

### Phi 270 F11 test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. Also restate your analyses using unrestricted quantifiers.

1. Everyone who called Bill praised Al.
2. The show didn't please any critic.
3. No bid met every specification.

Synthesize an English sentence that has the following logical form; that is, devise a sentence that would have the following analysis:

4.  $(\forall x: (Tx \wedge Nxs) \wedge Pxr) \neg Ftx$   
 F: [ \_ found \_ ]; N: [ \_ was in \_ ]; P: [ \_ predated \_ ]; T: [ \_ was a tree ]; r: the storm s: the stand t: Tom

Use derivations to show that the following arguments are valid. You may use any rules.

5. 
$$\frac{\forall x (Fx \wedge Gx)}{\forall x Gx}$$
6. 
$$\frac{\forall x (Rxa \rightarrow \forall y Rxy)}{\forall x Rxx}$$
  

$$\frac{\forall x Rxx}{\forall x Rax}$$

Use a derivation to show that the following argument is not valid and present a counterexample that lurks in an open gap. (You may present the counterexample either by a diagram or by tables.)

7. 
$$\frac{\forall x Rxa}{\forall x \forall y Rxy}$$

### F11 test 4 answers

1. Everyone who called Bill praised Al  
 Everyone who called Bill is such that (he or she praised Al)  
 $(\forall x: x \text{ is a person who called Bill}) x \text{ praised Al}$   
 $(\forall x: x \text{ is a person} \wedge x \text{ called Bill}) x \text{ praised Al}$   
 $(\forall x: Px \wedge Cxb) Rxa$   
 $\forall x ((Px \wedge Cxb) \rightarrow Rxa)$   
 C: [ \_ called \_ ]; P: [ \_ is a person ]; R: [ \_ praised \_ ]; a: Al; b: Bill
2. The show didn't please any critic  
 every critic is such that (the show didn't please him or her)  
 $(\forall x: x \text{ is a critic}) \text{ the show didn't please } x$   
 $(\forall x: x \text{ is a critic}) \neg \text{ the show pleased } x$   
 $(\forall x: Cx) \neg Pxs$   
 $\forall x (Cx \rightarrow \neg Pxs)$   
 C: [ \_ is a critic ]; P: [ \_ pleased \_ ]; s: the show
3. No bid met every specification  
 No bid is such that (it met every specification)  
 $(\forall x: x \text{ is a bid}) \neg x \text{ met every specification}$   
 $(\forall x: Bx) \neg \text{ every specification is such that } (x \text{ met it})$   
 $(\forall x: Bx) \neg (\forall y: y \text{ is a specification}) x \text{ met } y$   
 $(\forall x: Bx) \neg (\forall y: Sy) Mxy$   
 $\forall x (Bx \rightarrow \neg \forall y (Sy \rightarrow Mxy))$   
 B: [ \_ is a bid ]; M: [ \_ met \_ ]; S: [ \_ is a specification ]
4.  $(\forall x: (x \text{ was a tree} \wedge x \text{ was in the stand}) \wedge x \text{ predated the storm}) \neg$   
 Tom found x  
 $(\forall x: x \text{ was a tree in the stand} \wedge x \text{ predated the storm}) \neg \text{ Tom found } x$   
 $(\forall x: x \text{ was a tree in the stand that predated the storm}) \neg \text{ Tom found } x$   
 no tree in the stand that predated the storm it such that (Tom found it)  
 every tree in the stand that predated the storm it such that (Tom didn't find it)  
 Tom found no tree in the stand that predated the storm  
 or: Tom didn't find any tree in the stand that predated the storm

incorrect: Tom didn't find every tree in the stand that predated the storm

### Phi 270 F10 test 4

#### F10 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class—for example,
  - the proper analysis of **every**, **no**, and **only** (see §7.2.2),
  - how to incorporate bounds on complementary generalizations (see §7.2.3),
  - ways of handling compound quantifier phrases (such as **only cats and dogs**, see §7.3.2),
  - the distinction between **every** and **any** (see §§7.3.3 and 7.4.2),
  - how to analyze multiple quantifier phrases with overlapping scope (see §7.4.1).

You should be able restate your analysis using unrestricted quantifiers (see §7.2.1), but you will not need to present it in English notation.

- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. Remember that the distinction between **every** and **any** can be important here, too.
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rules introduced in §7.8.1.

