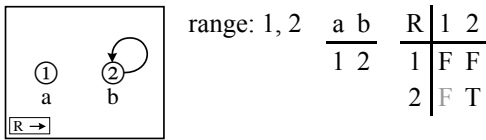## Phi 270 F11 test 5

F11 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 or 10 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

- *Analysis*. This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of course, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues that show up (e.g., how to handle there is or else).
- *Synthesis*. You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you may be asked to present a counterexample, which will involve describing a structure. You will not be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (i.e., PCh+, etc.) used to find finite counterexamples.
- *Earlier material*. These questions will concern the following topics.
  - *Basic concepts*. You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, relative inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1.)
  - *Calculations of truth values*. You should be able to complete a row of a truth table for a sentence formed using truth-functional connectives. (That is, you should be able to carry out the sort of calculation used to complete the confirmation of a counterexample in chs. 2-5.)
  - *Using abstracts to analyze sentences involving pronouns*. You might be asked to represent pronouns using abstracts and variables (i.e., in the way that was introduced in 6.2).

F11 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. A road was closed.

   [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]

2. Al hadn't read any book by Kant.

   [Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]

3. Every philosopher has read a certain book by Kant.

   [On one way of understanding this sentence, it would be false if there is no one book by Kant that all philosophers have read. Analyze it according to that interpretation.]

4. Bob spoke to Al and also to at least two other people.

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's analysis of definite descriptions as quantifier phrases as well as one that uses the description operator to analyze the definite description.

5. Al read the book that Bob read.

Use a derivation to show that the following argument is valid. You may use any rules.

6.   ∃x ∀y Rxy
   ─────────────
      ∃x Rxx

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that lurks in an open gap of your derivation.

7.   ∃x Rxx
   ─────────────
      ∀x ∃y Rxy

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

8. A pair of sentences φ and ψ are logically equivalent (i.e., φ ≃ ψ) if and only if ...

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). A letter standing for an individual term should appear in your analysis only as often as the individual term appears in the original sentence.

9. Ann called Bill, who called Carol, who called Dave.

F11 test 5 answers

1. A road was closed

    Some road is such that (it was closed)

    (∃x: x is a road) x was closed

    (∃x: Rx) Cx   ∃x (Rx ∧ Cx)

    C: [_ was closed]; S: [_ is a road]

2. Al hadn't read any book by Kant

    ¬ Al had read a book by Kant

    ¬ some book by Kant is such that (Al had read it)

    ¬ (∃x: x is a book by Kant) Al had read x

    ¬ (∃x: x is a book ∧ x is by Kant) Rax

    ¬ (∃x: Bx ∧ Yxk) Rax

    B: [_ is a book]; R: [_ had read _]; Y: [_ is by _]; a: Al; k: Kant

    The analysis (∃x: Bx ∧ Yxk) ¬ Rax would be incorrect, saying instead that there is some book by Kant that Al hadn't read—i.e., that he hadn't read all of Kant's books

3. Every philosopher has read a certain book by Kant

    some book by Kant is such that (every philosopher has read it)

    (∃x: x is a book by Kant) every philosopher has read x

    (∃x: Bx ∧ Yxk) every philosopher is s.t. (he or she has read x)

    (∃x: Bx ∧ Yxk) (∀y: y is a philosopher) y has read x

    (∃x: Bx ∧ Yxk) (∀y: Py) Ryx

    B: [_ is a book]; P: [_ is a philosopher]; R: [_ had read _]; Y: [_ is by _]; k: Kant

    The sentence Every philosopher is such that (he or she has read a book by Kant) expresses a possible interpretation, but it could be true when there is no one book by Kant that has been read by all philosophers

4. Bob spoke to Al and also to at least two other people

    Bob spoke to Al ∧ Bob spoke to at least two people other than Al

    Sba ∧ at least two people other than Al are such that (Bob spoke to them)

    Sba ∧ (∃x: x is a person other than Al) (∃y: y is a person other than Al ∧ ¬ y = x) (Bob spoke to x ∧ Bob spoke to y)

    Sba ∧ (∃x: x is a person ∧ x is not Al) (∃y: (y is a person ∧ y is not Al) ∧ ¬ y = x) (Sbx ∧ Sby)

    Sba ∧ (∃x: Px ∧ ¬ x = a) (∃y: (Py ∧ ¬ y = a) ∧ ¬ y = x) (Sbx ∧ Sby)

    P: [_ is a person]; S: [_ spoke to _]; a: Al; b: Bob

    There are many other equivalent analyses—for example:

    ∃x ∃y ((¬ x = a ∧ ¬ y = a ∧ ¬ y = x) ∧ (Px ∧ Py) ∧ (Sba ∧ Sbx ∧ Sby))

5. *Using Russell's analysis:*

    Al read the book that Bob read

    The book that Bob read is such that (Al read it)

    (∃x: x is a book that Bob read ∧ only x is a book that Bob read) Al read x

    (∃x: x is a book that Bob read ∧ (∀y: ¬ y = x) ¬ y is a book that Bob read) Rax

    (∃x: (x is a book ∧ Bob read x) ∧ (∀y: ¬ y = x) ¬ (y is a book ∧ Bob read y)) Rax

    (∃x: (Bx ∧ Rbx) ∧ (∀y: ¬ y = x) ¬ (By ∧ Rby)) Rax

    or: (∃x: (Bx ∧ Rbx) ∧ ¬ (∃y: ¬ y = x) (By ∧ Rby)) Rax
    or: (∃x: (Bx ∧ Rbx) ∧ (∀y: By ∧ Rby) x = y) Rax

    Using the description operator:

    Al read the book that Bob read

    [_ read _] Al the book that Bob read

    Ra(ɩx x is a book that Bob read)

    Ra(ɩx (x is a book ∧ Bob read x))

    Ra(ɩx (Bx ∧ Rbx))

    B: [_ is a book]; R: [_ read _]; a: Al; b: Bob

6.

			3 UI						
	∃x ∀y Rxy	1	*or*		∃x ∀y Rxy	1			
	ⓐ				ⓐ				
	∀y Ray	a:4			∀y Ray	a:2			
3 UI	∀x ¬ Rxx	a:3		2 UI	Raa	(3)			
4 UI	¬ Raa	(5)		3 EG	∃x Rxx	X,(4)			
	Raa	(5)			●				
	●			4 QED	∃x Rxx	1			
5 Nc	⊥	2		1 PCh	∃x Rxx				
2 NCP	∃x Rxx	1							
1 PCh	∃x Rxx								

7.

$\exists x\, Rxx \qquad 4$

$\quad\quad$ ⓐ

$\quad\quad$ $\forall y\, \neg\, Ray \quad a{:}3,\ b{:}5$

3 UI $\quad$ $\neg\, Raa$

$\quad\quad$ ⓑ

$\quad\quad$ Rbb

5 UI $\quad$ $\neg\, Rab$

$\quad\quad$ ○ $\quad\quad \neg\, Rab,\ Rbb,\ \neg\, Raa \nvDash \bot$

$\quad\quad$ $\bot \qquad 4$

4 PCh $\quad$ $\bot \qquad\quad 2$

2 NCP $\quad$ $\exists y\, Ray \qquad 1$

1 UG $\quad$ $\forall x\, \exists y\, Rxy$

range: 1, 2

a	b
1	2

R	1	2
1	F	F
2	F	T

8. A pair of sentences φ and ψ are logically equivalent if and only if there is no possible world in which φ and ψ have different truth values

$\qquad$ *or*

A pair of sentences φ and ψ are logically equivalent if and only if, in each possible world, φ has the same truth value as ψ

9. Ann called Bill, who called Carol, who called Dave.

Bill and Carol are such that (Ann called the former, who called the latter, who called Dave)

[Ann called x, who called y, who called Dave]$_{xy}$ Bill Carol

[Ann called x ∧ x called y ∧ y called Dave]$_{xy}$ Bill Carol

$\qquad$ [Cax ∧ Cxy ∧ Cyd]$_{xy}$bc

C: [_ called _]; a: Ann; b: Bill; c: Carol; d: Dave

Also correct: [Cxy ∧ Cyz ∧ Czw]$_{xyzw}$abcd

F10 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

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- *Analysis.* This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of course, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues that show up (e.g., how to handle there is or else).

- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)

- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you may be asked to present a counterexample, which will involve describing a structure. You will not be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (i.e., PCh+, etc.) used to find finite counterexamples.

- *Earlier material.* These questions will concern the following topics.

  - *Basic concepts.* You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, relative inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1.)

  - *Calculations of truth values.* You should be able to complete a row of a truth table for a sentence formed using truth-functional connectives.

  - *Using abstracts to analyze sentences involving pronouns.* You might be asked to represent pronouns using abstracts and variables (i.e., in the way introduced in 6.2).

F10 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. Sam saw a supernova.

[Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]

2. None of the flights Al was on were delayed.

[Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]

3. Someone ate every cookie.

[On one way of understanding this sentence, it would be false if the cookies were eaten by several people. Analyze it according to that interpretation.]

4. Fred had to make at least two connections.

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator to analyze the definite description.

5. Al opened the package.

Use a derivation to show that the following argument is valid. You may use any rules.

6.   ∀x (Fx ∨ Gx)

$~~~~~$ ∃x ¬ Fx

$~~~~~~~~~~$——————————

$~~~~~~~~~~$ ∃x Gx

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that lurks in an open gap of your derivation.

7.   ∃x (Fx ∧ Gx)

$~~~~~~~~$ Ha

$~~~~~~~~~$——————————

$~~~~~~~~$ ∃x (Fx ∧ Hx)

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

8. A set Γ entails a sentence φ (i.e., Γ ⊨ φ) if and only if ...

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). A letter standing for an individual term should appear in your analysis only as often as the individual term appears in the original sentence.

9. Al called both Bill, who called him back, and Carol, who didn't.

F10 test 5 answers

1. Sam saw a supernova

A supernova is such that (Sam saw it)

(∃x: x is a supernova) Sam saw x

$~~~~~~~~~~~$ (∃x: Nx) Ssx

$~~~~~~~~~~~$ ∃x (Nx ∧ Ssx)

N: [_ is a supernova]; S: [_ saw _]; s: Sam

2. None of the flights Al was on were delayed

¬ some flight Al was on was delayed

¬ some flight Al was on is such that (it was delayed)

¬ (∃x: x is flight Al was on) x was delayed

¬ (∃x: x is a flight ∧ Al was on x) x was delayed

$~~~~~~~~~~~$ ¬ (∃x: Fx ∧ Nax) Dx

D: [_ was delayed]; F: [_ is a flight]; N: [_ was on _]; a: Al

The analysis (∃x: Fx ∧ Nax) ¬ Dx would say that Al was on at least one flight that wasn't delayed (i.e., that not all the flights he was on were delayed)

3. Someone ate every cookie

someone is such that (he or she ate every cookie)

(∃x: x is a person) x ate every cookie

(∃x: Px) every cookie is such that (x ate it)

(∃x: Px) (∀y: y is a cookie) x ate y

$~~~~~~~~~~~$ (∃x: Px) (∀y: Cy) Axy

A: [_ ate _]; C: [_ is a cookie]; P: [_ is a person]

The alternative interpretation Every cookie is such that (someone ate it) would be true even if the cookies were eaten by several people (i.e., even if no one person ate all of them)

4. Fred had to make at least two connections

at least two connections are such that (Fred had to make them)

(∃x: x is an connection) (∃y: y is an connection ∧ ¬ y = x) (Fred had to make x ∧ Fred had to make y)

$~~~~~~~~~$ (∃x: Cx) (∃y: Cy ∧ ¬ y = x) (Mfx ∧ Mfy)

$~~~~$ *or:* ∃x (∃y: ¬ y = x) ((Cx ∧ Mfx) ∧ (Cy ∧ Mfy))

$~~~~$ *or:* ∃x ∃y (((¬ x = y) ∧ (Cx ∧ Cy)) ∧ (Mfx ∧ Mfy))

C: [_ is a connection]; M: [_ had to make _]; f: Fred

5. *Using Russell's analysis:*

Al opened the package

The package is such that (Al opened it)

(∃x: x is a package ∧ only x is a package) Al opened x

(∃x: x is a package ∧ (∀y: ¬ y = x) ¬ y is a package) Oax

(∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) Oax

or: (∃x: Px ∧ ¬ (∃y: ¬ y = x) Py) Oax
or: (∃x: Px ∧ (∀y: Py) x = y) Oax

Using the description operator:

Al opened the package

[_ opened _] Al the package

Oa(lx x is a package)

Oa(lx Px)

O: [_ opened _]; P: [_ is a package]; a: Al

6.

