7.3. Quantifiers and connectives

7.3.0. Overview

Many of the logical properties of quantifiers come from their interactions with other logical constants, both connectives and quantifiers; in this section, we will look at their interaction with connectives.

Glen Helman 11 Jul 2012

7.3.1. Generalizations and counterexamples

So far we have concentrated on quantifier phrases that are used to state generalizations, but there are others that are used to deny them. For example, Not every dog barks denies that the attribute of barking holds universally for the domain of dogs, and the phrase not every dog may always be used to deny affirmative generalizations concerning dogs. Another example of a denied generalization is Not only trucks were advertised, and the phrase not only trucks serves to deny negative generalizations concerning domains complementary to the class of trucks.

If we remove the common noun phrases from these quantifier phrases, we are left with the phrases not every and not only. These have a function comparable (though opposed) to that of the quantifier words every and only. Thus we might think of not every and not only signs indicating the kind of generalization being denied and develop an approach to the analysis of sentences containing them that is parallel to the one we developed for kinds of generalization. However, it is easier to deal with these sentences by treating the word not separately as a sign for negation (though it is worth noting that not functions in the non-standard way in such cases since it does not modify the main verb). So we can regard the sentences first as truth functional compounds and later analyze their negated components as generalizations. Here are the analyses

Not every dog barks
¬ every dog barks
¬ (∀x: x is a dog) x barks
¬ ∀x (x is a dog → x barks)

Not only trucks were advertised
¬ only trucks were advertised
¬ (∀x: ¬ x is a truck) ¬ x was advertised
¬ ∀x (¬ x is a truck → ¬ x was advertised)

Negations definitely pile up in the second. There are equivalent forms with fewer (think, for example, of the analysis of Not everything that was advertised was a truck) but such sentences miss some of the indirection of the original English.

Although we have extracted the word not from the units not every and not only in these analyses, it is grammatically a part of them. We might ask if there is a word or phrase that plays a role analogous to not every and not only in the case of negative direct generalizations. We can see more clearly the sort of expression this would be by recalling that to deny a generalization is to claim the existence of a counterexample to it. For example, Not every dog barks claims the existence of a dog that does not bark; and Not only new listings were distributed claims the existence of something other than a new listing that was nonetheless distributed. So, to find an expression that can be used to deny a negative direct generalization, we should look for an expression that can be used to claim the existence of counterexamples to such generalizations. Take the example No dog climbs trees. This says that the attribute of not climbing trees holds universally for dogs. A counterexample to such a claim would be a dog that does climb trees, so we are looking for a way of claiming that such a counterexample exists. English has an especially rich supply of ways of doing this, among them are Some dog climbs trees, There is a dog that climbs trees, and simply A dog climbs trees. None of these contain the word not, but they are still contradictory to generalizations and thus may be expressed as negations of those generalizations.

We might then treat a sentence using such an expression as negative and carry out the following sort of analysis:

Some dog climbs trees
¬ no dog climbs trees
¬ (∀x: x is a dog) ¬ x climbs trees
¬ ∀x (x is a dog → ¬ x climbs trees).

Although it is important to note that this analysis is possible, we will not have much occasion to employ it because we will introduce a more direct symbolic representation of claims like this in the next chapter. This is entirely analogous to something in truth-functional logic. We can express any disjunction φ or ψ using only conjunction and negation—as $\neg(\neg\varphi \land \neg\psi)$—but the sign $\lor$ provides a clearer representation.

The indefinite article a is one device used to make the claims that can be analyzed in this way; but, oddly, it also can be used to state direct affirmative generalizations. For example, Every dog barks could be restated as A dog barks. Thus A dog barks is ambiguous and might be interpreted as either Some dog barks or Every dog barks. However, the use of a to state generalizations occurs only in a certain grammatical contexts, and there would be no ambiguity in A dog climbed trees, which cannot be understood to state a generalization. But, however special in form, generalizations stated using the indefinite article are quite common in use and, since we will be studying this use of the indefinite article first, you may need to remind yourself that the indefinite article is also used to claim the existence of examples.