#### F10 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Also restate your analyses using unrestricted quantifiers.*

1. **No one was disappointed.**
2. **If any part was missing, the set wasn't assembled.**
3. **Only cartoons appealed to everyone.**

Synthesize an English sentence that has the following logical form; that is, devise a sentence that would have the following analysis:

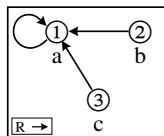
4.  $\neg (\forall x: Jx \wedge \neg Sx) \neg Fx$

F: [ **\_ was finished** ]; J: [ **\_ is a job** ]; S: [ **\_ is small** ]

Use derivations to show that the following arguments are valid. You may use

|    |  |  |  |
|----|--|--|--|
| 5. | $\forall x (Fx \wedge Gx)$ a:2<br>2 UI $Fa \wedge Ga$ 3<br>3 Ext $Fa$<br>3 Ext $Ga$ (4)<br>●<br>4 QED $Ga$ 1<br>1 UG $\forall x Gx$  |  |  |
| 6. | $\forall x (Rxa \rightarrow \forall y Rxy)$ a:2<br>$\forall x Rxx$ a:3<br>●<br>2 UI $Raa \rightarrow \forall y Ray$ 4<br>3 UI $Raa$ (4)<br>4 MPP $\forall y Ray$ b:5<br>5 UI $Rab$ (6)<br>●<br>6 QED $Rab$ 1<br>1 UG $\forall x Rax$ |  |  |
| 7. | $\forall x Rxa$ a:3, b:4, c:5<br>●<br>3 UI $Raa$<br>4 UI $Rba$<br>5 UI $Rca$<br>$\neg Rbc$<br>○ $\neg Rbc, Rca, Rba, Raa \neq \perp$<br> <br>⊥    6<br>6 IP $Rbc$ 2<br>2 UG $\forall y Rby$ 1<br>1 UG $\forall x \forall y Rxy$      |  |  |

Counterexample presented by a diagram    Counterexample presented by tables



|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| a | b | c | R | 1 | 2 | 3 |
| 1 | 2 | 3 | 1 | T | F | F |
|   |   |   | 2 | T | F | F |
|   |   |   | 3 | T | F | F |

any rules.

5. 
$$\frac{\forall x Mx \quad \forall x (Mx \rightarrow Qx)}{\forall x Qx}$$
6. 
$$\frac{\forall x \forall y (Fx \rightarrow Gy)}{Fa \rightarrow \forall x Gx}$$

Use a derivation to show that the following argument is not valid and present a counterexample that lurks in an open gap. (You may present the counterexample either by a diagram or by tables.)

7. 
$$\frac{Rab \quad \forall x Rxa}{\forall x Rxb}$$

**F10 test 4 answers**

- no one was disappointed.  
 no one is such that (he or she was disappointed)  
 $(\forall x: x \text{ is a person}) \neg x \text{ was disappointed}$   
 $(\forall x: Px) \neg Dx$   
 $\forall x (Px \rightarrow \neg Dx)$

D: [ \_ was disappointed ]; P: [ \_ is a person ]
- if any part was missing, the set wasn't assembled  
 every part is such that (if it was missing, the set wasn't assembled)  
 $(\forall x: x \text{ is a part}) \text{ if } x \text{ was missing, the set wasn't assembled}$   
 $(\forall x: Px) (x \text{ was missing} \rightarrow \text{the set wasn't assembled})$   
 $(\forall x: Px) (Mx \rightarrow \neg \text{the set was assembled})$   
 $(\forall x: Px) (Mx \rightarrow \neg As)$   
 $\forall x (Px \rightarrow (Mx \rightarrow \neg As))$

A: [ \_ was assembled ]; M: [ \_ was missing ]; P: [ \_ is a part ]; s: the set
- only cartoons appealed to everyone  
 only cartoons were such that (they appealed to everyone)  
 $(\forall x: \neg x \text{ is a cartoon}) \neg x \text{ appealed to everyone}$   
 $(\forall x: \neg Cx) \neg \text{everyone is such that } (x \text{ appealed to him or her})$   
 $(\forall x: \neg Cx) \neg (\forall y: y \text{ is a person}) x \text{ appealed to } y$   
 $(\forall x: \neg Cx) \neg (\forall y: Py) Axy$   
 $\forall x (\neg Cx \rightarrow \neg \forall y (Py \rightarrow Axy))$

A: [ \_ appealed to \_ ]; C: [ \_ is cartoon ]; P: [ \_ is a person ]

- $\neg (\forall x: x \text{ is a job} \wedge \neg x \text{ is small}) \neg x \text{ was finished}$   
 $\neg (\forall x: x \text{ is a job} \wedge x \text{ isn't small}) x \text{ was unfinished}$   
 $\neg (\forall x: x \text{ is a job that isn't small}) x \text{ was unfinished}$   
 $\neg \text{every job that isn't small it such that (it was unfinished)}$   
 $\neg \text{every job that isn't small was unfinished}$   
 not every job that isn't small was unfinished  
 or: among jobs not only small ones were finished  
 or: not only small jobs were finished  
 or: it's false that no jobs that are not small were finished

5. 
$$\frac{\forall x Mx \quad \forall x (Mx \rightarrow Qx)}{\forall x Qx}$$
 a:2 a:3

2 UI Ma (4)  
 3 UI Ma → Qa 4  
 4 MPP Qa (5)  
 5 QED Qa 1  
 1 UG  $\forall x Qx$

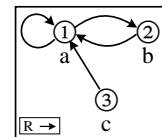
6. 
$$\frac{\forall x \forall y (Fx \rightarrow Gy)}{Fa \rightarrow \forall x Gx}$$
 a:3

