be described as a sentence with interrogative force)—see 1.3.2, 1.3.3
- formal logic: the study of the logical properties of sentences that can be attributed to their logical form—see 1.1.8
- formal validity: validity attributable to the logical forms of the premises and conclusion of an argument; an argument may be valid without being formally valid for a given analysis of logical form

  —see 2.3.8
- formula: an expression whose proper extension is a truth value; a formula either is a sentence or would be a sentence but for the occurrence of free variables—see 6.2.2
- free variable: a variable is free in a given expression if it occurs in the expression but is not bound to

any operator within the expression (so a variable that is not free in an expression may be free in a component of the expression)—see 6.2.2

Frege, Gottlob (1848-1925)—see 1.1.8, 4.1.3, 6.1.3, 6.2.1, 7.1.1, 7.1.1, 7.4.1, 8.4.3, 8.5.4

*fully developing path*: a path in a derivation that does not close but to which every rule that might be applied is applied at some point; a dead-end open gap falls at the end of a fully developing path, but, in derivations for quantifiers, a fully developing path need not have an end—*see* 7.7.1

function: a way of associating entities that uniquely determines an output for each choice of input; often used as a shortened term for reference functions (i.e., for the extensions of functors)—see 6.1.7—see also reference function

functor. an operator that forms individual terms from individual terms—see 6.1.7

functor abstract: an abstract whose body is an individual term; a functor abstract constitutes a compound functor—see 6.2.1

*functor application*: the formation of a compound term from a functor and individual terms filling its places—see 6.1.7

### G

gap: space, at a give stage of a derivation, between a goal and its resources; a gap is closed (and thus made to disappear) only by applying one of a special group of rules—see 2.2.3

*general argument*: an argument for a particular instance of a generalization is general if it could be replicated for every other instance—see 7.5.3

general exemplification: the content of a generalization whose instances are claims of exemplification—see 8.2.1

general term: a term (such as a common noun) that is designed to refer to a class of things rather than to pick out a particular thing—see 6.1.6—compare with singular term

*generalization*: the content of a universal sentence; also, the transition to a universal or existential from one of its instances—see 7.5.3

generalization over pairs: a generalization any of whose instances is a generalization over individuals—i.e., symbolically, a sentence of the form  $(\forall x ...)(\forall y ...) ... x ... y ... (where <math>(\forall x ...)$  indicates a quantifier that may be either restricted or unrestricted)—see 7.4.1

Gentzen, Gerhard (1909-1945)—see 2.2.3

*goal*: the conclusion being sought at a particular point (i.e., in a particular gap at a particular stage) in a derivation—see 2.2.3

Gödel, Kurt (1906-1978)—see 7.7.1, 8.5.4

grade (of a resource or goal): a measure of a sentence's distance from the sort that would appear in a dead-end gap of a derivation (once the rule IP is part of the system of derivations)—see 3.4.1

grammatical predicate: a predicate in the grammatical sense—see 6.1.1—compare with predicate, logical

grammatical subject: a subject in the grammatical sense—see 6.1.1—compare with individual term Grice, H. Paul (1913-1988)—see 1.3.4, 4.1.2, 5.1.3

#### Н

*hedging*: weakening a claim so that it covers additional possibilities, especially when this is done by adding further vocabulary (e.g., through use of disjunction or a conditional)—*see* 4.1.1

higher-order logic: logic employing an analysis of logical form that includes quantifiers for predicates or functors in addition to ordinary quantifiers for individuals, where these predicates and functors may themselves apply to predicates or functors—see 8.5.4—see also second-order logic hypothetical: a hypothetical argument is an argument whose assumptions include a supposition; a hypothetical sentence is a conditional—see 4.2.2

- ID of a reference value: a label for the reference value that is used to describe a structure it is part of—see 6.4.1
- identity: a predicate (represented symbolically by the equal sign =) that holds for those pairs of reference values that consist of a single value taken twice; also the relation expressed by this predicate—see 6.1.4
- if-conditional: a truth-functional compound of an antecedent and a consequent that is false only when its antecedent is true and its consequent is false and whose content thus trims to the content of its consequent to cases where the antecedent is true; also the connective that forms such a compound; as the paradigm example of a conditional in the broad sense, it may be referred to as "the conditional"—see 5.1.1—compare with only-if-conditional, unless-conditional
- immediate component: in an analysis of an expression, a component (perhaps one of a group) from which the expression is derived directly by the application of a single connective or other operator (e.g., the immediate components of a conjunction are its conjuncts)—see 2.1.7
- *immediate inference*: an unmediated inference, one in which a conclusion is drawn from a single premise (and thus without the mediation of further premises)—see 1.1.3—see also syllogism
- *implication*: the relation of entailment restricted to cases where there is a single assumption (and thus the relation between the premise and conclusion of a valid immediate inference)—*see* 1.2.3
- *implication, relative*: one sentence implies a conclusion relative to, or *given*, further assumptions when the conclusion is entailed by it together with those further assumptions; relative implication is thus entailment with one assumption singled out for special attention—*see* 5.3.1
- *implicature*: something that must hold in order for an utterance to be appropriate in context; also the relation between utterances and their implicatures—see 1.3.2, 1.3.4
- inclusive disjunction: disjunction in the ordinary sense when it is being distinguished from exclusive disjunction—see 4.1.2—see also disjunction, exclusive disjunction
- *inconsistency, relative*: a sentence is consistent relative to, or *with*, a set of assumptions when the result of adding it to those assumptions is an inconsistent set; a sentence is thus inconsistent with a set of assumptions when it is excluded by them—*see* 1.1.7, 1.4.5—*see also*—exclusion
- *inconsistent*: a set of assumptions is inconsistent when there is no possible world in which its members are all true—see 1.1.7, 1.4.5
- inconsistent with—see inconsistency, relative
- indefinite article: the word a (or an); it is important to distinguish from the definite article the because the indefinite article, while leading to a noun phrase that is singular in number, does not form an individual term—see 7.3.1
- independence, logical: a pair of sentences is logically independent when neither is tautologous or absurd and the two do not stand in any deductive relation; more generally, any set of sentences is logically independent if every assignment of truth values to its members appears in some possible world—see 1.2.8, 1.4.5
- independent term: a term is independent with respect to an argument if it is unanalyzed and appears in neither the premises nor conclusion of the argument; an independent term can be used to establish a generalization through a general argument—see 7.5.3, 8.5.1
- indexicality (or deixis): the dependence of the meaning of an expression on the context of its use (as the objects referred to by this and that depend on what is being pointed to)—see 1.3.2, 1.3.5
- indicated class: the class picked out by the class indicator of a quantifier phrase—see 7.1.4—see also class indicator
- *indicative conditional*: an ordinary truth-functional conditional, as opposed to a counterfactual conditional, which is often stated in subjunctive mood—*see* 5.1.3
- indirect proof. proof of a conclusion by a reduction to absurdity of its denial (or another sentence contradictory to it)—see 3.3.1—see also IP under Rule Labels