It may seem strange that the same word should have acquired two such different roles; but, from one perspective, they have something in common. In both uses, the indefinite article can be taken as a sign that a free choice may be made. The two uses differ in whether this choice lies with the speaker or the audience. If I state the generalization A dog likes bones, I claim that, whatever dog *you* pick, we will have something that likes bones. On the other hand, if I assert A dog was digging in the garden, I do not give you leave to choose any dog you please but claim only that it would be possible for *me* to pick a dog that was gardening.

Glen Helman 11 Jul 2012

7.3.2. Generalizations as components

We can wait to analyze sentences into predicates and individual terms until after we have completed all analysis by truth-functional connectives. On the other hand, we have already seen a number of cases where it is not possible to defer analysis as a generalization until all truth-functional connectives have been dealt with. For example, the sentence Everyone stood at the port or starboard rail is not a disjunction, and or can be dealt with only after we have analyzed it as a universal. Still, analysis by truth-functional connectives will often precede the analysis of generalizations into quantifiers and predicates, and we have already seen the simplest case of this: the denial of a generalization. We will now go on to consider some other examples.

In some cases, this sort of analysis is a straightforward matter. Here is an example:

Everyone was contacted, but no one responded

Everyone was contacted $\land$ no one responded

$(\forall x\text{: } x \text{ is a person})\ x \text{ was contacted}$

$\land\ (\forall x\text{: } x \text{ is a person})\ \neg\ x \text{ responded}$

$$(\forall x\text{: } Px)\ Cx \land (\forall x\text{: } Px)\ \neg\ Rx$$

$$\forall x\ (Px \to Cx) \land \forall x\ (Px \to \neg\ Rx)$$

C: [_ was contacted]; P: _ is a person; R: [_ responded]

The variable x is used in both generalizations here. This is quite legitimate since the pattern of binding can be understood as follows:

$$(\forall x\text{: } Px)\ Cx \land (\forall x\text{: } Px)\ \neg\ Rx$$

$$\forall x\ (Px \to Cx) \land \forall x\ (Px \to \neg\ Rx)$$

Since the abstractors have been absorbed in the quantifier-plus-variable ∀x, variables are bound to this expression under the same conditions that would lead them to be bound to an abstractor. The two occurrences of ∀x in each sentence apply to different expressions and can bind variables only in the expressions in their scopes, so neither can interfere with the operation of the other. Of course, this also means that the sentences could have been written just as well using different variables for the two quantifiers—e.g., as

$$(\forall x\text{: } Px)\ Cx \land (\forall y\text{: } Py)\ \neg\ Ry$$

$$\forall x\ (Px \to Cx) \land \forall y\ (Py \to \neg\ Ry)$$

It will depend on the context whether it is clearer to use the same letters for variables bound to different quantifiers or to use different letters, but you can expect to see examples of both sorts.

When generalizations appear as components of conditionals or disjunctions, it usually will be obvious that the sentence as a whole is a truth-functional compound. However, there are cases where an analysis as a conjunction is possible even though the sentence does not so clearly have this form. In particular, it is often possible to understand a generalization whose class indicator or quantified predicate is logically complex as a conjunction of generalizations that share a domain or share an attribute. For example, Everything is fine and dandy could be understood as a more compact equivalent of Everything is fine and everything is dandy. In making this restatement, we have repeated a quantifier phrase, and such a restatement does not always preserve meaning. However, in this case it does work and, in general, we can take a universal whose quantified predicate is formed by conjunction and restate it as a conjunction of universals. So we have the two alternative analyses:

$$\forall x\ (x \text{ is fine} \land x \text{ is dandy})$$
$$\forall x\ x \text{ is fine} \land \forall x\ x \text{ is dandy}$$

The first of these is preferable because it mirrors the form of the English sentence more closely, but the two are equivalent and we can claim a general equivalence between pairs of this sort:

$$\forall x\ (\rho x \land \theta x) \simeq \forall x\ \rho x \land \forall x\ \theta x.$$

A similar principle holds for the restricted universal quantifier.