3 UI Fa (5)  
 4 UI  $\forall y (Fa \rightarrow Gy)$  b:4  
 4 UI Fa → Gb 5  
 5 MPP Gb (6)  
 6 QED Gb 2  
 2 UG  $\forall x Gx$  1  
 1 CP Fa →  $\forall x Gx$

7. 
$$\frac{Rab \quad \forall x Rxa}{\forall x Rxb}$$
 a:2, b:3, c:4

2 UI Raa  
 3 UI Rba  
 4 UI Rca  
 $\neg Rcb$   
 $\neg Rcb, Rca, Rba, Raa, Rab \neq \perp$   
 $\perp$  5  
 5 IP Rcb 1  
 1 UG  $\forall x Rxb$

Counterexample presented by a diagram Counterexample presented by tables



| a | b | c | R | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | T | T | F |
|   |   |   | 2 | T | F | F |
|   |   |   | 3 | T | F | F |

## Phi 270 F09 test 4

### F09 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class—for example, the proper analysis of **every**, **no**, and **only** (see § 7.2.2), how to incorporate bounds on complementary generalizations (see § 7.2.3), ways of handling compound quantifier phrases (such as **only cats and dogs**, see § 7.3.2), the distinction between **every** and **any** (see §§ 7.3.3 and 7.4.2), how to represent multiple quantifier phrases with overlapping scope (see § 7.4.1). You should be able restate your analysis using unrestricted quantifiers (see § 7.2.1), but you will not need to present it in English notation.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. Remember that the distinction between **every** and **any** can be important here, too.
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rules introduced in § 7.8.1.

### F09 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Also restate your analyses using unrestricted quantifiers.*

1. **Everyone saw the eclipse.**
2. **Al didn't find any book that he was looking for.**
3. **No one ate only potato chips.**

Synthesize an English sentence that has the following logical form; that is, devise a sentence that would have the following analysis:

$$4. \quad (\forall x: \neg Sbx) Sax$$

S: [   saw   ]; a: **Al**; b: **Bill**

Use derivations to show that the following arguments are valid. You may use any rules.

$$5. \quad \begin{array}{l} \forall x (Gx \rightarrow Hx) \\ \forall x (Fx \wedge Gx) \end{array}$$

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$$\forall x Hx$$

$$6. \quad \frac{\forall y \forall x (Px \rightarrow \neg Fxy)}{\forall x \forall y (Fyx \rightarrow \neg Py)}$$

Use a derivation to show that the following argument is not valid and present a counterexample that lurks in an open gap.

$$7. \quad \frac{\forall x Rxa}{\forall x Rxx}$$

### F09 test 4 answers

1. **everyone saw the eclipse**  
**everyone is such that (he or she saw the eclipse)**  
 ( $\forall x$ : x is a person) x saw the eclipse  
 $(\forall x: Px) Sxe$   
 $\forall x (Px \rightarrow Sxe)$   
 P: [   is a person   ]; S: [   saw   ]; e: **the eclipse**
2. **Al didn't find any book that he was looking for**  
**every book that Al was looking for is such that (he didn't find it)**  
 ( $\forall x$ : x is a book that Al was looking for) **Al didn't find** x  
 ( $\forall x$ : x is a book  $\wedge$  Al was looking for x)  $\neg$  Al found x  
 $(\forall x: Bx \wedge Lax) \neg Fax$   
 $\forall x ((Bx \wedge Lax) \rightarrow \neg Fax)$   
 B: [   is a book   ]; F: [   found   ]; L: [   was looking for   ]; a: **Al**
3. **no one ate only potato chips**  
**no one is such that (he or she ate only potato chips)**  
 ( $\forall x$ : x is a person)  $\neg$  x ate only potato chips  
 ( $\forall x: Px) \neg$  only potato chips are such that (x ate them)  
 ( $\forall x: Px) \neg (\forall y: \neg y$  is a potato chip)  $\neg$  x ate y  
 $(\forall x: Px) \neg (\forall y: \neg Cy) \neg Axy$   
 $\forall x (Px \rightarrow \neg \forall y (\neg Cy \rightarrow \neg Axy))$   
 A: [   ate   ]; C: [   is a potato chip   ]; P: [   is a person   ]
4. ( $\forall x: \neg$  **Bill saw** x) **Al saw** x  
 ( $\forall x$ : **Bill didn't see** x) **Al saw** x  
**everything that Bill didn't see is such that (Al saw it)**  
**Al saw everything that Bill didn't see**

Phi 270 F08 test 4

F08 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class—for example, the proper analysis of **every**, **no**, and **only** (see §7.2.2), how to incorporate bounds on complementary generalizations (see §7.2.3), ways of handling compound quantifier phrases (such as **only cats and dogs**, see §7.3.2), the distinction between **every** and **any** (see §§7.3.3 and 7.4.2), how to represent multiple quantifier phrases with overlapping scope (see §7.4.1). You should be able restate your analysis using unrestricted quantifiers (see §7.2.1), but you will not need to present it in English notation.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. Remember that the distinction between **every** and **any** can be important here, too.
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rules introduced in §7.8.1.

