5.2. Only if and unless

5.2.0. Overview

The simple conditional is one of a group of connectives whose other members can be expressed using it together with negation.

5.2.1. Only if

If the simple conditional trims the content of an unconditional assertion, a second sort of conditional offers a trimmed denial, ruling out the truth of the main clause in cases where the subordinate clause is false.

5.2.2. Necessary and sufficient conditions

The implicatures of an only-if conditional are associated with the idea of necessary conditions while the implicatures of an if conditional are associated with the idea of sufficient conditions.

5.2.3. Unless

Although it may be embellished with implicatures, the basic content of unless is provided by the phrase if not, a common dictionary definition for it.

5.2.4. Three forms compared

The implicatures associated with conditionals can make it difficult to distinguish the three conditionals but, once they are distinguished, some mnemonic devices point to their symbolic forms.

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5.2.1. Only if

The bare word if is not the only way of making a conditional claim. Compare the following forecasts:

It will rain tomorrow if the front moves through.
It will rain tomorrow only if the front moves through.

The first was our original example of hedging a claim with an if-clause. The second differs in the substitution of only if for if. This makes quite a difference, though, for the second does not hedge the claim that it will rain but instead puts up a fence around it by placing a limit on the cases in which it might be true. While the first conditional leaves open some possibilities its main clause rules out, the second rules out some possibilities that its main clause leaves open. A forecaster who asserts the second sentence is committed to it *not* raining in cases where the front does not move through. That is, the force of only if is to offer a limited denial of the main clause rather than a limited assertion of it.

These considerations suggest the table below for sentences of the form ψ only if φ, sentences we will speak of as *only-if-conditionals*. This conditional form rules out ψ in cases where φ fails; that is, ψ only if φ is false only in a case where ψ is true even though φ is false. This means that, where the condition φ holds, the claim cannot go wrong. The form ψ only if φ thus provides information only about cases where φ fails and, in these, its truth value is opposite that of ψ. This is what makes it a limited denial of ψ; it rules out possibilities left open by ψ, but it rules out only those in which the condition φ does not hold. Or to put it in still other terms, it limits the truth of ψ to cases where φ is true; it does not assert ψ in those cases but excludes it in others.

φ	ψ	ψ only if φ
T	T	T
T	F	T
F	T	F
F	F	T

Diagrams of propositions may be of some help here, too. Figure 5.2.1-1 should be compared to Figure 5.1.2-1 and also to Figure 3.1.2-1 . In the example we have been using, 5.2.1-1B represents the proposition expressed by The number shown by the die is less than 4 only if it is odd.

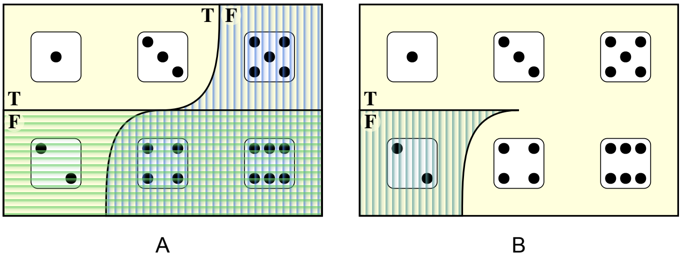

Fig. 5.2.1-1. Propositions expressed by two sentences (A) and an only-if-conditional (B) whose main clause leaves open the possibilities at the left in A.

Like the if-conditional, the only-if-conditional is a weak claim, leaving open possibilities in three of the four regions shown in Figure 5.2.1-1A; but it narrows the possibilities left open by the main clause (the area at the left in 5.2.1-1A) to those also left open by the subordinate clause. This is the reason for saying the function of an only-if-conditional is to fence in. Comparison with Figure 2.1.2-1 shows that it provides exactly the further information needed to move from the possibilities left open by the main clause ψ to the narrower range left open by $\psi \wedge \varphi$.

We will not introduce a new symbol for the connective marked by only if. A claim of the form ψ only if φ can be seen as a claim $\neg\,\psi$ hedged to be conditional on the truth of $\neg\,\varphi$; and that means we can express ψ only if φ as $\neg\,\psi \leftarrow \neg\,\varphi$. (You should check that this form has the correct table.)