It happens often enough that principles hold for both quantifiers that we will employ special notation to indicate that. Let us write "$(\forall x \ldots)$" to indicate the *possibility* of a restriction so that a quantifier $(\forall x \ldots)$ might take either of the forms $\forall x$ or $(\forall x\colon \rho x)$. Using this notation, we can write the more general principle as follows:

$$(\forall x \ldots)\ (\rho x \land \theta x) \simeq (\forall x \ldots)\ \rho x \land (\forall x \ldots)\ \theta x.$$

The ellipsis dots may be replaced by nothing at all here (as in the example above); but if they are replaced by something, they should be replaced by the same formula in all three occurrences.

Although the analysis as conjoined generalizations was not the most natural one in the case of Everything is fine and dandy, there is another sort of case where it is more natural. Consider the sentence All boys and girls are invited. This claims that the attribute of being invited holds universally for boys and also for girls. That is, it could be stated as a conjunction of two generalizations: All boys are invited ∧ all girls are invited. The sentence All boys and girls are invited can be analyzed also as a single generalization, but care must be taken in stating the restricting predicate. It must express membership in the class consisting of all boys and all girls; that is, we need a predicate that is true of any child of either sex. Of course, [_ is a child] would do; but if we are to employ the vocabulary of the original sentence, the best we can do is [x is a boy ∨ x is a girl]$_x$. Thus, we have the following pair of equivalent analyses:

$$(\forall x\text{: } Bx \lor Gx)\ Ix$$
$$(\forall x\text{: } Bx)\ Ix \land (\forall x\text{: } Gx)\ Ix$$

B: [_ is a boy]; G: _ is a girl; I: [_ is invited]

Here the second has the advantage of reflecting the use of and in the English sentence by a use of conjunction.

This pair of analyses is an instance of a general equivalence:

$$(\forall x\text{: } \rho x \lor \pi x)\ \theta x \simeq (\forall x\text{: } \rho x)\ \theta x \land (\forall x\text{: } \pi x)\ \theta x$$

It is enlightening to state this using unrestricted universal quantifiers

$$\forall x\ (\ (\rho x \lor \pi x) \to \theta x) \simeq \forall x\ (\rho x \to \theta x) \land \forall x\ (\pi x \to \theta x)$$

because we can then justify it by the following general equivalence for the conditional (which is closely associated with the idea behind proofs by cases):

$$(\varphi \lor \psi) \to \chi \simeq (\varphi \to \chi) \land (\psi \to \chi)$$

Together with the equivalence for the universal and conjunction noted above, this equivalence for the conditional and disjunction allows us to argue as follows:

$$\forall x\ (\ (\rho x \lor \pi x) \to \theta x) \simeq \forall x\ (\ (\rho x \to \theta x) \land (\pi x \to \theta x)\ )$$
$$\simeq \forall x\ (\rho x \to \theta x) \land \forall x\ (\pi x \to \theta x)$$

This argument locates the source of the change from disjunction to conjunction in the features of restricted universal generalizations that make them analogous to conditionals.

While it is possible to analyze All boys and girls are invited so that the word and in the class indicator turns out to mark the overall form of the sentence, things do not always work out like this—as the next few examples will show. Consider first a direct negative generalization with the same domain as the generalization above. Suppose, for example, we wish to say the property of having been forgotten fails for all boys and girls. We can state this as a conjunction of generalizations (e.g., No boy was forgotten and no girl was either)—or with a conjunctive class indicator if we make an affirmative generalization whose predicate incorporates negation (e.g., All boys and girls were unforgotten). But if we want a compound quantifier phrase using the quantifier word no, we will be forced to employ or—as in No boy or girl was forgotten. The closest we could come to this while using and with a negative quantifier word would be something like None of the boys and girls was forgotten. (The sentence No boys and girls were forgotten may *sound* fine, but its meaning is elusive.) An analysis of the negative generalization No boy or girl was forgotten as a universal quantification whose restricting predicate contains disjunction is probably the most natural one in this case because it preserves the connective appearing in the original sentence.