	∀x (Fx ∨ Gx)	a:2	*or*		∀x (Fx ∨ Gx)	a:3
	∃x ¬ Fx	1			∃x ¬ Fx	2
	(a)				∀x ¬ Gx	a:4
	¬ Fa	(3)			(a)	
2 UI	Fa ∨ Ga	3			¬ Fa	(6)
3 MTP	Ga	(4)	3 UI		Fa ∨ Ga	5
4 EG	∃x Gx	X, (5)	4 UI		¬ Ga	(5)
	●		5 MTP		Fa	(6)
5 QED	∃x Gx	1			●	
1 PCh	∃x Gx		6 Nc		⊥	2
			2 PCh		⊥	1
			1 NCP	∃x Gx		

7.

	∃x (Fx ∧ Gx)	1
	Ha	(5)
	ⓑ	
	Fb ∧ Gb	2
2 Ext	Fb	(7)
2 Ext	Gb	
	∀x ¬ (Fx ∧ Hx)	a:4, b:6
4 UI	¬ (Fa ∧ Ha)	5
5 MPT	¬ Fa	
6 UI	¬ (Fb ∧ Hb)	7
7 MPT	¬ Hb	
	○	¬ Fa, Fb, Gb, Ha, ¬ Hb ⊭ ⊥
	⊥	3
3 NCP	∃x (Fx ∧ Hx)	1
1 PCh	∃x (Fx ∧ Hx)	
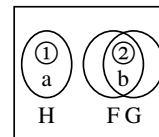
range: 1, 2

a	b	τ	Fτ	τ	Gτ	τ	Hτ
1	2	1	F	1	F	1	T
		2	T	2	T	2	F

8. A set Γ entails a sentence φ if and only if there is no possible world in which φ is false while every member of Γ is true

  or

A set Γ entails a sentence φ if and only if φ is true in every possible world in which every member of Γ is true

9. Al called both Bill, who called him back, and Carol, who didn't

Al, Bill, and Carol are such that (the first called both the second, who called him back, and the third, who didn't)

[x called both y, who called x back, and z, who didn't call x back]xyz Al Bill Carol

[x called y, who called x back ∧ x called z, who didn't call x back]xyz abc

[(x called y ∧ y called x) ∧ (x called z ∧ ¬ z called x)]xyz abc

[(Cxy ∧ Cyx) ∧ (Cxz ∧ ¬ Czx)]xyz abc

C: [_ called _]; a: Al; b: Bill; c: Carol

F09 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 or 10 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

- *Analysis*. This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of course, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues (e.g., how to handle there is or else) that show up.
- *Synthesis*. You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you may be asked to present a counterexample, which will involve describing a structure. You will not be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (i.e., PCh+, etc.) used to find finite counterexamples.
- *Earlier material*. These questions will concern the following topics.
  - *Basic concepts*. You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, conditional inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1.)
  - *Calculations of truth values*. You should be able to complete a row of a truth table for a sentence formed using truth-functional connectives.
  - *Using abstracts to analyze sentences involving pronouns*. You might be asked to represent pronouns using abstracts and variables (i.e., in the way introduced in 6.2).

F09 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. Someone spoke. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]

2. Al didn't run into anyone he knew. [Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]

3. Every child was visited by someone. [On one way of understanding this sentence, it could be true even though no one person visited all children. Analyze it according to that interpretation.]

4. Ed's ship came close to at least two icebergs.

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator to analyze the definite description.

5. The agent that Ed spoke to spoke to Fred.

Use a derivation to show that the following argument is valid. You may use any rules.

6.   ∃x ¬ Gx
      ∀x (¬ Fx → Gx)
      ────────────
        ∃x Fx

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that lurks in an open gap of your derivation.

7.   ∃x (Fx ∧ Rxx)
      ∀x (Fx → Rax)
      ────────────
        ∃x Rxa

Complete the following to give a definition of tautologousness in terms of truth values and possible worlds:

8. A sentence φ is a tautology (in symbols, ⊨ φ) if and only if ...

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). A letter standing for an individual term should appear in your analysis only as often as the individual term appears in the original sentence.

9. Al congratulated himself.

F09 test 5 answers

1. Someone spoke

   Someone is such that (he or she spoke)

   (∃x: x is a person) x spoke

   $$(∃x: Px) Sx$$
   $$∃x (Px ∧ Sx)$$

   P: [_ is a person]; S: [_ spoke]

2. Al didn't run into anyone he knew

   ¬ Al ran into someone he knew

   ¬ someone that Al knew is such that (Al ran into him or her)

   ¬ (∃x: x is a person Al knew) Al ran into x

   ¬ (∃x: x is a person ∧ Al knew x) Al ran into x

   $$¬ (∃x: Px ∧ Kax) Rax$$

   K: [_ knew _]; P: [_ is a person]; R: [_ ran into _]

   The analysis (∃x: Px ∧ Kax) ¬ Rax would say that there was someone Al knew who he didn't run into

3. Every child was visited by someone

   every child is such that (he or she was visited by someone)

   (∀x: x is a child) x was visited by someone

   (∀x: Cx) someone is such that (x was visited by him or her)

   (∀x: Cx) (∃y: y is a person) x was visited by y

   $$(∀x: Cx) (∃y: Py) Vxy$$

   C: [_ is a child]; P: [_ is a person]; V: [_ was visited by _]

   The alternative interpretation Someone is such that (every child was visited by him or her) would not be true unless some one person visited all children

4. Ed's ship came close to at least two icebergs

   at least two icebergs are such that (Ed's ship came close to them)

   (∃x: x is an iceberg) (∃y: y is an iceberg ∧ ¬ y = x) (Ed's ship came close to x ∧ Ed's ship came close to y)

   (∃x: Ix) (∃y: Iy ∧ ¬ y = x) (C(Ed's ship)x ∧ C(Ed's ship)y)

   $$(∃x: Ix) (∃y: Iy ∧ ¬ y = x) (C(se)x ∧ C(se)y)$$

   C: [_ came close to _]; I: [_ is an iceberg]; e: Ed; s: [_'s ship]

5. *Using Russell's analysis:*

   The agent that Ed spoke to spoke to Fred

   The agent that Ed spoke to is such that (he or she spoke to Fred)

   (∃x: x is an agent that Ed spoke to ∧ only x is an agent that Ed spoke to) x spoke to Fred

   (∃x: (x is an agent ∧ Ed spoke to x) ∧ (∀y: ¬ y = x) ¬ (y is an agent ∧ Ed spoke to y)) Sxf

   $$(∃x: (Ax ∧ Sex) ∧ (∀y: ¬ y = x) ¬ (Ay ∧ Sey)) Sxf$$

   also correct: (∃x: (Ax ∧ Sex) ∧ ¬ (∃y: ¬ y = x) (Ay ∧ Sey)) Sxf
   also correct: (∃x: (Ax ∧ Sex) ∧ (∀y: Ay ∧ Sey) x = y) Sxf

   Using the description operator:

   The agent that Ed spoke to spoke to Fred

   [_ spoke to _] the agent that Ed spoke to Fred

   S(ɪx x is an agent that Ed spoke to)f

   S(ɪx (x is an agent ∧ Ed spoke to x))f

   $$S(ɪx (Ax ∧ Sex))f$$

   A: [_ is an agent]; S: [_ spoke to _]; e: Ed; f: Fred

6.

	∃x ¬ Gx	1
	∀x (¬ Fx → Gx)	a:2
	ⓐ	
	¬ Ga	(3)
2 UI	¬ Fa → Ga	3
3 MTT	Fa	(6)
	∀x ¬ Fx	a:5
5 UI	¬ Fa	(6)
	●	
6 Nc	⊥	4
4 NCP	∃x Fx	1
1 PCh	∃x Fx	

   or

	∃x ¬ Gx	1
	∀x (¬ Fx → Gx)	a:2
	ⓐ	
	¬ Ga	(3)
2 UI	¬ Fa → Ga	3
3 MTT	Fa	(4)
4 EG	∃x Fx	X, (5)
	●	
5 QED	∃x Fx	1
1 PCh	∃x Fx	

7.

$\exists x\,(Fx \wedge Rxx)$ 1
$\forall x\,(Fx \rightarrow Rax)$ b:3, a:7

ⓑ
| $Fb \wedge Rbb$ 2

2 Ext | Fb (4)
2 Ext | Rbb
3 UI | $Fb \rightarrow Rab$ 4
4 MPP | Rab

        | | $\forall x \neg Rxa$ a:6, b:9

6 UI | | $\neg Raa$ (8)
7 UI | | $Fa \rightarrow Raa$ 8
8 MTT | | $\neg Fa$
9 UI | | $\neg Rba$
        | | ○ $\neg Rba, \neg Fa, \neg Raa, Rab, Fb, Rbb \nvDash \bot$

        | | $\bot$ 5

5 NCP | $\exists x\, Rxa$ 1

1 PCh $\exists x\, Rxa$
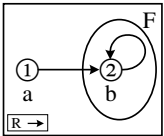
range: 1, 2

a b	τ	Fτ	R	1 2
1 2	1	F	1	F T
	2	T	2	F T

8. A sentence φ is a tautology if and only if there is no possible world in which φ is false

   or

   A sentence φ is a tautology if and only if φ is true in every possible world

9. Al congratulated himself

   Al is such that (he congratulated himself)

   [x congratulated x]$_x$ Al

           [Cxx]$_x$a

   C: [_ congratulated _]; a: Al

F08 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 or 10 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

- *Analysis*. This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of course, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues (e.g., how to handle there is or else) that show up.
- *Synthesis*. You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you may be asked to present a counterexample, which will involve describing a structure. You will not be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (i.e., PCh+, etc.) used to find finite counterexamples.
- *Earlier material*. These questions will concern the following topics.
  - *Basic concepts*. You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, relative inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1.)
  - *Calculations of truth values*. You should be able to complete a row of a truth table for a sentence formed using truth-functional connectives.
  - *Using abstracts to analyze sentences involving pronouns*. You might be asked to represent pronouns using abstracts and variables. (You will not find many questions of this sort in the old exams, but exercise 2 for 6.2 and your homework on 6.2 provide examples as do test 3 for F06 and F08 and test 5 for F06.)
  - *Describing structures*. Describing a structure that is a counterexample lurking an open gap is the last step in a derivation that fails, but I may ask you simply to describe a structure that makes certain sentences true. The derivation exercises in chapters 7 and 8 provide simple examples, and you can find more complex ones in the examples of 6.4.3 (as well as among the old tests—in old versions of both test 3 and test 5).

F08 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. Dave found a coin. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]

2. There is an elf who neglects no one. [Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]

3. Everyone watched a movie. [On one way of understanding this sentence, it would not be true unless everyone watched the same movie. Analyze it according to that interpretation.]

4. Someone sang to someone else.

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator to analyze the definite description.

5. Rudolph guided the sleigh that flew.

Use a derivation to show that the following argument is valid. You may use any rules.

6.   ∃x Gx
         ∀x Fx
         ─────────
         ∃x (Fx ∧ Gx)

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that lurks in an open gap of your derivation.

7.   ∃x ∀y Rxy
         ─────────
         ∀x ∃y Rxy

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

8. A pair of sentences φ and ψ entails a sentence χ (in symbols, φ, ψ ⊨ χ) if and only if ...

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). A letter standing for an individual term should appear in your analysis only as often as the individual term appears in the original sentence.