Because the only-if-conditional is analyzed using the simple conditional and negation, there is no need to state further principles of implication or equivalence for it. But there is one consequence for it of the principles for the simple conditional and negation that is worth noting. While ψ if φ is covariant with its main clause ψ and contravariant with its subordinate clause φ, the conditional ψ only if φ will have the opposite relation to its component clauses (because they are negated before being combined with a simple conditional). This is in keeping with the role of ψ only if φ as a limited denial of ψ. It says more as ψ says less because a weaker claim is harder to deny; and it says more as φ says more because strengthening φ narrows the range of possibilities to which the truth of ψ has been limited. For example, The package will arrive in the next week only if you pay extra says more than does The package will arrive tomorrow only if you pay extra while The package will arrive in the next week says less than does The package will arrive tomorrow. And strengthening You will pay extra to You will pay a lot extra changes The package will arrive tomorrow only if you pay extra to the stronger The package will arrive tomorrow only if you pay a lot extra.

While the interpretation of English sentences stated using only if raises most of the same issues as if-sentences, these arise with different severity and in different ways. For example, it is possible to move an only-if-clause to the front of a sentence, but this is done only in rather formal contexts. The sentence Only if the front moves through will we have rain tomorrow is perfectly grammatical, but you would not expect it to be used by a television weather forecaster. And, while there are only-if-conditionals in the subjunctive that we must leave unanalyzed (for example, We would be able to see the eclipse only if we were near the equator), they are less common than subjunctive if-conditionals. Only-if-conditionals in English do have one special feature that is linked to the use of negations in their analysis. It is rare for any sort of conditional to be negated in English, perhaps because of the difficulty of knowing what to make of the implicatures in that sort of context. Now any conditional appearing as either component of an only-if-conditional would not negated on our analysis and, in fact, it is also very rare for a conditional to appear as a component of an only-if-conditional.

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5.2.2. Necessary and sufficient conditions

Like if-conditionals, only-if-conditionals in the indicative voice carry implicatures, but their implicatures are different. This difference can be captured by the phrases *necessary condition* and *sufficient condition*. Consider the following sentences:

The match burned only if oxygen was present.
The match burned if it was struck.

Each carries, as an implicature, the suggestion of a connection between the burning of the match and some other state or event. In the first, the suggestion is that the presence of oxygen was required for the match to burn, that it was a necessary condition without which combustion could not occur. The suggestion of the second is that the striking of the match would have been enough for it to burn, that it would have been a sufficient condition. These necessary and sufficient conditions might be described as *causal*; they concern states whose absence can prevent an event from occurring or other events which are enough to bring it about.

Another kind of necessary and sufficient conditions could be described as *epistemic* since they concern grounds for reasonable belief. For example, we might say this.

If the match burned, oxygen was present.

In making this assertion, we do not mean to suggest that the burning of the match would have brought about the presence of oxygen but rather that the burning would be evidence of oxygen's presence. Combustion would give us sufficient grounds for believing that oxygen was present, so it is epistemically sufficient. On the other hand, we might say this:

The switch was thrown only if the light was on.

To see the force of this example, suppose it is known that the switch is in a different room from the light. The sentence would not suggest that the light was required for the switch to be thrown but rather that the light being on served as a test of the belief that the switch was thrown. That is, seeing that the light was not on would lead us to reject a belief that the switch was thrown, so it is an epistemically necessary condition for the belief. Epistemic conditions of both sorts are sometimes referred to as *signs* or *marks*.

Now, statements of necessary and sufficient conditions can themselves be understood as connectives, ones that we might express more explicitly in the following way:

The truth of φ is a necessary condition for the truth of ψ
The truth of φ is a sufficient condition for the truth of ψ.

A compound of either of these forms is plainly not truth-functional. Knowing, for example, that φ and ψ are both true will not tell us whether either is a necessary or a sufficient condition for the other. So necessary and sufficient conditions are not strictly within our purview. But, since they attach to indicative if- and only-if-conditionals as implicatures, we need to be aware of them because they can make certain ways of restating such conditionals more natural than others.