The conjoined noun phrase boys and girls can be used also in stating a complementary negative generalization—e.g., Only boys and girls are invited. The domain of this generalization is the class of everything that is not a boy or girl. This suggests the analysis

$$(\forall x: \neg (x \text{ is a boy} \lor x \text{ is a girl})) \neg x \text{ is invited}$$

$$(\forall x: \neg (Bx \lor Gx)) \neg Ix$$
$$\forall x\, (\neg (Bx \lor Gx) \to \neg Ix)$$

B: [_ is a boy]; G: _ is a girl; I: [_ is invited]

Of course, we could restate $\neg (Bx \lor Gx)$ as $\neg Bx \land \neg Gx$ by one of De Morgan's laws and in this way eliminate disjunction in favor of conjunction. The form we would get would be expressed more directly in English by Nothing that is not a boy and not a girl is invited. But the claim these sentences make cannot be analyzed as a conjoined pair of generalizations. In particular, the conjunction Only boys are invited ∧ only girls are invited is quite different in its implications: you could reasonably conclude from it that no one at all is invited.

Somewhat similar (and related) problems concern the quantified predicates of negative generalizations: compounding with and cannot be captured by a pair of conjoined generalizations while or gives rise to conjoined rather than disjoined ones. For example, No plane landed in either Detroit or Windsor amounts to No plane landed in Detroit ∧ no plane landed in Windsor; but it would be better to analyze it more directly as a single generalization whose quantified formula is a disjunction.

Glen Helman 11 Jul 2012

7.3.3. Any and every

We will conclude with some issues concerning the word any. It was noted in 3.1.3 that this word should be replaced (usually by some) when a sentence is analyzed as truth functional compound. Thus Tom didn't see anything becomes ¬ Tom saw something and If anyone backs out the trip will be canceled becomes Someone will back out → the trip will be canceled. But sentences containing any can—in most cases—also be understood to state direct affirmative generalizations and can be analyzed using a universal quantifier as the main logical operator. When they are seen in this way, the truth-functional structure that appears to give the overall form of the sentence will be confined to the quantified predicate. Thus the examples above could be analyzed as follows:

Tom didn't see anything
Everything is such that (Tom didn't see it)
∀x (Tom didn't see x)
∀x ¬ Tom saw x

$\forall x \neg Stx$

S: [_ saw _]; t: Tom

If anyone backs out, the trip will be canceled
Everyone is such that (if he or she backs out, the trip will be canceled)
(∀x: x is a person) (if x backs out, the trip will be canceled)
(∀x: Px) (x will back out → the trip will be canceled)

$(\forall x: Px)\ (Bx \rightarrow Ct)$
$\forall x\ (Px \rightarrow (Bx \rightarrow Ct))$

B: [_ will back out]; C: _ will be canceled; P: [_ is a person]; t: the trip

These analyses are, for the time being at least, preferable to analyses as truth-functional compounds since we do not yet have a perspicuous way of analyzing quantifier phrases containing some.

The indefinite article a is interchangeable with any in many cases like these—e.g., Tom didn't see a thing—so they constitute another sort of case (on top of those noted in 7.3.1) in which a may be used to state a generalization. (It's also true that any is interchangeable with a in many cases like those noted in 7.3.1—e.g., Any dog likes bones.) But a cannot be used very successfully in place of any in the second example above. Something like If even one person backs out, the trip will be canceled does work, but that is comparable to replacing anyone by someone.

It would be grammatical to put every in place of any in the examples above; but the meaning would be quite different, and the new meaning could be captured only by an analysis as truth-functional compounds:

Tom didn't see everything
¬ Tom saw everything
¬ ∀x (Tom saw x)

$\neg \forall x\ Stx$

S: [_ saw _]; t: Tom

If everyone backs out, the trip will be canceled
everyone will back out → the trip will be canceled
(∀x: x is a person) x will back out → the trip will be canceled

$(\forall x{:}\ Px)\ Bx \to Ct$
$\forall x\ (Px \to Bx) \to Ct$

B: [_ will back out]; C: _ will be canceled; P: [_ is a person]; t: the trip

These two sets of examples can be generalized to a rule of thumb: in contexts where any and every convey a different meaning, the significance of any can be captured by a generalization having a scope wider than some other operator while the significance of every will be captured by generalization having a scope narrower than this operator. The contexts in the examples above, negations and the antecedents of conditionals, are the most common ones where any and every convey different meanings; but we will encounter another such context in the next section. Contexts like these (along with some others where the operators are not ones we will study) are the chief contexts in which any can be used grammatically. Thus any can serve to avoid a potential ambiguity in the relative scope of generalization and other operators.