9. Bill called Carol and mentioned his father to her.

F08 test 5 answers

1. Dave found a coin
   A coin is such that (Dave found it)
   (∃x: x is a coin) Dave found x

$$(\exists x: Cx)\ Fdx$$
$$\exists x\ (Cx \wedge Fdx)$$

   C: [_ is a coin]; F: [_ found _]; d: Dave

2. There is an elf who neglects no one
   Something is an elf who neglects no one
   ∃x x is an elf who neglects no one
   ∃x (x is an elf ∧ x neglects no one)
   ∃x (x is an elf ∧ ¬ x neglects someone)
   ∃x (Ex ∧ ¬ someone is such that (x neglects him or her))
   ∃x (Ex ∧ ¬ (∃y: y is a person) x neglects y)

$$\exists x\ (Ex \wedge \neg (\exists y: Py)\ Nxy)$$

   E: [_ is an elf]; N: [_ neglects _]; P: [_ is a person]

3. Everyone watched a movie
   some movie is such that (everyone watched it)
   (∃x: x is a movie) everyone watched x
   (∃x: Mx) everyone is such that (he or she watched x)
   (∃x: Mx) (∀y: y is a person) y watched x

$$(\exists x: Mx)\ (\forall y: Py)\ Wyx$$

   M: [_ is a movie]; P: [_ is a person]; W: [_ watched _]

   The alternative interpretation Everyone is such that (he or she watched a movie) could be true even if there was no one movie that everyone watched

4. Someone sang to someone else
   Someone is such that (he or she sang to someone else)
   (∃x: x is a person) x sang to someone else
   (∃x: Px) someone other than x is such that (x sang to him or her)
   (∃x: Px) (∃y: y is a person ∧ ¬ y = x) x sang to y

$$(\exists x: Px)\ (\exists y: Py \wedge \neg y = x)\ Sxy$$

   P: [_ is a person]; S: [_ sang to _]

5. *Using Russell's analysis:*

Rudolph guided the sleigh that flew

the sleigh that flew is such that (Rudolph guided it)

(∃x: x is a sleigh that flew ∧ only x is a sleigh that flew) Rudolph

   guided x

(∃x: (x is a sleigh ∧ x flew) ∧ (∀y: ¬ y = x) ¬ (y is a sleigh ∧ y flew)) Grx

             (∃x: (Sx ∧ Fx) ∧ (∀y: ¬ y = x) ¬ (Sy ∧ Fy)) Grx

also correct: (∃x: (Sx ∧ Fx) ∧ ¬ (∃y: ¬ y = x) (Sy ∧ Fy)) Grx
also correct: (∃x: (Sx ∧ Fx) ∧ (∀y: Sy ∧ Fy) x = y) Grx

Using the description operator:

Rudolph guided the sleigh that flew

[_ guided _] Rudolph the sleigh that flew

Gr(ɩx x is a sleigh that flew)

Gr(ɩx (x is a sleigh ∧ x flew))

                  Gr(ɩx (Sx ∧ Fx))

F: [_ flew]; G: [_ guided _]; S: [_ is a sleigh]; r: Rudoph

6.

	∃x Gx	1
	∀x Fx	a:2
	ⓐ Ga	(6)
2 UI	Fa	(5)
	∀x ¬ (Fx ∧ Gx)	a:4
4 UI	¬ (Fa ∧ Ga)	5
5 MPT	¬ Ga	(6)
	●	
6 Nc	⊥	3
3 NCP	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	

or

	∃x Gx	1
	∀x Fx	a:2
	ⓐ Ga	(3)
2 UI	Fa	(3)
3 Adj	Fa ∧ Ga	X, (4)
4 EG	∃x (Fx ∧ Gx)	X, (5)
	●	
5 QED	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	

7.

	∃x ∀y Rxy	1
	ⓐ ∀y Ray	a:4, b:5
	ⓑ ∀y ¬ Rby	a:6, b:7
4 UI	Raa	
5 UI	Rab	
6 UI	¬ Rba	
7 UI	¬ Rbb	
	○	Raa, Rab, ¬ Rba, ¬ Rbb ⊭ ⊥
	⊥	3
3 NCP	∃y Rby	2
2 UG	∀x ∃y Rxy	1
1 PCh	∀x ∃y Rxy	

range: 1, 2

a	b
1	2

R	1	2
1	T	T
2	F	F

8. A pair of sentences φ and ψ entails a sentence χ if and only if there is no possible world in which both φ and ψ are true and χ is false

    or

A pair of sentences φ and ψ entails a sentence χ if and only if χ is true in every possible world in which both φ and ψ are true

9. Bill called Carol and mentioned his father to her

Bill and Carol are such that (he called her and mentioned his father to her)

[x called y and mentioned x's father to y]$_{xy}$ Bill Carol

[x called y ∧ x mentioned x's father to y]$_{xy}$bc

[Cxy ∧ Mx(x's father)y]$_{xy}$bc

                [Cxy ∧ Mx(fx)y]$_{xy}$bc

C: [_ called _]; M: [_ mentioned _ to _]; b: Bill; c: Carol; f: [_'s father]

F06 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 or 10 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

• *Analysis.* This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of consider, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues (e.g., how to handle there is or else) that show up.

• *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)

• *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (PCh+, etc.) used to find finite counterexamples.

• *Earlier material.* These questions will concern the following topics.

  • *Basic concepts.* You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, relative inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1.)

  • *Calculations of truth values.* That is, you should be able to calculate the truth value of a symbolic sentence on an extensional interpretation of it. This means you must know the truth tables for connectives and also how to carry out the sort of calculation from tables introduced in ch. 6—see exercise 2 of 6.4).

  • *Using abstracts to analyze sentences involving pronouns.* You might be asked to represent pronouns using abstracts and variables. (You will not find questions of this sort in the old exams, but your homework on this topic and exercise 2 for 6.2 provide examples.)

  • *Describing structures.* Describing a structure that is a counterexample lurking an open gap is the last step in a derivation that fails, but I may ask you simply to describe a structure that makes certain sentences true. The derivation exercises in chapters 7 and 8 have led only to very simple structures, but you can find more complex ones in the examples of 6.4.3 (as well as among the old tests—in old versions of both test 3 and test 5).

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. Someone called Tom. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]

2. Not a crumb was left, but there was a note from Santa. [Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]

3. A card was sent to each customer. [On one way of understanding this sentence, it would be true even if no two customers were sent the same card. Analyze it according to that interpretation.]

4. At most one size was left.

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator.

5. Ann found the note that Bill left.

Use a derivation to show that the following argument is valid. You may use any rules.

6.   ∃x (Fx ∧ Gx)
      ∀x (Gx → Hx)
      ─────────────
        ∃x Hx

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that lurks in an open gap of your derivation.

7.   ∃x ∃y (Rxa ∧ Ray)
      ─────────────
        ∃x Rxx

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

8. A pair of sentences φ and ψ are logically equivalent (in symbols, φ ≃ ψ) if and only if ...

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). An individual term should appear in your analysis only as often as it appears in the original sentence.

9. Ann wrote to Bill and he called her.

1. Someone called Tom

Someone is such that (he or she called Tom)

(∃x: x is a person) x called Tom

$$(\exists x: Px)\ Cxt$$
$$\exists x\ (Px \land Cxt)$$

C: [_ called _]; P: [_ is a person]; t: Tom

2. Not a crumb was left, but there was a note from Santa

Not a crumb was left ∧ there was a note from Santa

¬ a crumb was left ∧ something was a note from Santa

¬ some crumb is such that (it was left) ∧ something is such that (it was a note from Santa)

¬ (∃x: x is a crumb) x was left ∧ ∃y (y was a note from Santa)

¬ (∃x: Cx) Lx ∧ ∃y (y was a note ∧ y was from Santa)

$$\neg\ (\exists x: Cx)\ Lx \land \exists y\ (Ny \land Fys)$$

C: [_ is a crumb]; F: [_ was from _]; L: [_ was left]; N: [_ was a note]; s: Santa

3. A card was sent to each customer

each customer is such that (a card was sent to him or her)

(∀x: x is a customer) a card was sent to x

(∀x: Cx) some card is such that (it was sent to x)

(∀x: Cx) (∃y: y is a card) y was sent to x

$$(\forall x: Cx)\ (\exists y: Dy)\ Syx$$

C: [_ is a customer]; D: [_ is a card]; S: [_ was sent to _]

Some card is such that (it was sent to each customer) would be true only if there was at least one card that was sent to all customers, so an analysis of it would not be a correct answer

4. At most one size was left

¬ at least two sizes were left

¬ at least two sizes are such that (they were left)

¬ (∃x: x is a size) (∃y: y is a size ∧ ¬ y = x) (x was left ∧ y was left)

$$\neg\ (\exists x: Sx)\ (\exists y: Sy \land \neg\ y = x)\ (Lx \land Ly)$$

S: [_ is a size]; L: [_ was left]

also correct: (∀x: Sx) (∀y: Sy ∧ ¬ y = x) ¬ (Lx ∧ Ly)
also correct: (∀x: Sx ∧ Lx) (∀y: Sy ∧ Ly) x = y

5. *Using Russell's analysis:*

Ann found the note that Bill left

the note that Bill left is such that (Ann found it)

(∃x: x is a note that Bill left ∧ only x is a note that Bill left) Ann found x

(∃x: (x is a note ∧ Bill left x) ∧ (∀y: ¬ y = x) ¬ (y is a note ∧ Bill left x)) Fax

$$(\exists x: (Nx \land Lbx) \land (\forall y: \neg\ y = x)\ \neg\ (Ny \land Lby))\ Fax$$

also correct: (∃x: (Nx ∧ Lbx) ∧ ¬ (∃y: ¬ y = x) (Ny ∧ Lby)) Fax
also correct: (∃x: (Nx ∧ Lbx) ∧ (∀y: Ny ∧ Lby) x = y) Fax

Using the description operator:

Ann found the note that Bill left

[_ found _] Ann (the note that Bill left)

Fa(lx x is note that Bill left)

Fa(lx (x is a note ∧ Bill left x))

$$Fa(lx\ (Nx \land Lbx))$$

F: [_ found _]; L: [_ left _]; N: [_ is a note]; a: Ann; b: Bill

6.

				or			
	∃x (Fx ∧ Gx)	1			∃x (Fx ∧ Gx)	1	
	∀x (Gx → Hx)	a: 3			∀x (Gx → Hx)	a: 3	
	ⓐ				ⓐ		
	Fa ∧ Ga	2			Fa ∧ Ga	2	
2 Ext	Fa			2 Ext	Fa		
2 Ext	Ga	(4)		2 Ext	Ga	(4)	
3 UI	Ga → Ha	4		3 UI	Ga → Ha	4	
4 MPP	Ha	(5)		4 MPP	Ha	(7)	
5 EG	∃x Hx	X,6			∀x ¬ Hx	a: 6	
	●			6 UI	¬ Ha	(7)	
6 QED	∃x Hx	1			●		
					⊥	5	
1 Pch	∃x Hx			7 Nc			
				5 NcP	∃x Hx	1	
				1 PCh	∃x Hx		

Many different orders are possible for the rules used. In particular, NcP could be used before PCh in the second.

7.