individual: anything that might be referred to—see 6.1.6

individual term: a term that refers to a particular thing in a definite way (so, for example, dog and a dog are not individual terms); variables and compound terms with free variables count as symbolic individual terms, but the phrase unanalyzed individual term will usually be limited to non-variables—see 6.1.1, 6.2.2

*inductive inference*: a variety of non-deductive inference; the paradigm example is inference to a generalization from multiple instances of it but the term is often applied more broadly to any inference whose premises render its conclusion probable—see 1.1.4

inference: the action of drawing a conclusion from certain premises or assumptions—see 1.1.2

inference, immediate—see immediate inference

inference ticket: a conditional seen as supporting an inference from it antecedent to its consequent—see 5.3.2—see also modus ponens

*instance*: an instance of a universal (or existential) sentence is a sentence predicating the quantifier predicate of an individual terms (e.g., Spot barks is an instance of All dogs bark)—see 7.5.1

intension (spelled with an s): traditionally, the meaning of a predicate (as opposed to the class of things it happens to be true of); more generally, an aspect of meaning that concerns more than a single possible world—e.g., the intension of a sentence is the proposition it expresses—see 2.1.8, 6.4.1—compare with extension

intensional entity: entities whose identity depends on more than one possible world (e.g., two official roles in an organization that are filled by the same person which might be held to be distinct entities because they would be filled by different people); it is controversial whether such differences should be seen as differences in entities or merely differences between ways that ordinary extensional entities are referred to—see 6.3.1

intensional interpretation: an interpretation that assigns intensions; an unanalyzed sentence may be given an intensional interpretation by associating an English sentence with it and other unanalyzed components can be given intensional interpretations by associating English words and phrases of the appropriate sort—see 2.1.8, 6.4.1

intensional property: a property whose application in a given possible world depends on other possibile worlds—i.e., a property that cannot be expressed by an extensional operator—see 6.3.1—compare with property in intension

*interpretation*: an assignment of meaning to the unanalyzed components of a logical form—see 2.1.8, 6.4.1—see also extensional interpretation, intensional interpretation

interpretation of a gap: an interpretation of all the non-logical vocabulary that appears in resources or goals of the gap or any of its ancestors—see 7.7.2

*intersection*: the intersection of two sets is a set consisting of the things that are members of bot sets—i.e., it consists of the overlap in their membership—see 7.1.5

intuitionism: a view (due to the mathematician L. E. J. Brouwer) of reality and truth in mathematics that tied both closely to mathematical proof; in particular, mathematical objects were held to exist only when constructed in "intuition" (in a technical sense derived from the 18th century philosopher Immanuel Kant) and proofs were held to describe and report such construction, and inferences were legitimate only when they reflected modes of construction—see 3.1.3

intuitionistic negation: a non-truth-fuctional connective having only the logical properties accepted by intuitionists; in particular, a doubled intuitionistic negation does not imply an affirmative claim because disproving the possibility of disproving the existence of something does not in itself serve to construct the thing claimed to exist—see 3.1.3

*irreflexivity:* a relation is irreflexive when nothing stands in the relation to itself (e.g., the relation expressed by is less than is irreflexive, but the relation expressed by is the square of is merely non-reflexive because 0 and 1 bear it to themselves)—see 7.8.2—compare with reflexivity

J

Jaskowski, Stanislaw (1906-1965)—see 2.2.3 jointly exhaustive—see exhaustiveness

# K

Kant, Immanuel (1724-1804)—see intuitionism

#### T

lambda abstraction: the operation of forming an abstract by binding variables; the term reflects a common notation for a variable-binding operator that uses the Greek lower case lambda λ—see 6.2.1—see also abstract

*leave open* a possible world: a sentence or proposition leaves open a possibility when the possibility is one of the ways in which it is true—*see* 1.2.2—*compare with* rule out

law—see list of laws

Leibniz, G. W. (1646-1716)—see 1.1.8, 1.2.1

*lemma*: a conclusion that is drawn not for its own sake but as a step along the way to other conclusions—see 1.4.7

Lemma (the rule)—see Lem under Rule Labels

Lewis, David K. (1941-2001)—see 1.3.1, 5.1.3, 8.4.1

*logic*: the normative study of reasoning—i.e., the study of when reasoning is good—see 1.1.1

*logical and*—see ∧ under Symbols

logical contingency—see contingency (logical)

logical equivalence—see equivalence (logical)

*logical form*: a pattern that is repeated in different sentences and is a basis for formulating generalizations about their logical properties; the forms considered here can all be described as combinations of operators that form compound expressions and can be uncovered by repeated analysis of expressions as compounded out of component expressions by these operators—see 1.1.8, 2.1.7

*logical implication*: a way of referring to implication that marks its difference from material implication—see 5.1.1—see also implication, material implication

logical independence—see independence (logical)

*logical possibility:* a possibility in the widest sense—i.e., anything that can be described without self-contradiction—see 1.2.1

logical predicate—see predicate (logical)

*logical space*: either of two ways that different directions or dimensions of variation appear in logic—specifically, (i) the space of all the different logical possibilities and (ii) the space of all the different propositions—see 1.2.6

*logical vocabulary*: logically significant elements of a sentence's vocabulary—specifically, vocabulary used to indicate its logical form—see 6.1.4

*lurking* counterexample: an interpretation that separates the active resources of a gap from its goal—see 2.3.3

*lurking throughout* a path (said of a counterexample): an interpretation that separates the active resources from the goal of each gap in a path—see 7.7.1