F08 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *State your analysis also in a form that expresses any generalizations using unrestricted quantifiers.*

1. **No cover fit the container.**
2. **Everyone who Sam spoke to had seen the movie.**
3. **Only dogs chewed every bone.**
4. **No one who everyone knew bought anything.**

Use derivations to show that the following arguments are valid. You may use any rules.

5.  $\frac{\forall x (Fx \rightarrow Hx)}{\forall x ((Fx \wedge Gx) \rightarrow Hx)}$       6.  $\frac{\forall x (Px \rightarrow \forall y (Rxy \rightarrow Txy))}{\forall x \forall y ((Px \rightarrow Rxy) \rightarrow (Px \rightarrow Txy))}$

Use a derivation to show that the following argument is not valid and present a counterexample by using a diagram to describe a structure that is a counterexample lurking an open gap.

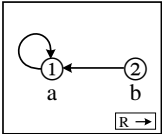
7.  $\frac{\forall x Rax}{\forall x (Rxx \rightarrow Rxa)}$

5.  $\frac{\forall x (Gx \rightarrow Hx) \quad a:2 \quad \forall x (Fx \wedge Gx) \quad a:3}{\begin{array}{|l} \text{①} \\ \hline 2 \text{ UI} \quad Ga \rightarrow Ha \quad 5 \\ 3 \text{ UI} \quad Fa \wedge Ga \quad 4 \\ 4 \text{ Ext} \quad Fa \\ 4 \text{ Ext} \quad Ga \quad (5) \\ 5 \text{ MPP} \quad Ha \quad (6) \\ \hline \bullet \\ \hline 6 \text{ QED} \quad Ha \quad 1 \\ \hline 1 \text{ UG} \quad \forall x Hx \end{array}}$

6.  $\frac{\forall y \forall x (Px \rightarrow \neg Fxy) \quad a:5}{\begin{array}{|l} \text{①} \\ \hline \text{②} \\ \hline Fba \quad (8) \\ \hline Pb \quad (7) \\ \hline \forall x (Px \rightarrow \neg Fxa) \quad b:6 \\ 5 \text{ UI} \quad Pb \rightarrow \neg Fba \quad 7 \\ 6 \text{ UI} \quad \neg Fba \quad (8) \\ 7 \text{ MPP} \\ \hline \bullet \\ \hline \perp \quad 4 \\ 8 \text{ Nc} \\ \hline \neg Pb \quad 3 \\ 4 \text{ RAA} \\ \hline Fba \rightarrow \neg Pb \quad 2 \\ 3 \text{ CP} \\ \hline \forall y (Fya \rightarrow \neg Py) \quad 1 \\ 2 \text{ UG} \\ \hline \forall x \forall y (Fyx \rightarrow \neg Py) \\ 1 \text{ UG} \end{array}}$

7.  $\frac{\forall x Rxa \quad a:2, b:3}{\begin{array}{|l} \text{①} \\ \hline \text{②} \\ \hline Raa \\ 2 \text{ UI} \quad Rba \\ 3 \text{ UI} \quad \neg Rbb \\ \hline \circ \quad \neg Rbb, Rba, Raa \not\equiv \perp \\ \hline \perp \quad 4 \\ 4 \text{ IP} \quad Rbb \quad 1 \\ \hline \forall x Rxx \\ 1 \text{ UG} \end{array}}$

Counterexample presented by a diagram



F08 test 4 answers

1. no cover fit the container

no cover is such that (it fit the container)

$(\forall x: x \text{ is a cover}) \rightarrow x \text{ fit the container}$

$$(\forall x: Cx) \rightarrow Fxc$$

$$\forall x (Cx \rightarrow \neg Fxc)$$

C: [ \_ is a cover ]; F: [ \_ fit \_ ]; c: the container

2. everyone who Sam spoke to had seen the movie

everyone who Sam spoke to is such that (he or she had seen the movie)

$(\forall x: x \text{ is a person who Sam spoke to}) \rightarrow x \text{ had seen the movie}$

$(\forall x: x \text{ is a person} \wedge \text{Sam spoke to } x) \rightarrow Sxm$

$$(\forall x: Px \wedge Ksx) \rightarrow Sxm$$

$$\forall x ((Px \wedge Ksx) \rightarrow Sxm)$$

K: [ \_ spoke to \_ ]; P: [ \_ is a person ]; S: [ \_ had seen \_ ]; m: the movie; s: Sam

3. only dogs chewed every bone

only dogs are such that (they chewed every bone)

$(\forall x: \neg x \text{ is a dog}) \rightarrow x \text{ chewed every bone}$

$(\forall x: \neg Dx) \rightarrow \text{every bone is such that } (x \text{ chewed it})$

$(\forall x: \neg Dx) \rightarrow (\forall y: y \text{ is a bone}) \rightarrow x \text{ chewed } y$

$$(\forall x: \neg Dx) \rightarrow (\forall y: By) \rightarrow Cxy$$

$$\forall x (\neg Dx \rightarrow \neg \forall y (By \rightarrow Cxy))$$

B: [ \_ is a bone ]; C: [ \_ chewed \_ ]; D: [ \_ is a dog ]