	∃x ∃y (Rxa ∧ Ray)	1
ⓑ	∃y (Rba ∧ Ray)	2
ⓒ	Rba ∧ Rac	3
3 Ext	Rba	
3 Ext	Rac	
	∀x ¬ Rxx	a:5, b:6, c:7
5 UI	¬ Raa	
6 UI	¬ Rbb	
7 UI	¬ Rcc	
	○	Rba, Rac, ¬ Raa, ¬ Rbb, ¬ Rcc ⊭ ⊥
	⊥	4
4 NcP	∃x Rxx	2
2 PCh	∃x Rxx	1
1 PCh	∃x Rxx	
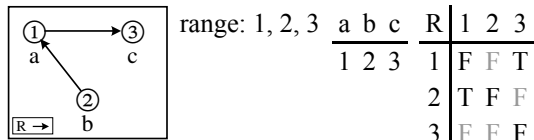
range: 1, 2, 3

a b c
1 2 3

R	1	2	3
1	F	F	T
2	T	F	F
3	F	F	F

8. A pair of sentences φ and ψ are logically equivalent if and only if there is no possible world in which φ and ψ have different truth values

   or

A pair of sentences φ and ψ are logically equivalent if and only if φ and ψ have the same truth value as each other in every possible world

9. Ann wrote to Bill and he called her

Ann and Bill are such that (she wrote to him and he called her)

[x wrote to y and y called x]ₓᵧ Ann Bill

[x wrote to y ∧ y called x]ₓᵧab

[Wxy ∧ Cyx]ₓᵧab

C: [_ called _]; W: [_ wrote to _]; a: Ann; b: Bill

F05 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 or 10 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

- *Analysis.* This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of consider, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues (e.g., how to handle there is or else) that show up.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (i.e., PCh+, etc.) used to find finite counterexamples.
- *Earlier material.* These questions will concern the following topics.
  - *Basic concepts.* You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, relative inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1 .)
  - *Calculations of truth values.* That is, you should be able to calculate the truth value of a symbolic sentence on an extensional interpretation of it. This means you must know the truth tables for connectives and also how to carry out the sort of calculation from tables introduced in ch. 6—see exercise 2 of 6.4.x).
  - *Describing structures.* Describing a structure that is a counterexample lurking an open gap is the last step in a derivation that fails, but I may ask you simply to describe a structure that makes certain sentences true. The derivation exercises in chapters 7 and 8 have led only to very simple structures, but you can find more complex ones in the examples of 6.4.3 (as well as among the old tests—in old versions of both test 3 and test 5).

F05 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. A bell rang. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]
2. There was a storm but no flight was delayed. [Avoid using ∀ in your analysis of any quantifier phrases in this sentence.]
3. Everyone was humming a tune. [On one way of understanding this sentence, it would be false if people were humming different tunes. Analyze it according to that interpretation.]
4. Tom saw at least two snowflakes.

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator.

5. Ann saw the play.

Use a derivation to show that the following argument is valid. You may use any rules.

6. $\dfrac{\exists x\,(Fa \rightarrow Gx)}{Fa \rightarrow \exists x\,Gx}$

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that lurks in an open gap of your derivation.

7. $\exists x\,Fx$

  $\dfrac{\exists x\,Rxa}{\exists x\,(Fx \wedge Rxa)}$

Complete the following to give a definition of inconsistency in terms of truth values and possible worlds:

8. A set Γ of sentences is inconsistent (in symbols, Γ ⊨ or, equivalently, Γ ⊨ ⊥) if and only if …

Complete the following truth table for the two rows shown. In each row, indicate the value of each compound component of the sentence on the right by writing the value under the main connective of that component (so, in each row, every connective should have a value under it); also circle the value that is under the main connective of the whole sentence.

9. $\dfrac{\text{A B C D}}{\begin{array}{l}\text{T F F F}\\\text{F F T T}\end{array}}\Big|\;(A \rightarrow \neg\, C) \wedge \neg\, (B \vee D)$

F05 test 5 answers

1. A bell rang
   Some bell is such that (it rang)
   (∃x: x is a bell) x rang

   $$(\exists x{:}\ Bx)\ Rx$$
   $$\exists x\,(Bx \wedge Rx)$$

   B: [_ is a bell]; R: [_ rang]

2. There was a storm but no flight was delayed
   There was a storm ∧ no flight was delayed
   Something was a storm ∧ ¬ some flight was delayed
   Something is such that (it was a storm) ∧ ¬ some flight is such that (it was delayed)
   ∃x x was a storm ∧ ¬ (∃x: x is a flight) x was delayed

   $$\exists x\,Sx \wedge \neg\,(\exists x{:}\ Fx)\ Dx$$

   D: [_ was delayed]; F: [_ is a flight]; S: [_ was a storm]

3. Everyone was humming a tune
   Some tune is such that (everyone was humming it)
   (∃x: x is a tune) everyone was humming x
   (∃x: Tx) everyone is such that (he or she was humming x)
   (∃x: Tx) (∀y: y is a person) (y was humming x)

   $$(\exists x{:}\ Tx)\ (\forall y{:}\ Py)\ Hyx$$

   H: [_ was humming _]; P: [_ is a person]; T: [_ is a tune]

   Everyone is such that (he or she was humming a tune) could be true even though people were humming different tunes, so an analysis of it would not be a correct answer.

4. Tom saw at least two snowflakes
   At least two snowflakes are such that (Tom saw them)
   (∃x: x is a snowflake) (∃y: y is a snowflake ∧ ¬ y = x) (Tom saw x ∧ Tom saw y)

   $$(\exists x{:}\ Fx)\ (\exists y{:}\ Fy \wedge \neg\, y = x)\ (Stx \wedge Sty)$$

   F: [_ is a snowflake]; S: [_ saw _]; t: Tom

5. *Using Russell's analysis:*

Ann saw the play

The play is such that (Ann saw it)

(∃x: x is a play ∧ (∀y: ¬ y = x) ¬ y is a play) Ann saw x

(∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) Sax

also correct:

(∃x: Px ∧ ¬ (∃y: ¬ y = x) Py) Sax

or:

(∃x: Px ∧ (∀y: Py) x = y) Sax

Using the description operator:

Ann saw the play

S Ann the play

Sa (Ιx x is a play)

Sa(Ιx Px)

P: [_ is a play]; S: [_ saw _]; a: Ann

6.

	∃x (Fa → Gx)	2	***or***		∃x (Fa → Gx)	2
	Fa	(3)			Fa	(3)
	ⓑ Fa → Gb	3			ⓑ Fa → Gb	3
3 MPP	Gb	(4)		3 MPP	Gb	(6)
4 EG	∃x Gx	X,(5)			∀x ¬ Gx	b:5
	●			5 UI	¬ Gb	(6)
5 QED	∃x Gx	2			●	
2 PCh	∃x Gx	1		6 Nc	⊥	4
1 CP	Fa → ∃x Gx			4 NcP	∃x Gx	2
				2 PCh	∃x Gx	1
				1 CP	Fa → ∃x Gx	

The order of CP and PCh can be reversed in these and the use of MPP in the second could come after NcP and UI.

7.

		∃x Fx	1
		∃x Rxa	2
		ⓑ Fb	(5)
		ⓒ Rca	(7)
		∀x ¬ (Fx ∧ Rxa)	b:4, c:6, a:8
4 UI		¬ (Fb ∧ Rba)	5
5 MPT		¬ Rba	
6 UI		¬ (Fc ∧ Rca)	7
7 MPT		¬ Fc	
8 UI		¬ (Fa ∧ Raa)	9
		¬ Fa	
		○	Fb,Rca,¬Rba,¬Fc,¬Fa ⊭ ⊥
		⊥	11
11 IP		Fa	10
		¬ Raa	
		○	Fb,Rca,¬Rba,¬Fc,¬Raa ⊭ ⊥
		⊥	12
12 IP		Raa	10
10 Cnj		Fa ∧ Raa	9
9 CR		⊥	3
3 NcP		∃x (Fx ∧ Rxa)	2
2 PCh		∃x (Fx ∧ Rxa)	1
1 PCh	∃x (Fx ∧ Rxa)		

range: 1, 2, 3

a b c		τ	Fτ	R	1	2	3
1 2 3		1	F	1	F	F	F
		2	T	2	F	F	F
		3	F	3	T	F	F
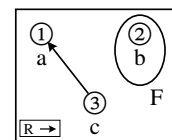
This counterexample lurks in both gaps; the value for F1 is needed only for the first gap and the value for R11 is needed only for the second.

8. A set Γ of sentences is inconsistent if and only if there is no possible world in which all members of Γ are true

  or

A set Γ of sentences is inconsistent if and only if, in each possible world, at least one member of Γ is false

9.

A B C D	(A → ¬ C) ∧ ¬ (B ∨ D)
T F F F	T T Ⓣ T F
F F T T	T F Ⓕ F T

F04 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 or 10 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

- *Analysis.* This will represent the majority of the questions on ch. 8. The homework assignments give a good sample of the kinds of issues that might arise but you should, of consider, consider examples and exercises in the text as well. In particular, pay attention to the variety of special issues (e.g., how to handle there is or else) that show up.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will have the option using the rules REP and REC (as well as RUP and RUC) in derivations for restricted quantifiers. You will *not* be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules (i.e., PCh+, etc.) used to find finite counterexamples.
- *Earlier material.* These questions will concern the following topics.
  - *Basic concepts.* You may be asked for a definition of a concept or asked questions about the concept that can be answered on the basis of its definition. You are responsible for: entailment or validity, equivalence, tautologousness, relative inconsistency or exclusion, inconsistency of a set, absurdity, and relative exhaustiveness. (These are the concepts whose definitions appear in Appendix A.1.)
  - *Calculations of truth values.* That is, you should be able to calculate the truth value of a symbolic sentence on an extensional interpretation of it. This means you must know the truth tables for connectives and also how to carry out the sort of calculation from tables introduced in ch. 6—see exercise 2 of 6.4.x).
  - *Describing structures.* Describing a structure that is a counterexample lurking an open gap is the last step in a derivation that fails, but I may ask you simply to describe a structure that makes certain sentences true. The derivation exercises in chapters 7 and 8 have led only to very simple structures, but you can find more complex ones in the examples of 6.4.3 (as well as among the old tests—in old versions of both test 3 and test 5).

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the special instructions given for **1** and **3**.

1. Someone was singing [Present your analysis also using an unrestricted quantifier.]
2. There is a package that isn't addressed to anyone.
3. An airline served each airport. [This sentence is ambiguous. On one way of interpreting it, it could be true even if no one airline served all airports. Analyze the sentence according to that interpretation of it.]
4. At least two people called.

Analyze the sentence below using each of the two ways of analyzing the definite description the sleigh Santa drove. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases and another analysis that uses the description operator.

5. The sleigh Santa drove was red.

Use derivations to show that the following arguments are valid. You may use any rules.

6. $\underline{\exists x\,(Fx \land Gx)}$

    $\exists x\,Gx$

7. $\underline{\exists x\,(Fx \land \exists y\,Rxy)}$

    $\exists x\,\exists y\,(Fy \land Ryx)$

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

8. A sentence φ is entailed by a set Γ (i.e., $\Gamma \vDash \varphi$) if and only if …

Complete the following truth table for the two rows shown. Indicate the value of each component of the sentence on the right by writing the value under the main connective of that component.

9. | A | B | C | D | $\neg\,(A \land B) \to (\neg\,C \lor D)$ |
    |---|---|---|---|---|
    | T | T | F | F | |
    | F | F | T | F | |

Use either tables or a diagram to describe a structure in which the following sentences are true. (That is, do what would be required to present a counterexample when a dead-end gap of a derivation had these sentences as its active resources.)

10. $a = c$, $fa = fb$, $\neg\,Ga$, Gb, $G(fc)$, $Ra(fb)$, $Rb(fa)$

1. Someone was singing
   Someone is such that (he or she was singing)
   ($\exists x$: x is a person) x was singing

   $$(\exists x{:}\ Px)\ Sx$$
   $$\exists x\,(Px \land Sx)$$

   P: [_ is a person]; S: [_ was singing]

2. There is a package that isn't addressed to anyone
   Something is a package that isn't addressed to anyone
   $\exists x$ x is a package that isn't addressed to anyone
   $\exists x$ (x is a package $\land$ x isn't addressed to anyone)
   $\exists x$ (Kx $\land \neg$ x is addressed to someone)
   $\exists x$ (Kx $\land \neg$ someone is such that (x is addressed to him or her))
   $\exists x$ (Kx $\land \neg$ ($\exists y$: y is a person) x is addressed to y)

   $$\exists x\,(Kx \land \neg\,(\exists y{:}\ Py)\ Axy)$$
   $$or:\ \exists x\,(Kx \land (\forall y{:}\ Py)\ \neg\,Axy)$$

   A: [_ is addressed to _]; K: [_ is a package]; P: [_ is a person]

3. An airline served each airport
   Every airport is such that (an airline served it)
   ($\forall x$: x is an airport) an airline served x
   ($\forall x$: Ax) some airline is such that (it served x)
   ($\forall x$: Ax) ($\exists y$: y is an airline) y served x

   $$(\forall x{:}\ Ax)\ (\exists y{:}\ Ly)\ Syx$$

   P: [_ is an airport]; L: [_ is an airline]; S: [_ served _]

   ($\exists x$: Lx) ($\forall y$: Ay) Sxy would be incorrect since it is true only if there is a single airline that serves all airports

4. At least two people called
   At least two people are such that (they called)
   ($\exists x$: x is a person) ($\exists y$: y is a person $\land \neg\ y = x$) (x called $\land$ y called)

   $$(\exists x{:}\ Px)\ (\exists y{:}\ Py \land \neg\ y = x)\ (Cx \land Cy)$$

   C: [_ called]; P: [_ is a person]

5. *Using Russell's analysis:*

The sleigh Santa drove was red

The sleigh Santa drove is such that (it was red)

(∃x: x is a sleigh Santa drove ∧ (∀y: ¬ y = x) ¬ y is a sleigh Santa drove) x was red

(∃x: (x is a sleigh ∧ Santa drove x) ∧ (∀y: ¬ y = x) ¬ (y is a sleigh ∧ Santa drove y)) x was red

$$(\exists x: (Sx \wedge Dsx) \wedge (\forall y: \neg y = x) \neg (Sy \wedge Dsy)) Rx$$

Using the description operator:

The sleigh Santa drove was red

R (the thing such that (it is a sleigh Santa drove))

R (lx x is a sleigh Santa drove)

R (lx (x is a sleigh ∧ Santa drove x))

$$R(lx (Sx \wedge Dsx))$$

D: [_ drove _]; R: [_ was red]; S: [_ is a sleigh]; s: Santa

6.