### M

main connective: the connective used last in forming a sentence, which thus marks it as having a particular sort of sentence (i.e., conjunction, negation, etc.); more generally, the main operator of an expression is the operator used last in forming the expression—see 2.1.5

main resource: the resource that is exploited when applying a detachment rule—see 4.3.1

material implication: the content of an (if-) conditional; calling this implication "material" derives, on the one hand, from philosophical views about a form/matter distinction that associates necessity with form and contingency with matter and, on the other hand, from the fact that (logical) implication amounts to the truth of a conditional in all possible worlds—see 5.1.1—compare with logical implication

mathematical logic: logic studied using mathematical methods—in particular, symbolic notation and rigorous proof; more narrowly, logic as applied to mathematical reasoning—see 1.1.8—see also metamathematics

meaning postulate (for a definite description): in general, a meaning postulate captures some aspect of meaning by asserting some sentence as a logical truth; in the case of definite descriptions, the sentence asserted specifies the reference of the description depending on whether or not something fits the description uniquely—see 8.6.3

meta-mathematics: the mathematics of mathematics—in particular, the logical properties of mathematical claims and theories studied mathematically—see 1.1.8—see also mathematical logic

middle term: the arguments counted as syllogisms (in the narrow sense) can each contain only three predicates as restricting or quantified predicates of its premises and conclusion; the predicate that does not appear in the conclusion is the middle term and is given this label because its relation to the restricting and quantified predicates of the conclusion, as stated in the premises, serve as the basis for the relation between them stated in the conclusion—see 7.5.6, 8.5.4

minimal sentence: a sentence of the sort that might appear in a dead end gap of a system of derivations including negation rules; a minimal sentence counts as having lowest "grade"—see 3.4.1—see also grade

misleading utterance: an utterance with a false presupposition (so that anyone who accommodated that utterance as appropriate would commit themselves to something that was false and thus be misled)—see 1.3.4

*modal logic*: the study of the logical properties of connectives associated with modal auxiliary verbs—especially, the logical properties of the connectives it is necessary that and it is possible that—see 3.1.2

modus *argument*: one of a group of argument patterns in which the affirmation or denial of one component of a weakened or hedged compound is concluded from the compound together with the affirmation or denial of the other component; names for the arguments fit the pattern modus \_\_endo \_\_ens, in which the blanks are filled with either pon to mark affirmation (think put) or toll to mark denial (think take) in the second premise and the conclusion, respectively—*see* 4.3.1, 5.3.2

modus ponendo ponens: a pattern of argument in which the consequent of a conditional is concluded from the conditional itself together with its antecedent—see 5.3.2—see also modus ponendo ponens under Laws, and MPP under Rule labels

modus ponendo tollens: a pattern of argument in which the denial of one conjunct of a conjunction is concluded from the denial of the conjunction together with the assertion of the other conjunct—see 4.3.1—see also modus ponendo tollens under Laws, and MPT under Rule labels

modus ponens—see modus ponendo ponens

modus tollendo ponens: a pattern of argument in which the one disjunct of a disjunction is concluded from the disjunction itself together with the denial of the other disjunct—see 4.3.1—see also modus tollendo ponens under Laws, and MTP under Rule labels

modus tollendo tollens: a pattern of argument in which the denial of the antecedent of a conditional is concluded from the conditional itself together with the denial of its consequent—see 5.3.2—see also modus tollendo tollens under Laws, and MTT under Rule labels

modus tollens—see modus tollendo tollens

monotonic: a trend that does not change direction; a form of inference is said to be monotonic when

the range of well-supported conclusions never decreases as further assumptions are added, so forms of inferences that hold normally or *ceteris paribus* (i.e., all things being equal) tend to be *non*-monotonic—*see* 1.4.7

mood: part of a traditional classification of syllogisms (in the narrow sense); the premises and conclusion of a syllogism in this sense must each have one of four logical forms—affirmative or negative direct generalization (labeled A and E, respectively) or existential with an affirmative or negative quantified predicate (labeled I and O)—and the mood of a syllogism is determined by which of the four appears in each of its component sentences (so a mood can be labeled by a series of three yowels)—see 7.5.6, 8.5.4

Morris, C. W. (1903-1979)—see 1.3.2

multiple ambiguity: a case of more than two possible meanings—see 7.1.1

multiple conjunction: a truth functional compound whose content accumulates the content of more than two components; a multiple conjunction can be analyzed (perhaps in several equivalent ways) as the result of repeated application of twofold conjunction—see 2.1.5

*multiple generality*: generality in several dimensions, often deriving from the presence of several quantifier phrases with overlapping scope; sentences exhibiting multiple generality are frequently, though not always, ambiguous—*see* 7.4.1

mutually exclusive: a pair of sentences are mutually exclusive when they cannot be both true; such sentences exclude each other, and form an inconsistent set—see 1.2.7, 1.4.5—see also exclusion, inconsistent

#### N

natural deduction system: a loosely defined term that tends to be applied to systems of proof that employ suppositions and thus reflect patterns of argument found in ordinary mathematical proofs—see 2.2.3

necessary condition: something that is required for something else; some necessary conditions are causal and others are epistemic—see 5.2.2—compare with sufficient condition—and see also causal condition, epistemic condition

negation: a truth-functional compound that is true only when its component is false and which is thus contradictory to its component; also the connective that forms such a compound—see 3.1.1

negative form (of a definition): a definition (of a deductive property or relation) in terms of the non-existence of certain sorts of possibility; such a definition might be stated using the form there is no possible world in which ...—see 1.4.1—compare with positive form

negative generalization: a generalization whose attribute is expressed by the denial of its quantified predicate—see 7.1.4

neither-nor form: a truth-functional compound that rejects both of a pair of alternatives; it can be expressed as the negation of a disjunction (i.e., as the negation of an either-or form)—see 4.1.4—compare with not-and-not form

the Nil: a nil value that is assumed to be the only such value and which therefore serves as the reference value of all undefined terms—see 6.1.3, 8.4.3, 8.6.1

nil value: a reference value that does not count as a reference to a genuine object—i.e., a reference value that implies a failure to refer—see 6.1.3

non-constructive proof: most specifically, a proof the establishes an existential conclusion without actually producing (i.e., without constructing) an example; more generally, a proof that employs techniques that must be ruled out if constructive proofs in the narrow sense are to be avoided—see 8.5.2—see also NcP under Rule labels