4. No one who everyone knew bought anything

everything is such that (no one who everyone knew bought it)

$\forall x$  no one who everyone knew bought x

$\forall x$  no one who everyone knew is such that (he or she bought x)

$\forall x (\forall y: y \text{ is a person who everyone knew}) \rightarrow \neg y \text{ bought } x$

$\forall x (\forall y: y \text{ is a person} \wedge \text{everyone knew } y) \rightarrow \neg Byx$

$\forall x (\forall y: Py \wedge \text{everyone is such that (he or she knew } y)) \rightarrow \neg Byx$

$\forall x (\forall y: Py \wedge (\forall z: z \text{ is a person}) \rightarrow z \text{ knew } y) \rightarrow \neg Byx$

$$\forall x (\forall y: Py \wedge (\forall z: Pz) \rightarrow Kzy) \rightarrow \neg Byx$$

$$\forall x \forall y ((Py \wedge \forall z (Pz \rightarrow Kzy)) \rightarrow \neg Byx)$$

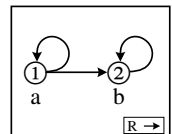
B: [ \_ bought \_ ]; K: [ \_ knew \_ ]; P: [ \_ is person ]

|       |   |     |
|-------|---|-----|
| 5.    | $\forall x (Fx \rightarrow Hx)$             | a:4 |
|       | ⓐ   |     |
|       | Fa ∧ Ga                                     | 3   |
|       | —   |     |
| 3 Ext | Fa  | (5) |
| 3 Ext | Ga  |     |
| 4 UI  | Fa → Ha                                     | 5   |
| 5 MPP | Ha  | (6) |
|       | ●   |     |
|       | —   |     |
| 6 QED | Ha  | 2   |
|       | —   |     |
| 2 CP  | (Fa ∧ Ga) → Ha                              | 1   |
| 1 UG  | $\forall x ((Fx \wedge Gx) \rightarrow Hx)$ |     |

|        |   |          |
|--------|---|----------|
| 6.     | $\forall x (Px \rightarrow \forall y (Rxy \rightarrow Txy))$                  | a:6      |
|        | ⓐ   |          |
|        | Pa → Rab  | 5        |
|        | —   |          |
|        | Pa  | (5), (7) |
|        | —   |          |
| 5 MPP  | Rab   | (9)      |
| 6 UI   | Pa → $\forall y (Ray \rightarrow Tay)$  | 7        |
| 7 MPP  | $\forall y (Ray \rightarrow Tay)$   | b:8      |
| 8 UI   | Rab → Tab   | 9        |
| 9 MPP  | Tab   | (10)     |
|        | ●   |          |
|        | —   |          |
| 10 QED | Tab   | 4        |
|        | —   |          |
| 4 CP   | Pa → Tab  | 3        |
|        | —   |          |
| 3 CP   | (Pa → Rab) → (Pa → Tab)   | 2        |
|        | —   |          |
| 2 UG   | $\forall y ((Pa \rightarrow Ray) \rightarrow (Pa \rightarrow Tay))$           | 1        |
| 1 UG   | $\forall x \forall y ((Px \rightarrow Rxy) \rightarrow (Px \rightarrow Txy))$ |          |

|      |                                   |                          |
|------|-----------------------------------|--------------------------|
| 7.   | $\forall x Rax$                   | a:3, b:4                 |
|      | ⓐ                                 |                          |
|      | Rbb                               |                          |
|      | —                                 |                          |
| 3 UI | Raa                               |                          |
| 4 UI | Rab                               |                          |
|      | —                                 |                          |
|      | ¬ Rba                             |                          |
|      | —                                 |                          |
|      | ○                                 | ¬ Rba, Rab, Raa, Rbb ≠ ⊥ |
|      | —                                 |                          |
|      | ⊥                                 | 5                        |
|      | —                                 |                          |
| 5 IP | Rba                               | 2                        |
|      | —                                 |                          |
| 2 CP | Rbb → Rba                         | 1                        |
| 1 UG | $\forall x (Rxx \rightarrow Rxa)$ |                          |

Counterexample presented by a diagram



## Phi 270 F06 test 4

### F06 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class—for example, the proper analysis of **every**, **no**, and **only** (see §7.2.2), how to incorporate bounds and exceptions (see §7.2.3), ways of handling compound quantifier phrases (such as **only cats and dogs**, see §7.3.2), the distinction between **every** and **any** (see §§7.3.3 and 7.4.2), how to represent multiple quantifier phrases with overlapping scope (see §7.4.1). You should be able to restate your analysis using unrestricted quantifiers (see §7.2.1), but you will not need to present it in English notation.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. Remember that the distinction between **every** and **any** can be important here, too.
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rules introduced in §7.8.1.