When operators of the relevant sorts are stacked up, any tends to mark wider scope than only the one of them with narrowest scope. For example, on its most natural interpretation,

If Tom didn't find anything, he was disappointed

amounts to

If everything is such that Tom didn't find it, he was disappointed so the generalization has a scope wider than the negation but narrower than the conditional. The statement made when the generalization has widest scope can be expressed using any, but it has a different form:

If there is anything that Tom didn't find, he was disappointed

We will look at the phrase there is in 8.1 . For now, it is enough to note that it permits us to use the relative clause that Tom didn't find. This clause serves grammatically to give any wider scope than the negation; and, as a result, the ability of any to assume a scope wider than some operator is held in reserve for the conditional.

There are other cases where we cannot analyze a sentence containing any as a truth functional compound even if we replace any by some. For example, If Alex hears anything, he'll tell us about it cannot be analyzed as a conditional because replacing the pronoun it by its antecedent would change the meaning; while it is not clear what claim is being made by If Alex hears anything, he'll tell us about anything, it is clear that it differs in meaning from the original sentence—as does If Alex hears something, he'll tell us about something. This means that we cannot get around the following analysis:

Everything is such that (if Alex hears it, he'll tell us about it)

∀x (if Alex hears x, he'll tell us about x)

∀x (<u>Alex</u> will hear x → <u>Alex</u> will tell <u>us</u> about x)

$\forall x\ (Hax \rightarrow Tasx)$

H: [_ will hear _]; T: _ will tell _ about _ ; a: Alex; s: us

Notice that this form is the restatement using an unrestricted universal of the restricted universal quantification $(\forall x\colon Hax)\ Tasx$. The latter symbolic form could turn up as the analysis of the sentence Alex will tell us about anything he hears, and this is a case where the word any cannot be replaced by some without changing the meaning (try it). In our original example, this replacement is possible (at least in colloquial speech), but it employs an exceptional use of some. The sentence we get—namely, If Alex hears something, he'll tell us about it—is used to state a generalization, not to claim the existence of an example.

7.3.s. Summary

1 The quantifier phrases not every and not only can be taken to mark negations of generalizations; they therefore cite the existence of counterexamples. Similarly, though less naturally, words like some and a can be taken to mark the negations of generalizations stated with no (although a may sometimes be used to the same effect as every).

2 Although some sentences containing both quantifier phrases and words marking connectives cannot be analyzed as truth-functional compounds, many can. It is clear how to do this when the sign for a connective is used to combine two separate generalizations, but the analysis may be more problematic in other cases. For example, every X and Y can be understood to indicate a conjunction of generalizations, but the same is true of no X or Y even though the connective is different. A claim of either sort can be analyzed as a single generalization, but its restricting predicate must then use disjunction (i.e., it amounts to the quantifier phrase everything that is X or Y). This recalls, and can be traced to, the properties of conjoined conditionals with a common consequent. Something similar happens when or appears in the quantified predicate of a negative generalization.

3 In sentences where any and every are alternatives that convey different meanings, the use of any can be understood to indicate a generalization whose scope is wider than some other operator, and the use of every will indicate a generalization whose scope is narrower than that same operator.

Glen Helman 11 Jul 2012

7.3.x. Exercise questions

1. Analyze the following in as much detail as possible:

a. Not everyone was enthusiastic but no one was disappointed.

b. Any defective unit will be repaired or replaced.

c. The bill will pass quickly if every member of the committee supports it.

d. Nothing suited both Ann and Bill.

e. Tom didn't sign up anyone; however, he didn't contact everyone.

f. If a bill arrives, it will be forwarded to you.

g. If the prize isn't won by anyone, it will be added to the next drawing.

h. Ralph looked in every closet and cabinet.

i. The alarm will sound if anyone who doesn't have the combination tries to open the door.