When checking that the form $\neg\,\psi \leftarrow \neg\,\varphi$ has that same truth table as ψ only if φ, you may have noticed that the simpler form $\psi \rightarrow \varphi$ also has the same table. This might suggest that as a first step in analyzing ψ only if φ we could rephrase it as If ψ then φ. However, to do so would often wreak such havoc on the implicatures that the paraphrase would sound crazy. In saying ψ only if φ, we suggest that the truth of φ is a necessary condition for the truth of ψ while in saying If ψ then φ, we suggest that the truth of ψ is a sufficient condition for the truth of φ. And sufficiency and necessity are not simple converse relations like *parent of* and *child of*.

As we saw in the examples above, a causally sufficient condition for an event may have the event as an epistemically necessary condition, and a causally necessary condition may have the event as an epistemically sufficient condition. However, in making such shifts we are changing the meaning of a sentence in a noticeable way, so a paraphrase of ψ only if φ by If ψ then φ will be at best awkward. This awkwardness becomes especially severe in the case of conditionals concerning the future, where causal and epistemic conditions tend to coincide. A meteorologist would certainly not be prepared to use the following interchangeably:

It will rain tomorrow only if the front moves through.
If it rains tomorrow, the front will move through.

We could do a bit better in this case by adjusting tenses to get If it rains tomorrow, then front will have moved through, but we would still have shifted from causal to epistemic implicatures.

The analysis only-if-conditionals that we do employ amounts to a paraphrase of ψ only if φ by It's not the case that ψ if it's not the case that φ. And this paraphrase tends to avoid such problems with implicatures. But it only *tends* to avoid them because our description of the implicatures of if- and only-if-conditionals in terms of necessary and sufficient conditions is still an oversimplified account of the relation between them.

For example, I might express my conviction that the temperature is high by using the sentence It's under 80° only if it's over 75°. Here the paraphrase If it's under 80°, it's over 75° works well even though it is the sort of paraphrase that failed in earlier examples; and a paraphrase of the sort we used in those examples—namely, If it isn't over 75°, then it isn't under 80°—sounds at least odd. The oddity here can be explained in a way that suggests it does not point to a widespread problem. It being over 75° could be a necessary condition for it being under 80° only if we take it for granted that it is hot. And the point of the initial sentence is more to commit the speaker to this presumption than to suggest the existence of a necessary condition. But the sentence If it isn't over 75°, then it isn't under 80° cannot play this role since it pointedly leaves open just the sort of case whose failure the original sentence is designed to suggest.

This sort of example shows that the implicatures of if- and only-if-conditionals can be sufficiently independent that the latter cannot be expressed in terms of the former. However, if we paraphrase using negation (rather than reversing main and subordinate clauses), the difference in implicatures will usually not be too great. The moral for our purposes is then that a paraphrase of ψ only if φ by $\neg\,\psi$ if $\neg\,\varphi$ will usually not be too jarring though if ψ then φ may be better in a few cases.

There is a final complication in dealing with if and only if that it is also a result of their implicatures. Conditionals of the two sorts can often be difficult to distinguish because a conditional of one sort carries a conditional of the other sort as an implicature. For example, imagine I were speaking of a farm in a year when corn yields have been affected by drought. If I were to assert the sentence

They will make a profit only if they get over $6.00 a bushel,

I would be understood to believe not only that this price was necessary for a profit but also that it was sufficient, and it seems that I would agree with the following:

They will make a profit if they get over $6.00 a bushel.

But this is only an implicature and, unlike the suggestion that the price is a necessary condition for making a profit, the suggestion that it is also sufficient is one that is easily canceled. If I wanted to avoid the implicature, I might have used the sentence

> They will make a profit only if they get over $6.00 a bushel, and even that might not be enough

and I would not have contradicted myself by saying this.

Moreover, the implicature of an if-conditional by an only-if-conditional, or vice versa, does not always arise. We would usually take the forecast It will rain tomorrow only if the front moves through to suggest that the passing of the front would produce rain; but during a severe drought, when rain seems very unlikely, a forecaster might not need to add the canceling clause and it might stay dry even if the front does move through. So, while implicatures may conceal the difference between them, ψ if φ and ψ only if φ really are different in content from each other.