non-deductive reasoning: reasoning that cannot be supported solely by the deductive properties and relations of the sentences in question; inductive and other sorts of non-monotonic reasoning are examples, as is reasoning based on pragmatic phenomena, such as implicature—see 1.1.4

non-empty: a set is non-empty when it has at least one member—i.e., when it is not the empty

set-see 8.1.1

- non-logical vocabulary: vocabulary that does not function to mark the presence of certain logical forms (and thus provides the content that fills out such forms); non-logical vocabulary remains as unanalyzed residue after an analysis of logical form—see 6.1.4, 6.4.1
- non-monotonic inference: an kind of inference whose quality can be damaged by adding further assumptions; for example, reasoning that holds in the normal case can be undermined by added assumptions indicating that a given case is exceptional—see 1.4.7
- non-restrictive relative clause: a relative clause that does not function to further specify (i.e., restrict the possibilities for) what is being spoken but instead to provide further information about it; non-restrictive relative clauses are set off by commas and do not use the relative pronoun that—see 6.1.8—compare with restrictive relative clause
- not-and-not form: a truth-functional compound that denies each of two claims; it can be expressed as a conjunction of negations—see 3.1.4—compare with non-and-not form
- not-but form: a truth-functional compound that denies one claim and asserts another; it can be expressed as a conjunction whose first conjunct is a negation—see 3.1.4

*not sign*—*see* ¬ *under* Symbols

- not-both form: a truth-functional compound that denies that both of a pair of claims are true (and thus asserts that at least one of the two is false); it can be expressed as the negation of a conjunction—see 3.1.4
- not-without *form*: a truth-functional compound that denies that one claim is true in the absence of another (i.e., it denies what is asserted by a certain but-not form); it can be expressed as the negation of a conjunction whose second conjunct is a negation—*see* 3.1.4
- numerical quantifier phrase: a quantifier phrase that is used to assert that at least, at most, or exactly some number n of things of the certain kind have a certain property—see 8.3.2

# O

object: anything that might be referred to—see 6.1.6

- obversion: a principle of equivalence for negated universals and negated existentials according to which one of these is equivalent to an unnegated quantified sentence of the opposite sort with a contradictory quantified formula; to put this another way, a universal or existential is contradictory to a sentence of the opposite sort if their quantified formulas are contradictory—see 8.1.2
- only-if *conditional*: a truth-functional compound that trims the content of a denial of one component to cases where the other is false; it may be expressed as an if-conditional formed from negations—see 5.2.1—compare with if-conditional, unless-conditional
- open term: a term that contains a variable that is free in the term (i.e., that is not bound by some operator within the term)—see 6.2.2
- operator. an operator has "places" and is thus looking for complete expressions or operators a input, given which it will yield as output a complete expression or another operator; in the simplest (and most common) case, an operator is an expression-with-blanks—see 6.1.1—compare with complete expression
- ordered pair. an object with two members in a given order (i.e., with a first and second member); the members of a pair may be identical, as is the case with the pair (1,1), which has the number 1 as both its first and its second member—see 6.4.2

#### P

- parent gap: after the initial stage of a derivation, every gap derives from some gap at the previous stage; that gap is its parent—see 2.3.3
- path: a complete line of descent (from parent to child gaps) beginning with the initial gap of a derivation—see 7.7.1

- performative verb: a verb that describes the speech act performed in using it in first-person present tense form under normal circumstances (e.g., I promise ..., I command ..., I declare ...)—see 1.3.3
- Philo (c. 300 BCE)—see 5.1.3
- place of an operator: an opening for input; when an operator is an expression-with-blanks a place corresponds roughly to a blank, but a single place in an abstract may correspond to several blanks in its body (as in [x introduced x to y]<sub>xy</sub> which has 2 places even though \_ introduced \_ to \_ has 3 blanks)—see 6.1.2
- planning for a goal: developing a derivation by analyzing what is required to reach the current goal, replacing it by one or more new goals and perhaps making other changes as well—see 2.2.4
- positive form (of a definition): a definition (of a deductive property or relation) in terms of a generalization about certain sorts of possibility; such a definition might be stated using the form ... in every possible world in which ...—see 1.4.1—compare with negative form
- possible world: a possibility spoken of as an entity for the sake of stating claims of necessity as generalizations about possibilities and claims of possibility as claims of the existence of possibilities; "world" in this sense should be understood to include the whole of the universe and its full history and future—see 1.2.1
- pragmatics: aspects of meaning—such speech acts, implicature, and indexicality—which, very loosely, concern the use of language; also the study of these aspects of meaning—see 1.3.2—compare with semantics
- predicate (logical): an operator that forms a sentence (or formula) from one or more individual terms; a predicate in this sense need not count as a predicate grammatically since it may amount to something like a transitive verb (with blanks for subject and object) or even something like [it was raining in \_ ], where the grammatical subject is included but the grammatical predicate is partly missing—see 6.1.1, 6.1.2
- predicate, zero-place: it is sometimes useful to group operators of a certain sort with the sort of things they yield as output (for example, we group the connectives with the sentences ⊤ and ⊥ for various purposes, and we group predicates with unanalyzed sentences—and functors with unanalyzed individual terms—in our notation for non-logical vocabulary); one way of conceiving of such a grouping is to regard the output of an operator as an "operator" that no longer requires the input the operator does and thus has no places for that input—see 6.2.2
- predicate abstract: an abstract whose body is a formula (so the abstract counts as an operator that applies to individual terms to yield a formula)—see 6.2.1
- *predication*: a sentence (or formula) analyzed as the result of applying a predicate to individual terms; also, the action of applying a predicate—*see* 6.1.2
- premise: a starting point of an inference; premise tends to be used in contrast with supposition to suggest that an assumption is accepted (and not merely considered for the sake of argument), but this should be regarded as only an implicature—see 1.1.2—see also assumption, supposition
- presenting a counterexample: to describe a counterexample (to a claim of entailment) using tables or a diagram—see 6.4.3
- presupposition, semantic (of a sentence): something that must hold in order for the sentence to have a truth value (e.g., John must "have" a car in order for John's car is red to be either truth or false)—see 1.3.2, 1.3.7
- presupposition of a question: something that must hold in order for the question to have a direct answer (e.g., Is John's car red? can be answered with a simple yes or no only if John "has" a car)—see 1.3.7
- progressive rule: a derivation rule whose use moves us along toward the final state of a derivation—see 2.3.6
- proof by cases: a proof in which a conclusion is established by showing that it holds in each of a number of alternatives (which are known to be jointly exhaustive given the assumptions)—see