2. Synthesize idiomatic English sentences that express the propositions that are associated with the logical forms below by the intensional interpretations that follow them. In some cases, you will have a choice between carrying connectives into the final English sentence and capturing them by the type of generalization you use. Do the former when possible, but answers of both sorts will be given.

a. $\neg (\forall x: Lx)\ Gx$

G: [_ is gold]; L: _ glitters

b. $(\forall x: Dx \land Nxc)\ Bx \land (\forall x: Dx \land Nxc)\ Wx$

B: [_ barked]; D: _ is a dog; N: [_ was in _]; W: _ wagged x 's tail; c: the cage

c. $\forall x \neg Ltxt$

L: [_ let _ stop _]; t: Tom

d. $(\forall x: Px \land \neg Rx) \neg Fx$

F: [_ is finished]; P: _ is a federal project; R: [_ is a road]

e. $\forall x\ (Oxr \to Gx)$

G: [_ is gone for good]; O: _ was left on _ ; r: the roof

f. $(\forall x: Px \land Mtx)\ (Ktx \lor Kxt)$

K: [_ knew _]; M: _ met _ ; P: [_ is a person]; t: Tom

7.3.xa. Exercise answers

1. **a.** Not everyone was enthusiastic but no one was disappointed

Not everyone was enthusiastic ∧ no one was disappointed

¬ everyone was enthusiastic ∧ (∀x: x is a person) ¬ x was disappointed

¬ (∀x: x is a person) x was enthusiastic ∧ (∀x: x is a person) ¬ x was disappointed

$$\neg (\forall x: Px)\ Ex \land (\forall x: Px) \neg Dx$$
$$\neg \forall x\ (Px \rightarrow Ex) \land \forall x\ (Px \rightarrow \neg Dx)$$

D: [_ was disappointed]; E: _ was enthusiastic; P: [_ is a person]

b. Any defective unit will be repaired or replaced

(∀x: x is a defective unit) x will be repaired or replaced

(∀x: x is a unit ∧ x is defective) (x will be repaired ∨ x will be replaced)

$$(\forall x: Ux \land Dx)\ (Px \lor Lx)$$
$$\forall x\ ((Ux \land Dx) \rightarrow (Px \lor Lx))$$

D: [_ is defective]; L: _ will be replaced; P: [_ will be repaired]; U: _ is a unit

c. The bill will pass quickly if every member of the committee supports it

The bill will pass quickly ← every member of the committee will support the bill

Pb ← (∀x: x is a member of the committee) x will support the bill

$$Pb \leftarrow (\forall x: Mxc)\ Sxb$$
$$(\forall x: Mxc)\ Sxb \rightarrow Pb$$
$$\forall x\ (Mxc \rightarrow Sxb) \rightarrow Pb$$

M: [_ is a member of _]; P: _ will pass quickly; S: [_ will support _]; b: the bill; c: the committee

d. Nothing suited both Ann and Bill.

∀x ¬ x suited both Ann and Bill

∀x ¬ (x suited Ann ∧ x suited Bill)

$$\forall x\ \neg (Sxa \land Sxb)$$

S: [_ suited _]; a: Ann; b: Bill

e. Tom didn't sign up anyone; however, he didn't contact every-one

Tom didn't sign up anyone ∧ Tom didn't contact everyone

everyone is such that (Tom didn't sign up him or her) ∧ ¬ Tom contacted everyone

(∀x: x is a person) ¬ Tom signed up x ∧ ¬ (∀x: x is a person) Tom contacted x

$$(\forall x\text{: } Px)\ \neg\ Stx \wedge \neg\ (\forall x\text{: } Px)\ Ctx$$
$$\forall x\ (Px \rightarrow \neg\ Stx) \wedge \neg\ \forall x\ (Px \rightarrow Ctx)$$

C: [_ contacted _]; P: _ is a person; S: [_ signed up _]

f. If a bill arrives, it will be forwarded to you

Every bill is such that (if it arrives, it will be forwarded to you)

(∀x: x is a bill) (if x arrives, x will be forwarded to you)

(∀x: Bx) (x will arrive → x will be forwarded to you)

$$(\forall x\text{: } Bx)\ (Ax \rightarrow Fxo)$$
$$\forall x\ (Bx \rightarrow (Ax \rightarrow Fxo))$$