	∃x (Fx ∧ Gx)	1	*or*		∃x (Fx ∧ Gx)	1
	ⓐ Fa ∧ Ga	2			ⓐ Fa ∧ Ga	2
2 Ext	Fa			2 Ext	Fa	
2 Ext	Ga	(3)		2 Ext	Ga	(5)
3 EG	∃x Gx	X, (4)			∀x ¬ Gx	a: 4
	●			4 UI	¬ Ga	(5)
					●	
4 QED	∃x Gx	1		5 Nc	⊥	3
1 PCh	∃x Gx			3 NcP	∃x Gx	1
				1 PCh	∃x Gx	

7.

	∃x (Fx ∧ ∃y Rxy)	1
	ⓐ Fa ∧ ∃y Ray	2
2 Ext	Fa	(4)
2 Ext	∃y Ray	3
	ⓑ Rab	(4)
4 Adj	Fa ∧ Rab	X, (5)
5 EG	∃y (Fy ∧ Ryb)	X, (6)
6 EG	∃x ∃y (Fy ∧ Ryx)	X, (7)
	●	
7 QED	∃x ∃y (Fy ∧ Ryx)	3
3 PCh	∃x ∃y (Fy ∧ Ryx)	1
1 PCh	∃x ∃y (Fy ∧ Ryx)	

or

	∃x (Fx ∧ ∃y Rxy)	1
	ⓐ Fa ∧ ∃y Ray	2
2 Ext	Fa	(9)
2 Ext	∃y Ray	
	ⓑ Rab	(10)
	∀x ¬ ∃y (Fy ∧ Ryx)	b: 5
5 UI	¬ ∃y (Fy ∧ Ryb)	6
	∀y ¬ (Fy ∧ Ryb)	a :8
8 UI	¬ (Fa ∧ Rab)	9
9 MPT	¬ Rab	(10)
	●	
10 Nc	⊥	7
7 NcP	∃y (Fy ∧ Ryb)	6
6 CR	⊥	4
4 NcP	∃x ∃y (Fy ∧ Ryx)	3
3 PCh	∃x ∃y (Fy ∧ Ryx)	1
1 PCh	∃x ∃y (Fy ∧ Ryx)	

8. A sentence φ is entailed by a set Γ if and only if there is no possible world in which φ is false while all members of Γ are true

or: A sentence φ is entailed by a set Γ if and only φ is true in every possible world in which all members of Γ are true

9.

A	B	C	D	¬ (A ∧ B) → (¬ C ∨ D)
T	T	F	F	F T Ⓣ T T
F	F	T	F	T F Ⓕ F F

10. range: 1, 2, 3

a b c	τ	fτ	τ	Gτ	R	1 2 3
1 2 1	1	3	1	F	1	F F T
	2	3	2	T	2	F F T
	3	3	3	T	3	F F F

The diagram provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.

alias sets	IDs	values
a	1	a: 1
c		c: 1
b	2	b: 2
fa	3	f1: 3
fb		f2: 3
fc		f1: 3

resources	values
¬ Ga	G1: **F**
Gb	G2: **T**
G(fc)	G3: **T**
Ra(fb)	R13: **T**
Rb(fa)	R23: **T**

F03 test 5 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

This test will have a few more questions than earlier ones (about 9 instead of about 7) and I will allow you as much of the 3 hour period as you want. The bulk of the questions (6 or 7 of the total) will be on ch. 8 but there will also be a few questions directed specifically towards earlier material (see below).

- *Analysis.* This will represent the majority of the questions on ch. 8. The homework assignment give a good sample of the kinds of issues that might arise but you should, of consider, consider examples and exercises in the text as well.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rule for the description operator introduced in §8.6 or for the supplemented rules used to find finite counterexamples.
- *Earlier material.* These questions will concern two topics.
  - *Basic concepts.* You may be asked for a definition or asked questions about them that can be answered by reasoning from their definitions. You are responsible for: entailment or validity, equivalence, tautologousness, inconsistency of a set, relative inconsistency or exclusion, absurdity, and relative exhaustiveness.
  - *Calculations of truth values.* That is, you should be able to calculate the truth value of a symbolic sentence on an extensional interpretation of it. This means you must know the truth tables for connectives and also how to carry out the sort of calculation from tables introduced in ch. 6--see exercise 2 of 6.4.x).

F03 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice theadditional instructions given for the first.

1. Tom sent something to Sue

2. Everyone heard a sound. [This is ambiguous but you need only analyze one interpretation; justchoose the one that seems most natural to you.]

3. There is someone who knows just one other person.

Analyze the sentence below using each of the two ways of analyzing the definite description the package. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The package rattled.

Use derivations to show that the following argument is valid. You may use any rules.

5.   ∃x Fx
       ∀x Gx
    _____
       ∃x (Fx ∧ Gx)

Use a derivation to show that the following argument is not valid and use either tables or a diagram to describe a counterexample lurking in an open gap.

6.   ∃x ∀y Rxy
    _____
       ∃x Rax

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

7. A sentence φ is equivalent to a sentence ψ (i.e., φ ≃ ψ) if and only if …

Answer the following question and explain your answer in terms of the definitions of the basic concepts it involves.

8. Suppose you are told that (i) φ ⊨ ψ and (ii) ψ is inconsistent with χ (i.e., the set formed of the two is inconsistent). What can you conclude about the relation between of φ and χ? That is, what patterns of truth values for the two are ruled out (if any are); and, if any are ruled out, what logical relation or relations holds as a result.

Complete the following truth table by calculating the truth value of the sentence on each of the given assignments. In each row, write under each connective the value of the component of which it is the main connective and circle the truth value of the sentence as a whole.

9. A B C D | (A ∧ ¬ B) ∨ ¬ (C → D)
       T T T T |
       F F T F |

F03 test 5 answers

1. Tom sent something to Sue
    ∃x Tom sent x to Sue
             ∃x Ntxs

    C: [_ sent _ to _]; s: Sue; t: Tom

2. Everyone heard a sound
    (∃x: x is a sound) everyone heard x
    (∃x: x is a sound) (∀y: y is a person) y heard x
             (∃x: Sx) (∀y: Py) Hyx

    H: [_ heard _]; P: [_ is a person]; S: [_ is a sound]

3. There is someone who knows just one other person
    ∃x x is a person who knows just one other person
    ∃x (x is a person ∧ x knows just one other person)
    ∃x (Px ∧ (∃y: Py ∧ ¬ y = x) x knows y and no other person besides y)
    ∃x (Px ∧ (∃y: Py ∧ ¬ y = x) (Kxy ∧ x knows no other person besides y))

    ∃x (Px ∧ (∃y: Py ∧ ¬ y = x) (Kxy
             ∧ (∀z: Pz ∧ ¬ z = x ∧ ¬ z = y) ¬ Kxz))
    or: ∃x (Px ∧ (∃y: Py ∧ ¬ y = x) (Kxy
             ∧ (∀z: Pz ∧ ¬ z = x ∧ Kxz) y = z))

    K: [_ knows _]; P: [_ is a person]

4. *using Russell's analysis:*
    The package rattled
    (∃x: x and only x is a package) x rattled
    (∃x: x is a package ∧ (∀y: ¬ y = x) ¬ y is a package) Rx
             (∃x: Px ∧ (∀y: ¬ y = x) ¬ Py) Rx
             or: (∃x: Px ∧ (∀y: Py) x = y) Rx

    using the description operator:
    The package rattled
    R(the package)
    R (lx x is a package)
             R(lx Px)

    P: [_ is a package]; r: [_ rattled]

5.

```
      | ∃x Fx              1
      | ∀x Gx              a: 2
      |──
      | ⓐ
      |   | Fa             (3)
      |   |──
2 UI  |   | Ga             (3)
3 Adj |   | Fa ∧ Ga        X, (4)
4 EG  |   | ∃x (Fx ∧ Gx)   X, (5)
      |   | ●
      |──
5 QED |   | ∃x (Fx ∧ Gx)   1
      |──
1 PCh | ∃x (Fx ∧ Gx)
```

6.

```
      | ∃x ∀y Rxy          1
      | ⓑ
      |   | ∀y Rby          a:3, b:4
      |   |──
      |   |   | ∀x ¬ Rax     a:5, b:6
      |   |   |──
3 UI  |   |   | Rba
4 UI  |   |   | Rbb
5 UI  |   |   | ¬ Raa
6 UI  |   |   | ¬ Rab
      |   |   | ○           Rba,Rbb,¬Raa,¬Rab ⊭ ⊥
      |   |   |──
      |   |   | ⊥           2
      |──
2 NcP |   | ∃x Rax          1
      |──
1 PCh | ∃x Rax
```


①←② (with self-loop on ②)   a   b   R →
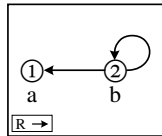
7. φ and ψ are equivalent if and only if there is no possible world in which they have different truth values (or: if and only, in every possible world, each has the same value as the other)

8. φ and χ are inconsistent. That is, φ and χ cannot be both true because ψ will be true when φ is, and ψ and χ cannot be both true. Other patterns of values for φ and χ are possible because they are not ruled out for ψ and χ by the fact that they are inconsistent and, for all weknow, φ and ψ may be equivalent.

9.

A	B	C	D	(A	∧	¬	B)	∨	¬	(C	→	D)
T	T	T	T		F	F		Ⓕ			F	
F	F	T	F		F	T		Ⓣ			T	

F02 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for the first.

1. Al received a card that made him laugh [Give this analysis also using an unrestricted quantifier.]

2. There is a toy that every child wanted

3. Santa left at least two packages

Analyze the sentence below using each of the two ways of analyzing the definite description the battery. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The battery is dead

Use derivations to show that the following argument is valid. You may use any rules.

5. ∃x (Fx ∧ Gx)
    ──────────────
    ∃x (Gx ∧ Fx)

Use a derivation to show that the following argument is not valid and use either tables or a diagram to describe a counterexample lurking in an open gap.

6. ∃x ∃y Rxy
    ──────────
    ∃x Rax

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

7. A set Γ entails a sentence φ (i.e., Γ ⊨ φ) if and only if …

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component, and circle the truth value of the sentence as a whole.

8.