4.2.1—see also PC under Rule labels

*proof by choice*: a proof that establishes a conclusion using an existential premise by choosing a name (new to the proof) for an example of the sort the existential claims to exist—see 8.5.1

property: the extension of a one-place predicate; more broadly, a quality that something may possess individually, in contrast to the relation, which involves reference to something to which it is related—see 6.1.3—compare with relation

property in intension: the intension of a one-place predicate—i.e., the way in which class of things a predicate is true of varies among possible worlds—see 6.3.1—compare with intensional property proposition: the intension of a sentence—i.e., a way of dividing the possible worlds into those in which the sentence is false (its content) and those in which it is true (its coverage)—see 1.2.2

proximate argument of a gap: the argument that has the goal of a gap as it conclusion and the active resources of the gap as its premises—see 2.2.3

# O

quantified formula: a formula to which an unrestricted quantifier (or a restricted quantifier, once restricted) is applied—e.g., the formula θx in the forms ∀x θx and (∀x: ρx) θx for the case of universal quantifiers; when a quantifier is understood as only the operator ∀ or ∃ (and not its combination with an abstractor), it is the body of an abstract to which a quantifier is applied (the second such abstract in the case of a restricted quantifier)—see 7.2.1—compare with restricting formula quantified predicate: in connection with an English sentence, it is the predicate to which a quantifier phrase is applied—i.e., when a sentence has been restated in expanded form with the quantifier phrase as subject, the quantified predicate will have the form [\_ is such that ...]; in connection with an analyzed form, it is the extension of the predicate to which an unrestricted quantifier is applied or the second of the two predicates to which an unrestricted quantifier is applied—see 7.1.4, 7.2.1, 8.1.1—compare with restricting predicate

*quantifier.* most strictly, an operator that applies to predicates to yield sentences; in the case of an unrestricted quantifier, our usual notation combines the quantifier in this sense with an abstractor, and we will refer to, also as a quantifier, such a combination (i.e., the formula-with-a-blank  $\forall x$ , which amounts to  $\forall [\ ]_x$ ); and, in the case of a restricted quantifier, we will also regard, as a quantifier, the formula-with-a-blank ( $\forall x: \rho x$ ) \_ (which amounts to  $\forall [\ ]_x$ )—see 7.2.1

quantifier phrase: a noun phrase that combines with a (logical) producte to make a claim about how many things of a certain sort the predicate is true of—see 6.1.6, 7.1.1

*quantifier word*: a word (such as every, no, and some) appearing in a quantifier phrase that serves to indicate the sort of quantificational claim the phrase is used to make—see 2.1.4, 7.1.1

quod erat demonstrandum: a phrase (meaning roughly, 'what was to be shown') that has been traditionally used to mark the end of a proof—see 2.2.1—see also QED under Rule labels

### R

range, referential: the collection of reference values that is the basis for an interpretation of logical forms once individual terms are part of their analysis—see 6.4.1

reductio argument: an argument that establishes a conclusion from given premises by showing that a sentence contradictory to it implies an absurd conclusion (and is thus "reduced to absurdity") relative to the given premises—see 2.4.3

reduced form: the simpler form of which an expanded form is an expansion—see 6.2.3, 6.2.4—compare with expanded form

reduction (of an expanded form): the transition from the expanded form to the corresponding reduced form; for example, the reduction of the English expanded form NP is such that (... it ...) is ... NP ... (where the noun phrase NP has been put in place of the pronouns that had it as antecedent in the expanded form), and the reduction the symbolic expanded form [ ... x ... ] $_{\tau}\tau$  is ...  $\tau$  ...

(where the term  $\tau$  has been put in place of the variables bound to the abstractor)—see 6.2.3, 6.2.4 reference function: a function whose input and output are reference values; the extension of a functor is a reference function—see 6.1.7

reference value: the extension of an individual term (i.e., the object it refers to or a nil value if it does not refer)—see 6.1.3

referential opacity: the failure of referential transparency; a case where the extension of an expression depends on more than the reference value of an individual term appearing in it (so that we cannot simply "look through" the individual term to its reference value when considering the extension of the expression)—see 6.1.3

referential transparency: an occurrence of an individual term in some expression is referentially transparent when only the reference value of the individual term matters for the extension of the expression (so we can "look through" the individual term, ignoring features other than its reference, when considering the extension of the expression)—see 6.1.3

reflexivity: a relation is reflexive when it holds between anything and itself (e.g., the relation ≤ is reflexive, as is implication)—see 1.2.3, 6.3.1

*reject* (a conditional): to show that a conditional is false, something that involves showing that its antecedent is true and that its conclusion is false—*see* 5.4.1

*relation*: the extension of a predicate with more than one place; more broadly, a quality that something may possess only with respect to one or more other things—*see* 6.1.3—*compare with* property

relative complement—see complement

relative exhaustiveness—see exhaustiveness, relative

relative implication—see implication, relative

relative inconsistencysee inconsistency, relative

*relative scope*: the relation between the scopes of two operators (i.e., the portions of an expression that have been formed using them); if the scopes of operators overlap, one will be wider than the other—see 2.1.5

render exhaustive—see exhaustiveness, relative

residual resources of a path: accumulated resources that are not exploited (not even partially) at any stage in the development of a path—i.e., resources that always remain unexploited—see 7.7.4