This means that the assertion of both conditionals, as in the form ψ if and only if φ, is not redundant. This sort of compound is known as the *biconditional*. Its analysis would lead us to the form

$$(\psi \leftarrow \varphi) \land (\neg\, \psi \leftarrow \neg\, \varphi)$$

or, with rightwards arrows,

$$(\varphi \rightarrow \psi) \land (\neg\, \varphi \rightarrow \neg\, \psi)$$

Biconditionals appear often in definitions, and calculating the truth table for this form will show why. A biconditional is true when the components φ and ψ are both true and also when they are both false, so this form enables us to say that two sentences have the same truth value without saying what that value is.

5.2.3. Unless

Yet another sort of conditional appears in this example:

They have run out of food unless they received new supplies.

Here the main clause is hedged, but in a different way than if the subordinate clause were introduced by if. The speaker's intent is to leave open some cases where the main clause fails (where they still have food) but to limit this failure to the sort of situation described in the subordinate clause. We can compare the function of unless to the function of only if by paraphrasing the sentence above as

They still have food only if they received new supplies.

The second sentence limits the truth of They still have food to cases where The received new supplies is true. So it asserts the truth of They have run out of food with the possible exception of such cases. Similarly, the first sentence asserts They have run out of food but hedges this by allowing the exception expressed by the subordinate clause.

φ	ψ	ψ unless φ
T	T	T
T	F	T
F	T	T
F	F	F

So, like an only-if-conditional, an *unless-conditional* is automatically true in cases where the subordinate clause is true; but unlike an only-if-conditional its truth value is the same as the main clause in cases where the subordinate clause is false. That is, the form ψ unless φ has the table at the right. This account of truth conditions appears also in Figure 5.2.3-1. Continuing the example of these diagrams, 5.2.3-1B represents the proposition expressed by The number shown by the die is less than 4 unless it is odd.

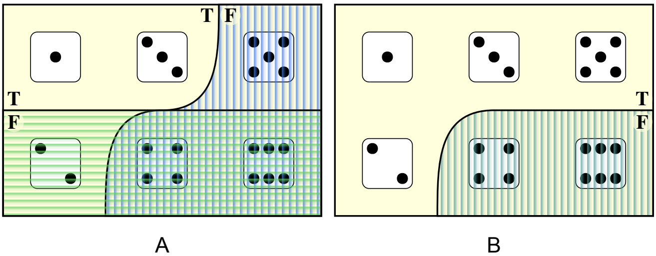

Fig. 5.2.3-1. Propositions expressed by two sentences (A) and an unless-conditional (B) whose main clause rules out the possibilities at the right in A.

There are two ways of describing the proposition on the right. First of all, it fences in the failure of the main clause. That is, it rules out some of the possibilities in which the main clause ψ fails, those that are ruled out by the subordinate clause φ. This is to see the conditional as the proposition expressed by $\neg\,\psi$ only if φ. But the possibilities left open by the denial of the main clause are those ruled out by the main clause itself. So the conditional can be seen also to whittle down the possibilities ruled out by the main clause to those left open by the denial of the subordinate clause. And this is to see the conditional as the hedge of the main clause expressed by ψ if $\neg\,\varphi$.

The same restatements appear if we trace our way back to if-conditionals in order to get a way of expressing this conditional symbolically. The form ψ unless φ amounts to $\neg\,\psi$ only if φ and we are treating the latter as $\neg\,\neg\,\psi \leftarrow \neg\,\varphi$. If we use the principle of double negation to simplify this last expression, we get $\psi \leftarrow \neg\,\varphi$ as a rendering of ψ unless φ. The corresponding English paraphrase of ψ unless φ as ψ if it is not the case that φ is usually pretty good (good enough that if not is a common dictionary definition of unless).

The negation used to analyze the subordinate clause of only-if-conditionals means that they are covariant with both their clauses. That will be no surprise if you have noticed that they have the same truth conditions as disjunctions, but it is also to be expected if it is regarded as an assertion of the main clause with the subordinate clause as a possible exception. Such a claim will say more as the main clause says more, and it will say more also as the subordinate clause says more because a narrower exception will apply in fewer cases.