A	B	C	D	(A	→	B)	∧	¬	(C	∨	¬	D)
T	F	F	T									

Give at least two restatements of the following sentence as an expansion on a term appearing in it (i.e., as an abstract applied to such a term):

9. Raba

1. Al received a card that made him laugh

some card that made Al laugh is such that (Al received it)

(∃x: x is a card that made Al laugh) Al received x

(∃x: x is a card ∧ x made Al laugh) Rax

(∃x: Cx ∧ Lxa) Rax

∃x ((Cx ∧ Lxa) ∧ Rax)

C: [_ is a card]; L: [_ made _ laugh]; R: [_ received _]; a: Al

2. There is a toy that every child wanted

Something is a toy that every child wanted

Something is such that (it is a toy that every child wanted)

∃x x is a toy that every child wanted

∃x (x is a toy ∧ every child wanted x)

∃x (Tx ∧ every child is such that (he or she wanted x))

∃x (Tx ∧ (∀y: y is a child) y wanted x)

∃x (Tx ∧ (∀y: Cy) Wyx)

C: [_ is a child]; T: [_ is a toy]; W: [_ wanted _]

3. Santa left at least two packages

at least two packages are such that (Santa left them)

(∃x: x is a package) (∃y: y is a package ∧ ¬ y = x) (Santa left
   x ∧ Santa left y)

(∃x: Px) (∃y: Py ∧ ¬ y = x) (Lsx ∧ Lsy)

L: [_ left _]; P: [_ is a package]; s: Santa

4. *using Russell's analysis:*

The battery is dead

The battery is such that (it is dead)

(∃x: x and only x is a battery) x is dead

(∃x: x is a battery ∧ (∀y: ¬ y = x) ¬ y is a battery) x is dead

(∃x: Bx ∧ (∀y: ¬ y = x) ¬ By) Dx

or: (∃x: Bx ∧ (∀y: By) x = y) Dx

using the description operator:

The battery is dead

D the battery

D(ıx x is a battery)

D(ıx Bx)

B: [_ is a battery]; D: [_ is dead]

5.

	∃x (Fx ∧ Gx)	1
ⓐ		
	Fa ∧ Ga	2
2 Ext	Fa	(6)
2 Ext	Ga	(5)
	∀x ¬ (Gx ∧ Fx)	a:4
4 UI	¬ (Ga ∧ Fa)	5
5 MPT	¬ Fa	(6)
	●	
6 Nc	⊥	3
3 NcP	∃x (Gx ∧ Fx)	1
1 PCh	∃x (Gx ∧ Fx)	

6.

	∃x ∃y Rxy	1
ⓑ		
	∃y Rby	2
ⓒ		
	Rbc	
	∀x ¬ Rax	a:4, b:5, c:6
4 UI	¬ Raa	
5 UI	¬ Rab	
6 UI	¬ Rac	
	○	Rbc,¬Raa, ¬Rab,¬Rac ⊭ ⊥
	⊥	3
3 NcP	∃x Rax	2
2 PCh	∃x Rax	1
1 PCh	∃x Rax	

```
 ①            ③
 a     ↗      c
       ②
 R→    b
```

7. A set Γ entails a sentence φ if and only if there is no possible world in which every member of Γ is true but φ is false (*or:* if and only if φ is true in every possible world in which all members of Γ are true)

8.

A	B	C	D	(A → B) ∧ ¬ (C ∨ ¬ D)
T	F	F	T	F Ⓕ T F F

9. Up to the choice of variables, the possibilities are the following:

[Rabx]$_x$a, [Rxba]$_x$a, [Rxbx]$_x$a, [Raxa]$_x$b

Phi 270 F00 test 5

F00 test 5 questions

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for each of the first two.

1. **There is a yak that someone yoked**. [Give this analysis also using an unrestricted quantifier.]

2. **Each explorer mapped a route**. [This sentence is ambiguous. Analyze it in two nonequivalent ways, and describe a situation in which the sentence is true on one of your analyses and false on the other.]

3. **Exactly one reindeer was red nosed**. [You may leave the predicate _ **was red nosed** unanalyzed.]

Analyze the sentence below using each of the two ways of analyzing the definite description **the fireplace**. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. **Santa gained entry through the fireplace.**

Use derivations to show that the following argument is valid. You may use any rules.

5. $$\underline{\exists x \, \forall y \, (Fy \rightarrow Rxy)}$$
$$\forall x \, (Fx \rightarrow \exists y \, Ryx)$$

That is: **Something is relevant to all findings** / **Each finding has something relevant to it** [Don't hesitate to ignore this English reading if it doesn't help you think about the argument.]

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in an open gap.

6. $$\underline{\exists x \, \exists y \, (\neg \, y = x \wedge Rxy)}$$
$$\exists x \, \neg \, Rxx$$

Complete the following to give a definition of inconsistency in terms of truth values and possible worlds:

7. A set Γ is inconsistent if and only if …

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component.

8.

A	B	C	D	(A ∨ ¬ B) ∧ ¬ (C → D)
T	F	T	F	

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the sentences below all true. (You may use either tables or a diagram.)

9. a = c, fc = b, d = e, Fc, Fd, ¬ Fb, Rab, Rea, R(fa)b, ¬ Re(fc)

F00 test 5 answers

1. **There is a yak that someone yoked**
   something is a yak that someone yoked
   something is such that (it is a yak that someone yoked)
   ∃x x **is a yak that someone yoked**
   ∃x (x **is a yak** ∧ **someone yoked** x)
   ∃x (Yx ∧ **someone is such that (he or she yoked** x))
   ∃x (Yx ∧ (∃y: y **is a person**) y **yoked** x)
   $$\exists x \, (Yx \wedge (\exists y: Py) \, Kyx)$$
   $$\exists x \, (Yx \wedge \exists y \, (Py \wedge Kyx))$$
   K: [_ **yoked** _]; P: [_ **is a person**]; Y: [_ **is a yak**]

2. *first analysis:*
   Each explorer mapped a route
   each explorer is such (he or she mapped a route)
   (∀x: x **is an explorer**) x **mapped a route**
   (∀x: Ex) **some route is such that (x mapped it)**
   (∀x: Ex) (∃y: y **is a route**) x **mapped** y
   $$(\forall x: Ex) \, (\exists y: Ry) \, Mxy$$

   second analysis:
   Each explorer mapped a route
   some route is st (each explorer mapped it)
   (∃x: x **is a route**) **each explorer mapped** x
   (∃x: Rx) **each explorer is such that (he or she mapped** x)
   (∃x: Rx) (∀y: y **is an explorer**) y **mapped** x
   $$(\exists x: Rx) \, (\forall y: Ey) \, Myx$$
   P: [_ **is an explorer**]; M: [_ **mapped** _]; R: [_ **is a route**]
   The first is true and the second false if every explorer mapped some route or other but no one route was mapped by all explorers

3. **Exactly one reindeer was red nosed**
   at least one reindeer was red nosed ∧ ¬ **at least two reindeer were red nosed**
   some reindeer is such that (it was red nosed) ∧ ¬ **at least two reindeer were such that (they were red nosed)**
   (∃x: x **is a reindeer**) x **was red nosed** ∧ ¬ (∃x: x **is a reindeer**) (∃y: y **is a reindeer** ∧ ¬ y = x) (x **was red nosed** ∧ y **was red nosed**)
   $$(\exists x: Rx) \, Nx \wedge \neg (\exists x: Rx) \, (\exists y: Ry \wedge \neg \, y = x) \, (Nx \wedge Ny)$$

or:

Exactly one reindeer was red nosed

some reindeer is such that (it was red nosed and no other reindeer was red nosed)

(∃x: x is a reindeer) (x was red nosed and no other reindeer was red nosed)

(∃x: Rx) (Nx ∧ no reindeer other than x was red nosed)

(∃x: Rx) (Nx ∧ no reindeer other than x is such that (it was red nosed)

(∃x: Rx) (Nx ∧ (∀y: y is a reindeer ∧ ¬ y = x) ¬ y was red nosed)

$\qquad$ (∃ x: Rx) (Nx ∧ (∀y: Ry ∧ ¬ y = x) ¬ Ny)

$\qquad$ *or:* (∃ x: Rx) (Nx ∧ (∀y: Ry ∧ Ny) x = y)

N: [_ was red nosed]; R: [_ is a reindeer]

The generalization using the variable y must be resricted to reindeer or else the sentence will say that some reindeer is the only and only thing that is red nosed—i.e., that there is exactly one red-nosed thing and it is a reindeer.

4.   *using Russell's analysis:*

Santa gained entry through the fireplace

the fireplace is such that (Santa gained entry through it)

(∃x: x and only x is a fireplace) Santa gained entry through x

(∃x: x is a fireplace ∧ (∀y: ¬ y = x) ¬ y is a fireplace) Gsx

$\qquad$ (∃x: Fx ∧ (∀y: ¬ y = x) ¬ Fy) Gsx

$\qquad$ *or:* (∃x: Fx ∧ (∀y: Fy) x = y) Gsx

using the description operator:

Santa gained entry through the fireplace

G s (the fireplace)

G s (ɪx x is a fireplace)

$\qquad$ Gs(ɪx Fx)

F: [_ is a fireplace]; G: [_ gained entry through _]; s: Santa

---

5.

	∃x ∀y (Fy → Rxy)	
	ⓐ ∀y (Fy → Ray)	b:4
	ⓑ Fb	(5)
4 UI	Fb → Rab	5
5 MPP	Rab	(6)
6 EG	∃y Ryb	X, (7)
	●	
7 QED	∃y Ryb	3
3 CP	Fb → ∃y Ryb	2
2 UG	∀x (Fx → ∃y Ryx)	1
1 PCh	∀x (Fx → ∃y Ryx)	

6.

	∃x ∃y (¬ y = x ∧ Rxy)	1
	ⓐ ∃y (¬ y = a ∧ Ray)	2
	ⓑ ¬ b = a ∧ Rab	3
3 Ext	¬ b = a	
3 Ext	Rab	
	∀x Rxx	a:5, b:6
5 UI	Raa	
6 UI	Rbb	
	○	¬ b = a, Rab, Raa, Rbb ⊭ ⊥
	⊥	4
4 NcP	∃x ¬ Rxx	2
2 PCh	∃x ¬ Rxx	1
1 PCh	∃x ¬ Rxx	
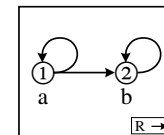
7.   A set Γ is inconsistent if and only if there is no possible world in which every member of Γ is true

8.

A	B	C	D	(A ∨ ¬ B) ∧ ¬ (C → D)
T	F	T	F	$\quad$ T T $\quad$ ⓉT $\quad$ F

9.

range: 1,	a b c d e	τ	fτ	τ	Fτ	R	1 2 3
2, 3	1 2 1 3 3	1	2	1	T	1	F T F
		2	2	2	F	2	F T F
		3	3	3	T	3	T F F
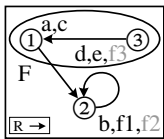
Diagram: a,c ① ③ d,e,f3 F ② b,f1,f2 R →

(The diagram above provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

alias sets IDs values

a	1	a: 1
c		c: 1
b	2	b: 2
fa		f1: 2
fc		f1: 2
d	3	d: 3
e		e: 3

resources values

Fc	F1: **T**
Fd	F3: **T**
¬ Fb	F2: **F**
Rab	R12: **T**
Rea	R31: **T**
R(fa)b	R22: **T**
¬ Re(fc)	R32: **F**

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for each of the first two.

1. Sam mentioned someone Tina didn't know. [Give this analysis also using an unrestricted quantifier.]

2. Every shoe fit someone. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]

3. Sam found at least two pieces.

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The elephant standing on Sam sighed.

5. [This question was on a topic not covered this year]

Use derivations to show that the following argument is valid. You may use attachment rules (but not replacement by equivalence).

6. $\forall x \, \forall y \, (Rxy \rightarrow (Ryx \rightarrow Rxx))$

        $\exists x \, \exists y \, (Rxy \wedge Ryx)$
        _____

              $\exists x \, Fxx$

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in an open gap.

7. $\exists x \, Fx$

        $\exists x \, (Gx \wedge Hx)$
        _____

        $\exists x \, (Fx \wedge Hx)$

Complete the following to give a definition of entailment by a single sentence (i.e., implication) in terms of truth values and possible worlds:

8. A sentence φ entails a sentence ψ if and only if …

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component.

9. | A B C D | ¬ (A ∧ B) → (C ∨ ¬ D) |
     |---|---|
     | T F F T | |

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the sentences below all true. (You may use either tables or a diagram.)