**resource**: assumptions and conclusions derived from them that are drawn on in reaching the goal of a gap—see 2.2.3

restricted existential: a quantifier used to claim the existence of an example drawn from a restricted domain (and not merely from the full range of reference values); also a sentence formed by such a quantifier—see 8.1.1—compare with unrestricted existential

restricted universal: a quantifier used to state a generalization about a restricted domain; also a sentence formed by such a quantifier—see 7.2.1—compare with unrestricted universal

restricting formula: the formula used to restrict a restricted quantifier—e.g., the formula  $\rho x$  in the forms  $(\exists x: \rho x) \theta x$  and  $(\forall x: \rho x) \theta x$ ; when a quantifier is understood as only the operator  $\forall$  or  $\exists$  (an not in combination with an abstractor), it is the body of the first abstract to which the quantifier is applied—see 7.2.1—compare with quantified formula

restricting predicate: the extension of the predicate to which an unrestricted quantifier is applied or the second of the two predicates to which an unrestricted quantifier is applied—see 7.2.1, 8.1.1—compare with quantified predicate

restriction by a relation: a restriction of a generalization over pairs (or a claim of exemplification that would be contradictory to one) to pairs whose members stand in a certain relation (rather than possessing certain properties)—see 7.4.1

restrictive relative clause: a relative clause that serves to further specific what is being referred to (rather than merely providing further information about it)—see 6.1.8—compare with non-re-

strictive relative clause

right turnstile, double—see ⊨ under Symbols

*rule out* a possible world: a sentence or proposition rules out a possibility when it is false in that possibility—*see* 1.2.2

run-on conjunction: a multiple conjunction stated in abbreviated notation without any grouping specifying the relative scope of the conjunctions involved—see 2.1.5—see also multiple conjunction

run-on disjunction: a multiple disjunction (i.e., a compound analyzable as a disjunction at least one of whose components is a disjunction) that is stated in abbreviated notation without any grouping specifying the relative scope of the disjunctions involved—see 4.1.3

Russell, Bertrand (1872-1970)—see 8.4.2, 8.5.4

# S

safe rule: a rule is safe when any counterexample lurking in any of the child gaps it introduces was already lurking in the parent to which it is applied—see 2.3.4—compare with sound rule

salience: a feature of the context of use that serves to specify the reference of a definite description the most salient of the things fitting its description—see 8.4.1

scope of an assumption: the portion of a proof within which the assumption is available for use—see 2.4.1

scope of a connective or other operator: the portion of a sentence that was formed by the operator; when the sentence has been analyzed, this is the component of which the operator is the *main* operator—see 2.1.5

scope ambiguity: a ambiguity in which the alternative meanings are associated with analyses that differ in the relative scope of operators (e.g., Al spoke and walked or run might be a conjunction claiming he spoke no matter what or instead be a disjunction with speaking only part of the first disjunct)—see 7.1.1

scope line: a vertical line in a derivation that marks the scope of a goal and perhaps also one or more suppositions or the independent term of a general argument—see 2.2.3

second-order logic: logic employing an analysis of logical form that includes quantifiers for predicates or functors applying to individuals in addition to ordinary quantifiers for individuals—see 8.5.4—see also higher-order logic

securing a compound term: developing a derivation in a way that insures that the compound term is in the same alias set as some non-compound term—see 7.8.1

securing a definite description: developing a derivation in a way that insures that the definite description is in the same alias set as some non-compound term—see 8.6.2

semantic presupposition—see presupposition, semantic

semantics: aspects of meaning—such as the propositions expressed by sentences—which concern the way language represents the world; also the study of these aspects of meaning—see 1.3.2 compare with pragmatics

sentence: most strictly, an expression that has a truth value on any interpretation of its non-logical vocabulary—i.e., a closed formula; this is the main usage of the term beginning with §6.2 but most references to sentences in earlier parts of the text concern any expressions whose proper extensions are truth values and thus apply to all formulas (on exception is in discussions of lines of derivations which cannot have free variables)—see 6.2.2—see also formula, closed expression

*separation* of a sentence from a sentence:  $\phi$  is separated from  $\psi$  when  $\phi$  is true and  $\psi$  is false—*see* 1.2.3

separation of a set from a set:  $\Gamma$  is separated from  $\Sigma$  when every member of  $\Gamma$  is true (i.e., none is false) and every member of  $\Sigma$  is false—see 1.4.2

serial conjunction: a use of and in English to combine a series of phrases or clauses (rather than merely two)—see 2.1.5

- serial disjunction: a use of or in English to combine a series of phrases or clauses (rather than merely two)—see 4.1.3
- Sextus Empiricus (2nd cent.)—see 5.1.3
- shared content of a set  $\Gamma$ : the set of possible worlds in which every member of  $\Gamma$  is false (same as the content of  $\Gamma$  as a set of alternatives)—see 1.4.3
- shared coverage of a set  $\Gamma$ : the set of possible worlds in which every member of  $\Gamma$  is true (same as the coverage of  $\Gamma$  as a set of assumptions)—see 1.4.3
- singular term: an individual term (the label singular term is the standard contrast to general term)—see 6.1.6—compare with general term
- sound rule: a derivation rule is sound when any counterexample lurking in the gap to which it is applied and lurking also in all ancestors of that gap lurks in at least one of the gap's children—see 2.3.4, 7.7.2—see also strict rule, safe rule
- sound system of derivations: a system of derivations that provides no proof for any invalid argument—see 2.3.7, 7.7.1—compare with complete system of derivations
- speech act. an act performed in using language (e.g., assertion, commanding, promising); all performative verbs mark speech acts—see 1.3.2, 1.3.3
- square of opposition: a pattern formed of two non-equivalent sentences, on of which implies the other, and sentences that are contradictory to the two; traditionally, the square was used only for certain cases of implication—see 1.2.8
- Stalnaker, Robert C. (1940-)—see 5.1.3
- statement: a term sometimes used for a sentence as used in a particular context—see 1.3.5
- strict rule: a derivation rule is strict when any counterexample lurking in the gap to which it is applied lurks in at least one of the gap's children; all strict rules are sound but not all sound rules are strict—see 2.3.4, 7.7.2—see also sound rule, safe rule
- stronger claim: a claim that has great content (and thus leaves open fewer possibilities) than another—see 1.2.3—compare with weaker claim
- structure: the information needed to interpret logical forms when these may involve individual terms, predicates, and functors—see 6.4.1
- subjunctive conditional—see counterfactual conditional
- substantive existential commitment: although the existential commitments of a sentence (or set of assumptions) lie in any existential that is entailed, in a more specific sense, substantive existential commitments lie in entailed existentials whose claims of exemplification are restricted to non-nil reference values—see 8.1.5
- subtraction: a set X with a set Y subtracted is the collection of members of X that are not members of Y—that is, it is the complement of Y relative to X—see 7.1.4—see also complement
- sufficient system of derivations: a system of derivations is sufficient when every dead-end gap has a lurking counterexample; that is, a system is sufficient when it has enough rules to develop or close and open gap with no lurking counterexample—see 2.3.2
- sufficient condition: something that is enough for something else; some sufficient conditions are causal and others are epistemic—see 5.2.2—compare with necessary condition—and see also causal condition, epistemic condition
- supposition (as a sort of assumption): a starting point of an inference; supposition tends to be used in contrast with premise to suggest that an assumption is not accepted (instead merely considered for the sake of argument), but this should be regarded as only an implicature—see 2.4.1—see also assumption, premise
- supposition (as a form of reference): the medieval theory of supposition was designed to handle quantifier phrases by way of an account of forms of reference—see 7.1.1
- syllogism: in a broad sense, any argument with multiple premises (i.e., an argument that presents its conclusion as the result of drawing together premises in reasoning); in a narrow sense, a specific