There are enough steps in the path from unless to $\leftarrow\neg$ to justify a fear that the implicatures are not all in order when we arrive, but this account of unless works better than using or. How far the synonymy of unless and or extends beyond truth conditions can be seen by considering a few examples. We might paraphrase the example above as

Either they have run out of food, or they received new supplies

and we would do so with reasonable success. But things do not work out as well in other cases, particularly with unless-conditionals concerning the future. The following two sentences have quite different implicatures:

We'll run out of gas unless we get to a town soon.
We'll either run out of gas or get to a town soon.

Disjunction is not symmetric when it comes to an implicated connection between its two components, and we could paraphrase the first sentence better by We'll either get to a town soon or run out of gas, but the need to change the order of the clauses reduces the advantages of or over if not as a paraphrase of unless.

The remaining issues regarding unless pretty well parallel those concerning if and only if. It is possible to find an unless-clause at the front of a sentence (e.g., Unless we get to town soon, we'll run out of gas). And the form ψ unless φ has, in addition to its core implicature that the truth of φ is necessary for the falsity of ψ, a secondary and easily canceled implicature of sufficiency. In our initial example (They have run out of food unless they received new supplies), this secondary implicature is rather weak if it is present at all, so there might be no need to add the canceling clause and they might have run out even if they got them. But, in other cases, the implicature is stronger. For example, in We'll go unless it rains, we would have to add and we might go even if it does if we did not want to suggest that rain would be enough to keep us from going.

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5.2.4. Three forms compared

Before going on to work through some sample analyses, let us bring together the key points about the three connectives:

English forms	Symbolic analyses	Truth conditions	Core implicatures	Secondary implicatures
ψ if φ if φ, ψ	$\psi \leftarrow \varphi$ $\varphi \rightarrow \psi$	same value as ψ when φ is **T**; otherwise **T**	φ is sufficient for ψ's truth	φ is necessary for ψ's truth
ψ only if φ	$\neg\,\psi \leftarrow \neg\,\varphi$ $\neg\,\varphi \rightarrow \neg\,\psi$	opposite value to ψ when φ is **F**; otherwise **T**	φ is necessary for ψ's truth	φ is sufficient for ψ's truth
ψ unless φ unless φ, ψ	$\psi \leftarrow \neg\,\varphi$ $\neg\,\varphi \rightarrow \psi$	same value as ψ when φ is **F**; otherwise **T**	φ is necessary for ψ's failure	φ is sufficient for ψ's failure

The core implicatures are the ones that can make an indicative conditional seem non-truth-functional. The secondary implicatures are the ones that can make it difficult to distinguish between different kinds of conditional. The latter implicatures are easily canceled.

It may help, when trying to recall the symbolic analysis of only if, that in response to the question Did they finish? the answers Only if the parts arrived and Not unless the parts arrived come to pretty much the same thing (give or take a few implicatures). Combining this idea with the paraphrase of unless as only if, we get the formula not if not for only if—that is, ψ only if φ amounts to Not ψ if not φ or $\neg\,\psi \leftarrow \neg\,\varphi$.

Here are some examples involving only if and unless.

If Dave didn't show up, they moved the piano only if it was a small one

Dave didn't show up $\rightarrow$ they moved the piano only if it was a small one

$\neg$ Dave showed up $\rightarrow$ ($\neg$ they moved the piano $\leftarrow$ $\neg$ the piano was a small one)

$$\neg\,D \rightarrow (\neg\,M \leftarrow \neg\,S)$$
$$\neg\,D \rightarrow (\neg\,S \rightarrow \neg\,M)$$

if not D then if not S then not M

D: Dave showed up; S: the piano was a small one; M: they moved the piano

Mike didn't hear from either Sue or Tom unless a call came through late

Mike didn't hear from either Sue or Tom ← ¬ a call came through late

¬ Mike heard from either Sue or Tom ← ¬ a call came through late

¬ (Mike heard from Sue ∨ Mike heard from Tom) ← ¬ a call came through late

$$\neg (S \vee T) \leftarrow \neg L$$

$$\neg L \rightarrow \neg (S \vee T)$$

if not L then not either S or T

L: a call came through late; S: Mike heard from Sue; T: Mike heard from Tom

Notice that the form assigned to the second example would do as well for Mike heard from either Sue or Tom only if a call came through late, a sentence that is a fair paraphrase of the one we analyzed. The first example shares its form with Unless Dave showed up, they moved the piano only if it was a small one, also a reasonable paraphrase.