### F06 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *State your analysis also in a form that expresses any generalizations using unrestricted quantifiers.*

1. **Every door was locked.**
2. **Only people who had witnessed the event were able to follow the description of it.**

[It is possible for the scope of **only** to change with emphasis; although varying interpretations are less likely with this sentence than with others, you may choose whichever scope seems most plausible to you.]

3. **No key opened every door.**

[You should understand this sentence to leave open the possibility that some key opened some door.]

Synthesize an English sentence with the following logical form; that is, find a sentence that would have the following analysis:

4.  $(\forall x: Px \wedge Nxa) (Dxm \vee Axm)$

A: [ **\_ was acted on at \_** ]; D: [ **\_ was discussed at \_** ]; N: [ **\_ was on \_** ]; P: [ **\_ was a proposal** ]; a: **the agenda**; m: **the meeting**

Use derivations to show that the following arguments are valid. You may use any rules.

5. 
$$\frac{\forall x (Fx \rightarrow (Gx \rightarrow Hx)) \quad \forall x Gx}{\forall x (Fx \rightarrow Hx)}$$
6. 
$$\frac{\forall x (Fx \rightarrow \forall y Rxy) \quad \forall x Fx}{\forall x \forall y Ryx}$$

Use a derivation to show that the following argument is not valid and present a counterexample by describing a structure that is a counterexample lurking an open gap. (You may describe the structure either by depicting it in a diagram, as answers in the text usually do, or by giving tables.)

7. 
$$\frac{\forall x Rax \quad \forall x Rxb}{\forall x Rxx}$$

### F06 test 4 answers

1. **Every door was locked**  
**Every door is such that (it was locked)**  
 ( $\forall x: x$  is a door) **x was locked**  
 $(\forall x: Dx) Lx$   
 $\forall x (Dx \rightarrow Lx)$
2. **only people who had witnessed the event were able to follow the description of it**  
**only people who had witnessed the event are such that (they were able to follow the description of it)**  
 ( $\forall x: \neg x$  is a person who had witnessed the event)  $\neg x$  was able to follow the description of the event  
 ( $\forall x: \neg (x$  is a person  $\wedge x$  had witnessed the event)  $\rightarrow Fx$ (the description of the event)

$$(\forall x: \neg (Px \wedge Wxe)) \rightarrow Fx(de)$$

$$\forall x (\neg (Px \wedge Wxe) \rightarrow \neg Fx(de))$$

F: [ **\_ was able to follow \_** ]; P: [ **\_ is a person** ]; W: [ **\_ had witnessed \_** ]; e: **the event**; d: [ **the description of \_** ]

Other possible (though less likely) interpretations:

$(\forall x: Px \wedge \neg Wxe) \rightarrow Fx(de)$  says **only people who had witnessed ...**

$(\forall x: \neg Px \wedge Wxe) \rightarrow Fx(de)$  says **only people who had witnessed ...**

Not a possible interpretation:  $(\forall x: \neg Px \wedge \neg Wxe) \rightarrow Fx(de)$



3. **No key opened every door**  
**No key is such that (it opened every door)**  
 $(\forall x: x \text{ is a key}) \neg x \text{ opened every door}$   
 $(\forall x: Kx) \neg \text{every door is such that (x opened it)}$   
 $(\forall x: Kx) \neg (\forall y: y \text{ is a door}) x \text{ opened } y$

$$(\forall x: Kx) \neg (\forall y: Dy) Oxy$$

$$\forall x (Kx \rightarrow \neg \forall y (Dy \rightarrow Oxy))$$

D: [ \_ is a door ]; K: [ \_ is a key ]; O: [ \_ opened \_ ]

Although there are equivalent analyses, one that differs only in the location of  $\neg$  is likely to be wrong. In particular,  $(\forall x: Kx) (\forall y: Dy) \neg Oxy$  rules out the possibility that some key opened some door.

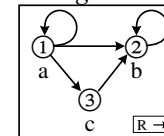
4.  $(\forall x: Px \wedge Nxa) (Dxm \vee Axm)$   
 **$(\forall x: x \text{ was a proposal} \wedge x \text{ was on the agenda}) (x \text{ was discussed at the meeting} \vee x \text{ was acted on at the meeting})$**   
 **$(\forall x: x \text{ was a proposal on the agenda}) (x \text{ was discussed or acted on at the meeting})$**   
**Every proposal on the agenda is such that (it was discussed or acted on at the meeting)**  
**Every proposal on the agenda was discussed or acted on at the meeting**

|       |  |      |
|-------|--|------|
|       | $\forall x (Fx \rightarrow (Gx \rightarrow Hx))$ | a: 3 |
|       | $\forall x Gx$                                   | a: 5 |
|       | ⓐ  |      |
|       | Fa   | (4)  |
| 3 UI  | Fa $\rightarrow (Ga \rightarrow Ha)$             | 4    |
| 4 MPP | Ga $\rightarrow Ha$                              | 6    |
| 5 UI  | Ga   | (6)  |
| 6 MPP | Ha   | (7)  |
|       | ●  |      |
|       | Ha   | 2    |
| 7 QED | Ha   | 2    |
|       | Fa $\rightarrow Ha$                              | 1    |
| 2 CP  | Fa $\rightarrow Ha$                              | 1    |
| 1 UG  | $\forall x (Fx \rightarrow Hx)$                  |      |