sort of two-premised argument form involving generalizations or existential claims satisfying certain further conditions (which are restrictive enough to limit the syllogisms to only 15 valid patterns)—see 1.1.3, 7.1.1, 7.5.6, 8.5.4—see also immediate inference

symbolic logic: logic which uses symbolic notation for logical forms—see 1.1.8

symmetry: a symmetric relation that holds between objects in one direction holds the other direction as well—see 1.2.3. 6.3.1

*synthesis*: the generation of an English sentence from a logical form and interpretation of that form—see 2.1.8—compare with analysis

system of derivations: definitions concerning a "system of derivations" are intended to apply to a system of proof like the one studied in this text (i.e., with the same general organization but possibly different rules)—see 2.2.3

#### T

*Tautology*: the particular tautology T—see 1.2.5

tautology: a sentence that is false in no possible world (and therefore has no content) —see 1.2.5

tense logic: the logic of (non-truth-functional) connectives that reflect the modifications of sentences produce by tense—see 3.1.2

term: when used in a technical sense in this text, the word term (without qualification) is short for individual term—see 6.1.6

there-is existential: an existential sentence stated using the syntactic form there is ...—see 8.1.3 thing: anything that might be referred to—see 6.1.6

thorough system of derivations: a system for which it is true that any derivation that does not close has some path that develops fully—see 7.7.1

tilde equal—see ≃ under Symbols

*transitivity*: the property of a relation that holds directly whenever it holds in several steps (as is an ancestor of does but is a parent of does not); more precisely, a relation is transitive if it holds between objects a and b whenever there is some object c such that it holds between a and b and also between c and b—see 1.2.3, 6.3.1

*true of*: the extension of a predicate is not a truth value since the predicate is not a complete sentence, but the extension is the sort of thing that will yield a truth value when it is applied to enough reference values to fill its places—*see* 6.4.2

truth conditions: the conditions under which a sentence is true—i.e., a specification of the possible worlds in which it is true—see 1.2.2

truth function: a function whose input and output are truth values—see 2.1.2

truth-functional completeness: a set of connectives is truth-functionally complete when it can be used to express any truth function—see 3.1.4

truth-functional connective: a connective whose extension is a truth function; the truth value of a compound formed by a truth-functional connective will depend only on the truth values of its immediate components—see 3.1.2

truth-functional logic: the study of logical forms that can be specified using truth-functional connectives (including the "zero-place connectives" T and \(\percap \))—see 1.1.8, 3.1.2

truth table: a table describing the output of a truth function for at least some of its possible input—see 2.1.2

truth value: one of the values truth and falsity—see 1.2.1

type: a system of types is a classification of reference values and a corresponding classification of individual terms and restriction of predicates, functors, and other operators to certain types or combinations of types—see 6.1.7

*type theory*: a system of types in which the types are ordinary individuals, sets, sets of sets, and the like; type theory is closely related to higher-order logic—see 8.5.4

ultimate argument of a derivation: the argument for which a derivation is constructed and whose proof (or invalidation) is the ultimate aim of any of its later gaps—see 2.2.3

ultimate component: a component in an analysis that is not analyzed further—see 2.1.7

unanalyzed component: a component that is not analyzed—i.e., an ultimate component of an analysis—see 2.1.7

undefined term: an individual term whose reference value is a nil value—see 6.1.3

uniformly general exemplification: content of a claim asserting not merely that an example may be found in all cases of a certain sort but that in fact the same example will serve in all such cases (so the example may be chosen uniformly)—see 8.2.1

uniqueness (requirement for a definite description to refer): a definite description refers only when the description suffices to single out a single reference value—i.e., when some value fits it uniquely—see 8.4.1

universal instantiation: the transition from a universal to any of its instances—see 7.5.2

universal predicate: a predicate that is true of an reference values (e.g.,  $[x = x]_v$ )—see 7.2.1

universal quantifier. a quantifier used to represent the logical form of generalizations, marked symbolically by ∀—see 7.2.1

universal sentence: a sentence whose main operator is a universal quantifier—see 7.2.1

unless-conditional: a truth-functional compound that trims the content of one component to cases where the other is false (and thus extends its coverage to cases where that component is true); it may be expressed as an if-conditional with a negative antecedent—see 5.2.3—compare with if-conditional, only-if-conditional

unrestricted existential: a quantifier used to claim the existence of an example among the full range of reference values; also a sentence formed by such a quantifier—see 8.1.1—compare with restricted existential

unrestricted universal: a quantifier used to state a generalization about the full range of reference values; also a sentence formed by such a quantifier—see 7.2.1—compare with restricted universal

*up tack*—see ⊥ *under* Symbols

# V

*vacuous abstraction*: abstraction is vacuous when the abstractor binds no variables in the body (as in the abstract  $[T]_x$ )—see 6.2.1

vacuous generalization: a generalization that is empty of content because it generalizes about an empty collection of objects—see 1.4.2

*vagueness*: an aspect of meaning exhibited by terms that in some cases do not clearly apply or fail to apply; the range of application of a vague term thus has fuzzy boundaries—see 1.3.2, 1.3.6