In general, the forms marked by unless conditionals can also be expressed by simple conditionals, and the form marked by an only-if conditional can be expressed by any of the three English forms. That means that there can be a number of different ways of synthesizing an English sentence with a given form. For example, the truth conditions of the analyzed sentence

¬ they ate outside ← ¬ it was warm

can be expressed by any of the following:

They didn't eat outside if it wasn't warm.

They didn't eat outside unless it was warm.

They ate outside only if it was warm.

And the differences among implicatures in this case are limited enough that these sentences would be equally appropriate in many situations.

5.2.s. Summary

1 The simple if-conditional is not the only conditional in English. The phrase only if is used to mark a compound which limits the possibilities for the truth of its main clause. It does this by asserting a denial of the main clause that is conditional on the failure of the subordinate clause, so it can be thought of as a hedged denial. As this suggests, the only-if-conditional can be paraphrased using the if-conditional and negation, with ψ only if φ expressed symbolically as $\neg \psi \leftarrow \neg \varphi$.

2 Like the if-conditional the only-if-conditional has implicatures. It suggests that the truth of its subordinate clause is a necessary condition for the truth of its main clause (while the if-conditional suggests that the truth of the subordinate clause is a sufficient condition). There is a secondary implicature of each conditional in which it suggests the truth of the other conditional, and this can make each seem to say that same thing as a conjunction of the two, a compound known as a biconditional. However, these secondary implicatures are easily canceled. The biconditional ψ if and only if φ can be expressed symbolically as $(\psi \leftarrow \varphi) \land (\neg \psi \leftarrow \neg \varphi)$, or $(\varphi \rightarrow \psi) \land (\neg \varphi \rightarrow \neg \psi)$ when arrows are reversed.

3 A third sort of conditional is marked by the English word unless. It hedges the main clause by asserting a limitation on the possibility of its failure, saying this can happen only when the subordinate clause is true. The effect is to assert the main clause conditional on the denial of the subordinate clause, and the unless-conditional can be stated using the if-conditional and negation, with ψ unless φ expressed as $\psi \leftarrow \neg \varphi$. Like the other two conditionals, the unless-conditional carries implicatures, both core implicatures and easily canceled secondary ones.

4 The symbolic analyses of the conditionals can be captured by the rough formulas: only if = not unless (i.e., ψ only if φ = not ψ unless φ) and unless = if not. In these terms, only if = not if not.

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5.2.x. Exercise questions

1. Analyze each of the following sentences in as much detail as possible.

- **a.** Tom was late unless he left early.
- **b.** You'll get a good picture only if you take the cap off the lens.
- **c.** Neither Ann nor Bill knew of it unless they both did.
- **d.** The bill will pass if the chairman supports it—unless public opinion runs heavily against it.
- **e.** Unless Ed is late, we'll get started on time and finish early if there isn't a lot of business.
- **f.** If Bob was under no obligation to help, he worked only if he was in a good mood and had nothing to do.

2. Synthesize idiomatic English sentences that express the propositions that are associated with the logical forms below by the intensional interpretations that follow them. These repeat **3 a**, **c**, and **d** of 5.1.x. This time, you should look for ways of stating the sentences using only if and unless.

- **a.** $\neg S \rightarrow \neg B$

  S: I'll see it; B: I'll believe it
- **b.** $\neg W \leftarrow \neg (P \land \neg B)$

  W: the set works; P: the set is plugged in; B: the set is broken
- **c.** $\neg (A \lor B) \rightarrow (G \leftarrow \neg (C \lor D))$

  A: Adams will back out; B: Brown will back out; G: the deal will go through; C: Collins will have trouble with financing; D: Davis will have trouble with financing

For more exercises, use the exercise machine.