|       |  |      |
|-------|--|------|
|       | $\forall x (Fx \rightarrow \forall y Rxy)$ | b: 3 |
|       | $\forall x Fx$                             | b: 4 |
|       | ⓐ  |      |
|       | ⓑ  |      |
| 3 UI  | Fb $\rightarrow \forall y Rby$             | 5    |
| 4 UI  | Fb   | (5)  |
| 5 MPP | $\forall y Rby$                            | a: 6 |
| 6 UI  | Rba  | (7)  |
|       | ●  |      |
|       | Rba  | 2    |
| 7 QED | Rba  | 2    |
|       | $\forall y Rya$                            | 1    |
| 2 UG  | $\forall y Rya$                            | 1    |
| 1 UG  | $\forall x \forall y Ryx$                  |      |

|      |                 |                  |
|------|-----------------|------------------|
|      | $\forall x Rax$ | a: 2, b: 3, c: 4 |
|      | $\forall x Rxb$ | a: 5, b: 6, c: 7 |
|      | ⓐ               |                  |
| 2 UI | Raa             |                  |
| 3 UI | Rab             |                  |
| 4 UI | Rac             |                  |
| 5 UI | Rab             |                  |
| 6 UI | Rbb             |                  |
| 7 UI | Rcb             |                  |
|      | ⊥               |                  |
|      | ⊥               | 8                |
|      | ⊥               |                  |
|      | ⊥               | 8                |
| 8 IP | Rcc             | 1                |
| 1 UG | $\forall x Rxx$ |                  |

Counterexample presented by a diagram



Counterexample presented by tables

|                |   |   |   |   |   |   |   |
|----------------|---|---|---|---|---|---|---|
| range: 1, 2, 3 | a | b | c | R | 1 | 2 | 3 |
|                | 1 | 2 | 3 | 1 | T | T | T |
|                | 2 | F | T | 2 | F | T | F |
|                | 3 | F | T | 3 | F | T | F |



## Phi 270 F05 test 4

### F05 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class--for example, the proper analysis of **every**, **no**, and **only** (§7.2), how to incorporate bounds and exceptions (§7.2), ways of handling compound quantifier phrases (such as **only cats and dogs**, §7.3), the distinction between **every** and **any** (§§7.3 and 7.4), how to represent multiple quantifier phrases with overlapping scope (§7.4). Be able to restate your analysis using unrestricted quantifiers, but you will not need to present it in English notation.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure (by either tables or a diagram). In derivations involving restricted universals you will have the option using the rules RUG, SB, SC, and MRC or instead using RUP and RUC along with rules for unrestricted universals and conditionals. You will *not* be responsible for the rules introduced in §7.8.

### F05 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Restate 1 using an unrestricted quantifier.*

1. **Everyone knew the tune.** [Remember to restate your answer to this using an unrestricted quantifier.]
2. **Sam heard only tunes that he knew.**  
[Remember to restate your answer in 2 using an unrestricted quantifier.]
3. **No one liked everything on the menu.**

Synthesize an English sentence with the following logical form; that is, produce a sentence that would have the following analysis:

4.  $(\forall x: Px) \neg Fsx$   
P: [ **\_ is a person\_** ]; F: [ **\_ fit \_** ]; s: **the shoe**

Use derivations to show that the following arguments are valid. You may use any rules.

5. 
$$\frac{\forall x (Fx \wedge Gx)}{\forall x (Gx \wedge Fx)}$$
6. 
$$\frac{\forall x \forall y (Gy \rightarrow Rxy)}{\forall x (Fx \rightarrow Gx)}$$
  
$$\forall x (Fx \rightarrow \forall y Ryx)$$

Use a derivation to show that the following argument is not valid and present a counterexample by describing a structure that is a counterexample lurking an open gap. (You may describe the structure either by depicting it in a diagram, as answers in the text usually do, or by giving tables.)