10.   a = fb, fb = fc, fa = c, Pa, Pb, ¬ Pc, Rab, Rbc, Rc(fb)

1. Sam mentioned someone Tina didn't know

   someone Tina didn't know is such that (Sam mentioned him or her)

   (∃x: x is a person Tina didn't know) Sam mentioned x

   (∃x: x is a person ∧ ¬ Tina knew x) Sam mentioned x

   $$(∃x: Px ∧ ¬ Ktx) \, Msx$$
   $$∃x \, ((Px ∧ ¬ Ktx) ∧ Msx)$$

   K: [_ knew _]; M: [_ mentioned _]; P: [_ is a person]; s: Sam;
   t: Tina

2. *first analysis:*

   Every shoe fit someone

   every shoe is such that (it fit someone)

   (∀x: x is a shoe) x fit someone

   (∀x: Sx) someone is such that (x fit him or her)

   (∀x: Sx) (∃y: y is a person) x fit y

   $$(∀x: Sx) (∃y: Py) \, Fxy$$

   second analysis:

   Every shoe fit someone

   someone is such that (every shoe fit him or her)

   (∃x: x is a person) every shoe fit x

   (∃x: Px) every shoe is such that (it fit x)

   (∃x: Px) (∀y: y is a shoe) y fit x

   $$(∃x: Px) (∀y: Sy) \, Fyx$$

   F: [_ fit _]; P: [_ is a person]; S: [_ is a shoe]

   The sentence is true on the first analysis and false on the second if every shoe could be worn but not all by the same person

3. Sam found at least two pieces

   at least two pieces are such that (Sam found them)

   (∃x: x is a piece) (∃y: y is a piece ∧ ¬ y = x) (Sam found x ∧ Sam found y)

   $$(∃ x: Px) (∃ y: Py ∧ ¬ y = x) (Fsx ∧ Fsy)$$

   F: [_ found _]; P: [_ is a piece]; s: Sam

4. *using Russell's analysis:*

   The elephant standing on Sam sighed

   The elephant standing on Sam is such that (it sighed)

   (∃x: x and only x is an elephant standing on Sam) x sighed

   (∃x: x is an elephant standing on Sam ∧ (∀y: ¬ y = x) ¬ y is an elephant standing on Sam) Sx

   (∃x: (x is an elephant ∧ x is standing on Sam) ∧ (∀y: ¬ y = x) ¬ (y is an elephant ∧ y is standing on Sam)) Sx

   $$(∃x: (Ex ∧ Txs) ∧ (∀y: ¬ y = x) ¬ (Ey ∧ Tys)) \, Sx$$

   or:

   $$(∃x: (Ex ∧ Txs) ∧ (∀y: Ey ∧ Tys) \, x = y) \, Sx$$

   using the description operator:

   The elephant standing on Sam sighed

   S (the elephant standing on Sam)

   S (lx x is an elephant standing on Sam)

   S (lx (x is an elephant ∧ x is standing on Sam))

   $$S(lx \, (Ex ∧ Txs))$$

   E: [_ is an elephant]; S: [_ sighed]; T: [_ is standing on _]; s: Sam

5. [This question was on a topic not covered this year]

6.

	∀x ∀y (Rxy → (Ryx → Rxx))	a:4
	∃x ∃y (Rxy ∧ Ryx)	1
ⓐ		
	∃y (Ray ∧ Rya)	2
ⓑ		
	Rab ∧ Rba	3
3 Ext	Rab	(6)
3 Ext	Rba	(7)
4 UI	∀y (Ray → (Rya → Raa))	b:5
5 UI	Rab → (Rba → Raa)	6
6 MPP	Rba → Raa	7
7 MPP	Raa	(8)
8 EG	∃x Rxx	X, (9)
	●	
9 QED	∃x Rxx	2
2 PCh	∃x Rxx	1
1 PCh	∃x Rxx	

7.

$\exists x \, Fx$ 1
$\exists x \, (Gx \wedge Hx)$ 2

ⓐ | Fa   (7)

    ⓑ | Gb ∧ Hb   3

3 Ext | Gb
3 Ext | Hb   (8)

     | ∀x ¬ (Fx ∧ Hx)   a:5, b:6

5 UI | ¬ (Fa ∧ Ha)   7
6 UI | ¬ (Fb ∧ Hb)   8
7 MPT | ¬ Ha
8 MPT | ¬ Fb
     | ○   Fa,Gb,Hb,¬Ha,¬Fb ⊭ ⊥

     | ⊥   4

4 NcP | ∃x (Fx ∧ Hx)   2

2 PCh | ∃x (Fx ∧ Hx)   1

1 PCh   ∃x (Fx ∧ Hx)
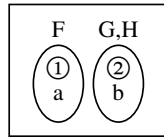
```
┌─────────────┐
│  F     G,H  │
│ ⎛①⎞   ⎛②⎞  │
│ ⎝a⎠   ⎝b⎠  │
└─────────────┘
```

8. A sentence φ entails a sentence ψ if and only if there is no possible world in which φ is true but ψ is false (*or:* if and only if ψ is true in every possible world in which φ is true)

9.

A	B	C	D	¬	(A	∧	B)	→	(C	∨	¬	D)
T	F	F	T	T		F		Ⓕ		F	F	

10. range: 1, 2, 3

a	b	c
1	2	3

τ	fτ
1	3
2	1
3	1

τ	Pτ
1	T
2	T
3	F

R	1	2	3
1	F	T	F
2	F	F	T
3	T	F	T

```
┌──────────────R→──┐
│P      ②           │
│      /b\          │
│  ①←     ↘③        │
│ a,f2,f3  c,f1     │
└──────────────────┘
```
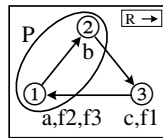
The diagram above provides a complete answer, as do the tables to its left. The tables below illustrate a way of finding this structure.

alias sets	IDs	values
a	1	a: 1
fb		f2: 1
fc		f3: 1
b	2	b: 2
c	3	c: 3
fa		f1: 3

resources	values
Pa	P1: **T**
Pb	P2: **T**
¬ Pc	P3: **F**
Rab	R12: **T**
Rbc	R23: **T**
Rc(fb)	R31: **T**

Phi 270 F98 test 5

F98 test 5 questions

(These questions are from the last of the 6 quizzes given in F98.)

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. George traveled to LA by way of some town in Wyoming. [Give this analysis also using an unrestricted quantifier.]

2. Everyone is afraid of something. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]

3. Spot knew exactly one trick.

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. Tom opened the letter from Bulgaria

Use derivations to show that the following argument is valid. You may use any rules.

5. $\exists x \, (Fx \wedge \exists y \, \neg \, x = y)$

$\exists x \, \exists y \, (\neg \, y = x \wedge Fy)$

That is: Some finding is different from something ⊨ Something is such that something different from it is a finding [but don't hesitate to ignore the English if it doesn't help].

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in an open gap.

6. $\exists x \, \exists y \, Rxy$

$\exists x \, Rxx$

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

7. A sentence φ is equivalent to a sentence ψ if and only if …

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the 8 sentences below all true.

8.   fab = fba, ga = fab, fba = c, Fb, F(ga), Rab, ¬ Rba, R(ga)c

9. [This question was on a topic not covered this year]

1. George traveled to LA by way of some town in Wyoming
    some town in Wyoming is such that (George traveled to LA by way of
    it)
    (∃x: x is a town in Wyoming) George traveled to LA by way of x
    (∃x: x is a town ∧ x is in Wyoming) George traveled to LA by way of x
    (∃x: Tx ∧ Nxm) Rglx
    ∃x ((Tx ∧ Nxm) ∧ Rglx)

    N: [_ is in _]; R: [_ traveled to _ by way of _]; T: [_ is a town]; g: George; l: LA; m: Wyoming

2. *first analysis:*
    Everyone is afraid of something
    everyone is such that (he or she is afraid of something)
    (∀x: x is a person) x is afraid of something
    (∀x: Px) something is such that (x is afraid of it)
    (∀x: Px) ∃y x is afraid of y
    (∀x: Px) ∃y Axy

    second analysis:
    Everyone is afraid of something
    something is such that (everyone is afraid of it)
    ∃x everyone is afraid of x
    ∃x everyone is such that (he or she is afraid of x)
    ∃x (∀y: y is a person) y is afraid of x
    ∃x (∀y: Py) Ayx

    A: [_ is afraid of _]; P: [_ is a person]
    The first is true and the second false if all people are fearful but not all fearful of the same thing

3. Spot knew exactly one trick
    Spot knew a trick ∧ ¬ Spot knew at least two tricks
    (∃ x: x is a trick) Spot knew x ∧ ¬ (∃ x: x is a trick) (∃ y: y is a
    trick ∧ ¬ y = x) (Spot knew x ∧ Spot knew y)
    (∃ x: Tx) Ksx ∧ ¬ (∃ x: Tx) (∃ y: Ty ∧ ¬ y = x) (Ksx ∧ Ksy)
    or
    (∃ x: Tx) (Ksx ∧ (∀ y: Ty ∧ ¬ y = x) ¬ Ksy)
    or
    (∃ x: Tx) (Ksx ∧ (∀ y: Ty ∧ Ksy) x = y)
    K: [_ knew _]; T: [_ is a trick]; s: Spot

4. *using Russell's analysis:*
    Tom opened the letter from Bulgaria
    the letter from Bulgaria is such that (Tom opened it)
    (∃x: x and only x is a letter from Bulgaria) Tom opened x
    (∃x: x is a letter from Bulgaria ∧ (∀y: ¬ y = x) ¬ y is a letter from
    Bulgaria) Otx
    (∃x: x is a letter ∧ x is from Bulgaria ∧ (∀y: ¬ y = x) ¬ y is a let-
    ter ∧ y is from Bulgaria) Otx
    (∃x: (Lx ∧ Fxb) ∧ (∀y: ¬ y = x) ¬ (Ly ∧ Fyb)) Otx
    or: (∃x: (Lx ∧ Fxb) ∧ (∀y: Ly ∧ Fyb) x = y) Otx

    using the description operator:
    Tom opened the letter from Bulgaria
    Ot(the letter from Bulgaria)
    Ot(lx x is a letter from Bulgaria)
    Ot(lx (x is a letter ∧ x is from Bulgaria))
    Ot(lx (Lx ∧ Fxb))

    F: [_ is from _]; L: [_ is a letter]; O: [_ opened _]; b: Bulgaria; t: Tom

5.

	∃x (Fx ∧ ∃y ¬ x = y)	1
	ⓐ Fa ∧ ∃y ¬ a = y	2
2 Ext	Fa	(4)
2 Ext	∃y ¬ a = y	3
	ⓑ ¬ a = b	(4)
4 Adj	¬ a = b ∧ Fa	X, (5)
5 EG	∃y (¬ y = b ∧ Fy)	X, (6)
6 EG	∃x ∃y (¬ y = x ∧ Fy)	X, (7)
	●	
7 QED	∃x ∃y (¬ y = x ∧ Fy)	3
3 PCh	∃x ∃y (¬ y = x ∧ Fy)	1
1 PCh	∃x ∃y (¬ y = x ∧ Fy)	

6.

∃x ∃y Rxy   1

ⓐ
  ∃y Ray   2

ⓑ
   Rab

    ∀x ¬ Rxx   a:4,b:5

4 UI   ¬ Raa
5 UI   ¬ Rbb
      ○   Rab, ¬Raa, ¬Rbb ⊭ ⊥

      ⊥   3

3 NcP ∃x Rxx   2

2 PCh ∃x Rxx   1

1 PCh ∃x Rxx

7. A sentence φ is equivalent to a sentence ψ if and only if there is no possible world in which φ and ψ have different truth values

8. range: 1, 2, 3

a b c	f	1 2 3		τ	gτ	τ	Fτ	R	1 2 3
1 2 3	1	1 3 1		1	3	1	F	1	F T F
	2	3 1 1		2	1	2	T	2	F F F
	3	1 1 1		3	1	3	T	3	F F T
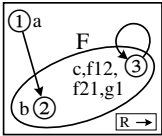
Only non-arbitrary values are shown for f and g

The diagram provides a complete answer, as do the tables above it. The tables below are a way of finding this structure.

alias sets	IDs	values		resources	values
a	1	a: 1		Fb	F2: **T**
b	2	b: 2		F(ga)	F3: **T**
c	3	c: 3		Rab	R12: **T**
fab		f12: 3		¬ Rba	R21: **F**
fba		f21: 3		R(ga)c	R33: **T**
ga		g1: 3			

9. [This question was on a topic not covered this year]

(These questions are from the last of the 6 quizzes given in F97.)