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5.2.xa. Exercise answers

1. **a.** Tom was late $\leftarrow \neg$ Tom left early

$$L \leftarrow \neg E$$
$$\neg E \rightarrow L$$

if not E then L

E: Tom left early; L: Tom was late

b. $\neg$ you'll get a good picture $\leftarrow \neg$ you'll take the cap off the lens

$$\neg P \leftarrow \neg C$$
$$\neg C \rightarrow \neg P$$

if not C then not P

C: you'll take the cap off the lens; P: you'll get a good picture

c. Neither Ann nor Bill knew of it $\leftarrow \neg$ Ann and Bill both knew of it

$\neg$ (Ann knew of it $\vee$ Bill knew of it) $\leftarrow \neg$ (Ann knew of it $\wedge$ Bill knew of it)

$$\neg (A \vee B) \leftarrow \neg (A \wedge B)$$
$$\neg (A \wedge B) \rightarrow \neg (A \vee B)$$

if not both A and B then not either A or B

A: Ann knew of it; B: Bill knew of it

d. The bill will pass if the chairman supports it $\leftarrow \neg$ public opinion will run heavily against the bill

(the bill will pass $\leftarrow$ the chairman will support the bill) $\leftarrow \neg$ A

$$(P \leftarrow S) \leftarrow \neg A$$
$$\neg A \rightarrow (S \rightarrow P)$$

if not A then if S then P

A: public opinion will run heavily against the bill; P: the bill will pass; S: the chairman will support the bill

e. $\neg$ Ed will be late $\rightarrow$ we'll get started on time and finish early if there isn't a lot of business

$\neg$ L $\rightarrow$ (we'll get started on time $\wedge$ we'll finish early if there isn't a lot of business)

$\neg$ L $\rightarrow$ (T $\wedge$ (we'll finish early $\leftarrow$ there won't be a lot of business))

$\neg$ L $\rightarrow$ (T $\wedge$ (F $\leftarrow \neg$ there will be a lot of business))

$$\neg L \rightarrow (T \wedge (F \leftarrow \neg B))$$
$$\neg L \rightarrow (T \wedge (\neg B \rightarrow F))$$

if not L then both T and if not B then F

B: there will be a lot of business; F: we'll finish early; L: Ed will be late; T: we'll get started on time

It would be possible to understand the sentence to make the whole of we'll get started on time and finish early conditional on there won't be a lot of business. On that interpretation, the form would be $\neg L \rightarrow (\neg B \rightarrow (T \wedge F))$. However, the interpretation used above fits better with common sense expectations concerning the content, and those are often the grounds on which ambiguous sentences are understood in a particular way.

f. Bob was under no obligation to help $\rightarrow$ Bob worked only if he was in a good mood and had nothing to do

$\neg$ Bob was under an obligation to help $\rightarrow$ ($\neg$ Bob worked $\leftarrow \neg$ Bob was in a good mood and had nothing to do)

$\neg O \rightarrow (\neg W \leftarrow \neg$ (Bob was in a good mood $\wedge$ Bob had nothing to do))

$\neg O \rightarrow (\neg W \leftarrow \neg (G \wedge \neg$ Bob had something to do))

$$\neg O \rightarrow (\neg W \leftarrow \neg (G \wedge \neg S))$$
$$\neg O \rightarrow (\neg (G \wedge \neg S) \rightarrow \neg W)$$

if not O then if not both G and not S then not W

O: Bob was under an obligation to help; G: Bob was in a good mood; S: Bob had something to do; W: Bob worked

2. **a.** $\neg$ I'll see it $\rightarrow \neg$ I'll believe it

Unless I see it, I won't believe it

or: I'll believe it only if I see it

b. $\neg$ the set works $\leftarrow \neg$ (the set is plugged in $\wedge \neg$ the set is broken)

$\neg$ the set works $\leftarrow \neg$ (the set is plugged in $\wedge$ the set isn't broken)

$\neg$ the set works $\leftarrow \neg$ (the set is plugged in and isn't broken)

The set works only if it is plugged in and isn't broken

or: The set doesn't work unless it is plugged in and isn't broken

c. $\neg$ (Adams will back out $\vee$ Brown will back out) $\rightarrow$ (the deal will go through $\leftarrow \neg$ (Collins will have trouble with financing $\vee$

Davis will have trouble with financing))

¬ Adams or Brown will back out → (the deal will go through ← ¬ Collins or Davis will have trouble with financing)

Unless Adams or Brown backs out, the deal will go through if neither Collins nor Davis has trouble with financing

or: If neither Adams nor Brown backs out, the deal will go through unless Collins or Davis has trouble with financing

or: Unless Adams nor Brown backs out, the deal will go through unless Collins or Davis has trouble with financing