A: [_ will arrive]; B: _ is a bill; F: [_ will be forwarded to _]; o: you

g. If the prize isn't won by anyone, it will be added to the next drawing

the prize won't be won by anyone → the prize will be added to the next drawing

everyone is such that (the prize won't be won by him or her) → Apn

(∀x: x is a person) the prize won't be won by x → Apn

(∀x: Px) ¬ the prize will be won by x → Apn

$$(\forall x\text{: } Px)\ \neg\ Wpx \rightarrow Apn$$
$$\forall x\ (Px \rightarrow \neg\ Wpx) \rightarrow Apn$$

A: [_ will be added to _]; P: _ is a person; W: [_ will be won by _]; n: the next drawing; p: the prize

h. Ralph looked in every closet and cabinet

Ralph looked in every closet ∧ Ralph looked in every cabinet

(∀x: x is a closet) Ralph looked in x ∧ (∀x: x is a cabinet) Ralph looked in x

$$(\forall x: Sx)\ Lrx \wedge (\forall x: Bx)\ Lrx$$
$$\forall x\ (Sx \rightarrow Lrx) \wedge \forall x\ (Bx \rightarrow Lrx)$$
$$\textit{or: } (\forall x: Sx \vee Bx)\ Lrx$$

B: [_ is a cabinet]; L: _ looked in _ ; S: [_ is a closet]; r: Ralph

i. The alarm will sound if anyone who doesn't have the combination tries to open the door

everyone who doesn't have the combination is such that (the alarm will sound if he or she tries to open the door)

(∀x: x is a person who doesn't have the combination) the alarm will sound if x tries to open the door

(∀x: x is a person ∧ x doesn't have the combination) (the alarm will sound ← x will try to open the door)

(∀x: x is a person ∧ ¬ x has the combination) (Sa ← Txd)

$$(\forall x: Px \wedge \neg Hxc)\ (Sa \leftarrow Txd)$$
$$(\forall x: Px \wedge \neg Hxc)\ (Txd \rightarrow Sa)$$
$$\forall x\ ((Px \wedge \neg Hxc) \rightarrow (Txd \rightarrow Sa))$$

H: [_ has _]; P: _ is a person; S: [_ will sound]; T: _ will try to open _ ; a: the alarm; c: the combination; d: the door

2. **a.** ¬ (∀x: x glitters) x is gold

¬ everything that glitters is gold

Not everything that glitters is gold

or: All that glitters is not gold

However, negating the main the verb is not always the clearest way of denying a generalization; for example, Everyone was not in the best of moods could be understood either as saying that not everyone was in the best of moods or as saying that no one was.

Note also that we here treat the restricting predicate x glitters as if it were x is a thing that glitters; this sort of use of the class indicator thing is always possible when the restricting predicate does not already provide a common noun.

b. (∀x: x is a dog ∧ x was in the cage) x barked ∧ (∀x: x is a dog ∧ x was in the cage) x wagged x's tail

(∀x: x is a dog that was in the cage) x barked ∧ (∀x: x is a dog that was in the cage) x wagged x's tail

Every dog that was in the cage barked ∧ every dog that was in the cage wagged it's tail

Every dog in the cage barked, and each wagged it's tail

or: Every dog in the cage barked and wagged it's tail

However, the latter sentence would be more naturally analyzed as having the form (∀x: Dx ∧ Nxc) (Bx ∧ Wx).

c. ∀x ¬ Tom let x stop Tom

∀x Tom didn't let x stop him

Tom didn't let anything stop him

or: Tom let nothing stop him

d. (∀x: x is a federal project ∧ ¬ x is a road) ¬ x is finished

(∀x: x is a federal project that is not a road) x is unfinished

Every federal project that is not a road is unfinished

or: No federal projects except roads are finished

The latter approach—capturing the negation by a negative generalization—helps to avoid ambiguity in cases were an explicit negation would have to apply to the main verb, as in (∀x: x is a federal project ∧ ¬ x is a road) ¬ x is under way

e. ∀x (x was left on the roof → x is gone for good)

∀x (if x was left on the roof then x is gone for good)

If anything was left on the roof then it is gone for good

or: Anything that was left on the roof is gone for good

f. (∀x: x is a person ∧ Tom met x) (Tom knew x ∨ x knew Tom)

(∀x: x is a person Tom met) Tom knew or was known by x

Tom knew or was known by everyone he met