*valid conclusion*: the conclusion of an argument is valid when it is entailed by the premises—*see* 

validity of an argument: an argument is valid when its premises entail its conclusion—see 1.1.6

validity, formal or in virtue of form—see formal validity

veil of ignorance: a scope line flagged by an independent term; the requirement that the term appear only to the right of the scope line is designed to insure that the argument to its right can draw no information about the term from outside—see 7.5.5

virgule—see / under Symbols

#### W

weakening: one of group of implications whose conclusions have less content their premises (although they are more complex forms)—see 4.3.2, 5.4.2—see also Wk under Rule labels weaker claim: a claim that has less content (and thus leaves open more possibilities) than another—see 1.2.3—compare with stronger claim
Wittgenstein, Ludwig (1889-1951)—see 1.2.6

# XYZ

yes-but answer. an affirmative answer with an attached qualification (the qualification may be added using the word but; however, it need not be)—see 1.3.4 yes-no question: a question that requests simply affirmative or negative response—see 1.3.4

# **Symbols**

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/ (virgule—U+002F)—see 1.1.3
\{a_1, ..., a_n\} (list notation for sets)—see 1.1.3
Ø (empty set—U+2205)—see 1.4.2
\models (double right turnstile—U+22A8)—see 1.1.6
\simeq (tilde equal—U+2243)—see 1.2.3
Δ (white up-pointing small triangle—U+25B5)—see 1.2.7
∇ (white up-pointing small triangle—U+25BF)—see 1.2.7
⋈ (bowtie—U+22C8)—see 1.2.7
T (down tack—U+22A4)—see 1.2.5
⊥ (up tack—U+22A5)—see 1.2.5
∧ (logical and—U+2227)—see 2.1.1
o (empty circle—U+25CB)—see 2.2.6

    (filled circle—U+25CF)—see 2.2.4

⊭ (negated double right turnstile—U+22AD)—see 2.3.1
¬ (not sign—U+00AC)—see 3.1.1
-± (negation or de-negation)—see 4.2.2
∨ (logical or—U+2228)—see 4.1.1
\rightarrow (rightwards arrow—U+2192)—see 5.1.1
← (leftwards arrow—U+2190)—see 5.1.1
∀ (for all—U+2200)—see 7.2.1
∃ (there exists—U+2203)—see 8.1.1
* (asterisk operator—U+2217)—see 8.4.3
l—see description operator
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zero-place predicate—see predicate, zero-place

# Laws

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⊥ as an alternative, law for—see 1.4.8

⊥ as a premise, law for—see 1.4.8

aliases, law for—see 6.3.2

alternatives via contradictory assumptions—see 1.4.6

associativity—see 2.1.2, 4.1.1

commutativity—see 2.1.2, 4.1.1

the conditional as a conclusion, law for—see 5.3.1

the conditional as a premise, law for—see 5.4.1

congruence for a functor—see 6.3.1

congruence for a predicate—see 6.3.1
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congruence for f-see 6.3.1 congruence for P-see 6.3.1 conjunction as a conclusion, law for-see 2.2.1 conjunction as a premise, law for—see 2.2.1 contravariance—see 3.1.2 covariance with the antecedent—see 5.1.2 covariance—see 2.1.2, 4.1.1 covariance with the consequent—see 5.1.2 Curry's law—see 5.1.2 De Morgan's laws—see 4.2.4 descriptions, law for-see 8.6.1 disjunction as a conclusion, law for—see 4.2.2 disjunction as a premise, law for—see 4.2.1 idempotence—see 2.1.2, 4.1.1 inconsistency via Absurdity—see 1.4.6 involution—see 3.1.2 lemmas, law for-see 1.4.7 modus ponendo ponens—see 5.3.2 modus ponendo tollens-see 4.3.1 modus tollendo ponens-see 4.3.1 modus tollendo tollens—see 5.3.2 monotonicity-see 1.4.7 negation as a conclusion, law for-see 3.2.1 negation as a premise, law for—see 3.2.1 non-contradiction, law of-see 3.2.2 premises, law for-see 1.4.7 T as a conclusion, law for—see 1.4.8 T as an alternative, law for—see 1.4.8 T as a premise, law for—see 1.4.8 unrestricted existential as a conclusion, law for—see 8.5.2 unrestricted existential as a premise, law for-see 8.5.1 unrestricted universal as a conclusion, law for—see 7.5.3 unrestricted universal as a premise, law for—see 7.5.2

# Rule labels

Adj (Adjunction)—see 2.4.4 CE (Co-alias Equation)—see 6.3.3 Cng (Congruence)—see 6.3.3 Cnj (Conjunction)—see 2.2.4 CP (Conditional Proof)—see 5.3.1 CR (Completing a Reductio)—see 3.3.2 DC (Distinguished Co-aliases)—see 6.3.3 EC (Equated Co-aliases)—see 6.3.3 EFQ (Ex Falso Quodlibet)—see 2.2.7 EG (Existential Generalization)—see 8.5.3 ENV (Ex Nihilo Verum)—see 2.2.7 Ext (Extraction)—see 2.2.4 IP (Indirect Proof)—see 3.3.1 Lem (Lemma)—see 2.4.2 LFR (Lemma for Reductio)—see 2.4.3 MPP (Modus Ponendo Ponens)—see 5.3.2

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MPT (Modus Ponendo Tollens)—see 4.3.1
MTP (Modus Tollendo Ponens)-see 4.3.1
MTT (Modus Tollendo Tollens)—see 5.3.2
Nc (Non-contradiction)—see 3.2.2
Nc= (Non-contradiction given Equations)—see 6.3.3
NcP (Non-constructive Proof)—see 8.5.3
PC (Proof by Cases)—see 4.2.1
PCh (Proof by Choice)—see 8.5.3
PCh+ (Supplemented Proof by Choice)—see 8.5.3
PE (Proof of Exhaustion)—see 4.2.2
RC (Rejecting a Conditional)—see 5.4.1
QED (Quod Erat Demonstrandum)—see 2.2.4
QED= (QED given Equations)—see 6.3.3
RAA (Reductio ad Absurdum)—see 3.2.2
SD (Securing a Description)—see 8.6.2
SD+ (Securing a Description Supplemented)—see 8.6.3
ST (Securing a Term)—see 7.8.1
UG (Universal Generalization)—see 7.5.5
UG+ (Supplemented Universal Generalization)—see 7.8.1
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UI (Universal Instantiation)—see 7.5.4 Wk (Weakening)—see 4.3.2, 5.4.2

Glen Helman 12 Jul 2012