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. Tom phoned someone who had left a message for him. [Give this analysis also using an unrestricted quantifier.]

2. Santa said something to each child. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]

3. Ron asked Santa for at least two things.

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. Bill lent the book Ann gave him to Carol

Use derivations to show that the following argument is valid. You may use any rules.

5.   ∃x ∃y (Rxy ∧ Sxy)

   ∃y ∃x (Sxy ∧ Rxy)

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in an open gap.

6.   ∃x Rax

   ∃x Rxa

Complete the following to give a definition of inconsistency in terms of truth values and possible worlds:

7. A set Γ is inconsistent if and only if …

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the list of 5 sentences below all true and use it to calculate a truth value for the sentence that follows them. (You may present the structure using either tables or a diagram.)

8. *make true:* b = ga, fa = f(ga), Rab, R(fa)a, ¬ R(fb)b

   calculate: (b = gb ∨ Ra(ga)) → (R(fa)(ga) ∧ f(gb) = g(fb))

Give two different restatements of the sentence below in expanded form as a complex predicate (i.e., an abstract) applied to a term.

9. ∃y Rayb

F97 test 5 answers

1. Tom phoned someone who had left a message for him

   someone who had left a message for Tom is such that (Tom phoned him or her)

   (∃x: x is a person who had left a message for Tom) Tom phoned x

   (∃x: x is a person ∧ x had left a message for Tom) Htx

   (∃x: Px ∧ some message is such (x had left it for Tom)) Htx

   (∃x: Px ∧ (∃y: y is a message) x had left y for Tom) Htx

   $$(\exists x: Px \wedge (\exists y: My)\ Lxyt)\ Htx$$
   $$\exists x\ ((Px \wedge \exists y\ (My \wedge Lxyt)) \wedge Htx)$$

   H: [_ phoned _]; L: [_ had left _ for _]; M: [_ is a message]; P: [_ is a person]; t: Tom

2. *first analysis:*

   each child is such that (Santa said something to him or her)

   (∀x: x is a child) Santa said something to x

   (∀x: Cx) something is such that (Santa said it to x)

   (∀x: Cx) ∃y Santa said y to x

   $$(\forall x: Cx)\ \exists y\ Dsyx$$

   second analysis:

   something is such that (Santa said it to each child)

   ∃x Santa said x to each child

   ∃x each child is such that (Santa said x to him or her)

   ∃x (∀y: y is a child) Santa said x to y

   $$\exists x\ (\forall y: Cy)\ Dsxy$$

   C: [_ is a child]; D: [_ said _ to _]; s: Santa

   The sentence is true on the first analysis and false on the second in a situation where Santa spoke to each child but said different things to different children

3. Ron asked Santa for at least two things

   ∃x (∃y: ¬ y = x) (Ron asked Santa for x ∧ Ron asked Santa for y)

   $$\exists x\ (\exists y: \neg\ y = x)\ (Arsx \wedge Arsy)$$

   A: [_ asked _ for _]; r: Ron; s: Santa

4. *using Russell's analysis:*

   Bill lent the book Ann gave him to Carol

   the book Ann gave Bill is such that (Bill lent it to Carol)

   (∃x: x and only x is a book Ann gave Bill) Bill lent x to Carol

   (∃x: x is a book Ann gave Bill ∧ (∀y: ¬ y = x) ¬ y is a book Ann gave Bill) Lbxc

   (∃x: (x is a book ∧ Ann gave Bill x) ∧ (∀y: ¬ y = x) ¬ (y is a book ∧ Ann gave Bill y)) Lbxc

   $$(\exists x: (Bx \wedge Gabx) \wedge (\forall y: \neg\ y = x)\ \neg\ (By \wedge Gaby))\ Lbxc$$
   or:
   $$(\exists x: (Bx \wedge Gabx) \wedge (\forall y: By \wedge Gaby)\ x = y)\ Lbxc$$

   using the description operator:

   Bill lent the book Ann gave him to Carol

   Lb(the book Ann gave Bill)c

   Lb(lx x is a book Ann gave Bill)c

   Lb(lx (x is a book ∧ Ann gave Bill x))c

   $$Lb(lx\ (Bx \wedge Gabx))c$$

   B: [_ is a book]; G: [_ gave _ _]; L: [_ lent _ to _]; a: Ann; b: Bill; c: Carol

5.

	∃x ∃y (Rxy ∧ Sxy)	1
ⓐ	∃y (Ray ∧ Say)	2
ⓑ	Rab ∧ Sab	3
3 Ext	Rab	(4)
3 Ext	Sab	(4)
4 Adj	Sab ∧ Rab	X, (5)
5 EG	∃x (Sxb ∧ Rxb)	X, (6)
6 EG	∃y ∃x (Sxy ∧ Rxy)	X, (7)
	●	
7 QED	∃y ∃x (Sxy ∧ Rxy)	2
2 PCh	∃y ∃x (Sxy ∧ Rxy)	1
1 PCh	∃y ∃x (Sxy ∧ Rxy)	

6.

$\exists x\ Rax$

ⓑ | Rab

| $\forall x\ \neg\ Rxa$ a:3,b:4

3 UI | $\neg\ Raa$
4 UI | $\neg\ Rba$
    ○   Rab, ¬Raa, ¬Rba ⊭ ⊥

    ⊥   2

2 NcP | $\exists x\ Rxa$ 1

1 PCh | $\exists x\ Rxa$

7. A set Γ is inconsistent if and only if there is no possible world in which every member of Γ is true.

8. range: 1, 2, 3

a b	τ	fτ	τ	gτ	R	1	2	3
1 2	1	3	1	2	1	F	T	F
	2	3	2	3	2	F	F	F
	3	2	3	3	3	T	F	F

$$\underline{(b\ =\ g\,b\ \lor\ R\ a\ (g\,a))\ \rightarrow\ (R\ (f\,a)\,(g\,a)\ \land\ f\,(g\,b)\ =\ g\,(f\,b))}$$
2 F 3 2 T T 1   2 1   Ⓕ   F   3 1   2 1   F   2 3 2   F 3 3 2

Your values for some of the compound terms and equations may differ from those shown here in gray, but your values for other predications and for truth-functional compounds should be the same as those shown.

The diagram above provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.

alias sets	*IDs*	*values*		*resources*	*values*
a	1	a: 1		Rab	R12: **T**
b	2	b: 2		R(fa)a	R31: **T**
ga		g1: 2		¬ R(fb)b	R32: **F**
fa	3	f1: 3			
fb		f2: 3			
f(ga)		f2: 3			

9. The following are 3 possibilities (up to choice of the variable) from which your two might be chosen; in the last, τ may be any term:

$$[\exists y\ Rxyb]_x a,\ [\exists y\ Rayx]_x b,\ [\exists y\ Rayb]_x \tau$$

F96 test 5 questions

(These questions are from the last of the 6 quizzes given in F96.)

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. Ned has visited a museum in Linden. [Give this analysis also using an unrestricted quantifier.]

2. Something blocked each route. [This sentence is ambiguous. Analyze it in two ways, as making a claim of *general exemplification* and as making the stronger claim of *uniformly general exemplification*, and indicate which analysis is which.]

3. At most one plan was implemented.

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The scout you saw saw you.

Use derivations to show that the following argument is valid. You may use any rules.

5. $\exists x\ Rax$
     $\underline{\forall x\ (\exists y\ Ryx\ \rightarrow\ Fx)}$
        $\exists x\ Fx$

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in an open gap.

6. $\exists x\ Fx$
      $\underline{Ga}$
  $\exists x\ (Fx\ \land\ Gx)$

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

7. A sentence φ is entailed by a set Γ if and only if …

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the following sentences all true. (You may present the structure using either tables or a diagram.)

8. a = b, fb = fc, Pa, ¬ P(fa), Rab, ¬ Rbc, Rb(fb)

Give two different restatements of the sentence below in expanded form as a complex predicate (i.e., an abstract) applied to a term.

9. Fa ∧ Ga

1. Ned has visited a museum in Linden

(∃x: x is a museum in Linden) Ned has visited x

(∃x: x is a museum ∧ x is in Linden) Ned has visited x

(∃x: Mx ∧ Nxl) Vnx

∃x ((Mx ∧ Nxl) ∧ Vnx)

M: [_ is a museum]; N: [_ is in _]; V: [_ has visited _]; l: Linden;

n: Ned

2. *general exemplication*

(∀x: x is a route) something blocked x

(∀x: Rx) ∃y y blocked x

(∀x: Rx) ∃y Byx

uniformly general exemplication

∃y y blocked each route

∃y (∀x: x is a route) y blocked x

∃y (∀x: Rx) Byx

B: [_ blocked _]; R: [_ is a route]

3. At most one plan was implemented

¬ at least two plans were implemented

¬ (∃x: x is a plan) (∃y: y is a plan ∧ ¬ y = x) (x was implemented ∧ y

was implemented)

¬ (∃x: Px) (∃y: Py ∧ ¬ y = x) (Ix ∧ Iy)

I: [_ was implemented]; P: [_ is a plan]

4. *using Russell's analysis:*

the scout you saw is such that (he or she saw you)

(∃x: x and only x is a scout you saw) Sxo

(∃x: x is a scout you saw ∧ (∀y: ¬y = x) ¬ y is a scout you saw) Sxo

(∃x: (Tx ∧ Sox) ∧ (∀y: ¬y = x) ¬ (Ty ∧ Soy)) Sxo

using the description operator:

the scout you saw saw you

S(the scout you saw)o

S(l x x is a scout you saw)o

S(l x (x is a scout ∧ you saw x))o

S(l x (Tx ∧ Sox))o

S: [_ saw _]; T: [_ is a scout]; o: you

5.

	∃x Rax	1
	∀x (∃y Ryx → Fx)	b:2
	ⓑ	
	Rab	(3)
2 UI	∃y Ryb → Fb	4
3 EG	∃y Ryb	X, (4)
4 MPP	Fb	(4)
5 EG	∃x Fx	X, (6)
	●	
6 QED	∃x Fx	1
1 PCh	∃x Fx	

6.

	∃x Fx	1
	Ga	(4)
	ⓑ	
	Fb	(6)
	∀x ¬ (Fx ∧ Gx)	a:3, b:5
3 UI	¬ (Fa ∧ Ga)	4
4 MPT	¬ Fa	
5 UI	¬ (Fb ∧ Gb)	6
6 MPT	¬ Gb	
	○	¬Fa, Fb, Ga, ¬Gb ⊭ ⊥
	⊥	2
2 NcP	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	
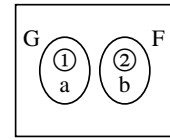
7. A sentence φ is entailed by a set Γ of sentences if and only if there is no possible world in which φ is false while each member of Γ is true.

8. range: 1,

a b c	τ	fτ	τ	Pτ	R	1	2	3
2, 3	1	2	1	T	1	T	T	F
1 1 3	2	1	2	F	2	F	F	F
	3	2	3	F	3	F	F	F
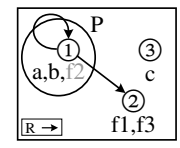
(The diagram provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

alias sets	IDs	values		resources	values
a	1	a: 1		Pa	P1: **T**
b		b: 1		¬ P(fa)	P2: **F**
fa	2	f1: 2		Rab	R11: **T**
fb		f1: 2		¬ Rbc	R13: **F**
fc		f3: 2		Rb(fb)	R12: **T**
c	3	c: 3			

9. The following are 4 possibilities (up to choice of the variable) from which your two might be chosen; in the last, τ may be any term:

$$[Fx \wedge Gx]_x a$$
$$[Fx \wedge Ga]_x a$$
$$[Fa \wedge Gx]_x a$$
$$[Fa \wedge Ga]_x \tau$$