7. 
$$\frac{\forall x (Fx \rightarrow Rax)}{Fa}$$
  
$$\forall x Rxa$$

### F05 test 4 answers

1. **Everyone knew the tune**  
**Everyone is such that (he or she knew the tune)**  
**( $\forall x: x$  is a person)  $x$  knew the tune**  
 $(\forall x: Px) Kxt$   
 $\forall x (P \rightarrow Kxt)$   
K: [ **\_ knew \_** ]; P: [ **\_ is a person\_** ]; t: **the tune**
2. **Sam heard only tunes that he knew**  
**only tunes that Sam knew are such that (Sam heard them)**  
**( $\forall x: \neg x$  is a tune that Sam knew)  $\neg$  Sam heard  $x$**   
**( $\forall x: \neg (x$  is a tune  $\wedge$  Sam knew  $x)) \neg Hsx$**   
 $(\forall x: \neg (Tx \wedge Ksx)) \neg Hsx$   
[ **\_ heard \_** ]; K: [ **\_ knew \_** ]; T: [ **\_ is a tune\_** ]; s: **Sam**  
A different but equally plausible interpretation would be to treat tunes as a bounds indicator; this interpretation would be analyzed as  $(\forall x: Tx \wedge \neg Ksx) \neg Hsx$ . This is also the analysis of **Sam heard no tunes he didn't know.**
3. **No one liked everything on the menu**  
**No one is such that (he or she liked everything on the menu)**  
**( $\forall x: x$  is a person)  $\neg x$  liked everything on the menu**  
**( $\forall x: Px) \neg$  everything on the menu is such that ( $x$  liked it)**  
**( $\forall x: Px) \neg (\forall y: y$  is on the menu)  $x$  liked  $y$**   
 $(\forall x: Px) \neg (\forall y: Oym) Lxy$   
L: [ **\_ liked \_** ]; O: [ **\_ is on \_** ]; P: [ **\_ is a person\_** ]; m: **the menu**

4.  $(\forall x: x \text{ is a person}) \neg \text{the shoe fit } x$   
 No one is such that (the shoe fit him or her)  
 The shoe fit no one

or

$(\forall x: x \text{ is a person}) \neg \text{the shoe fit } x$   
 $(\forall x: x \text{ is a person}) \text{ the shoe didn't fit } x$   
 Everyone is such that (the shoe didn't fit him or her)  
 The shoe didn't fit anyone

The sentence *The shoe didn't fit everyone* is not the best synthesis since it is likely to be understood as the denial of *The shoe fit everyone*—i.e., as  $\neg(\forall x: Px) Fsx$ .

5.  $\forall x (Fx \wedge Gx)$  a:2

|       |   |                            |     |
|-------|---|----------------------------|-----|
| 2 UI  | ⓐ | Fa ∧ Ga                    | 3   |
| 3 Ext |   | Fa                         | (6) |
| 3 Ext |   | Ga                         | (5) |
|       |   | ●                          |     |
| 5 QED |   | Ga                         | 4   |
|       |   | ●                          |     |
| 6 QED |   | Fa                         | 4   |
| 4 Cnj |   | Ga ∧ Fa                    | 1   |
| 1 UG  |   | $\forall x (Gx \wedge Fx)$ |     |

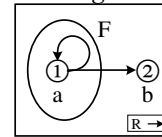
6.  $\forall x \forall y (Gy \rightarrow Rxy)$  b:6  
 $\forall x (Fx \rightarrow Gx)$  a:4

|       |   |  |      |
|-------|---|--|------|
|       | ⓐ | Fa   | (5)  |
|       | ⓑ | Fa → Ga                                    | 5    |
| 4 UI  |   | Ga   | (8)  |
| 5 MPP |   | $\forall y (Gy \rightarrow Rby)$           | a: 7 |
| 6 UI  |   | Ga → Rba                                   | 8    |
| 7 UI  |   | Rba  | (9)  |
| 8 MPP |   | ●  |      |
| 9 QED |   | Rba  | 3    |
| 3 UG  |   | $\forall y Rya$                            | 2    |
| 2 CP  |   | Fa → $\forall y Rya$                       | 1    |
| 1 UG  |   | $\forall x (Fx \rightarrow \forall y Ryx)$ |      |

7.  $\forall x (Fx \rightarrow Rax)$  a:1, b:4  
 Fa (2)

|       |   |                 |                        |
|-------|---|-----------------|------------------------|
| 1 UI  |   | Fa → Raa        | 2                      |
| 2 MPP |   | Raa             |                        |
|       | ⓑ | Fb → Rab        | 6                      |
| 4 UI  |   | ¬ Rba           |                        |
|       |   | ¬ Fb            |                        |
|       |   | ○               | Fa, Raa, ¬Rba, ¬Fb ≠ ⊥ |
|       |   | ⊥               | 7                      |
| 7 IP  |   | Fb              | 6                      |
|       |   | Rab             |                        |
|       |   | ○               | Fa, Raa, ¬Rba, Rab ≠ ⊥ |
|       |   | ⊥               | 6                      |
| 6 RC  |   | ⊥               | 5                      |
| 5 IP  |   | Rba             | 3                      |
| 3 UG  |   | $\forall x Rxa$ |                        |

Counterexample presented by a diagram



Counterexample presented by tables

range: 1, 2

| a | b | τ | Fτ | R | 1 | 2 |
|---|---|---|----|---|---|---|
| 1 | 2 | 1 | T  | 1 | T | T |
| 2 | 1 | 2 | F  | 2 | F | F |

This counterexample lurks in both gaps; but the specific value for F2 is needed only for the first gap and the specific value for R12 is needed only for the second.

## Phi 270 F04 test 4

### F04 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class--for example, the proper analysis of **every**, **no**, and **only** (§7.2), how to incorporate bounds and exceptions (§7.2), ways of handling compound quantifier phrases (such as **only cats and dogs**, §7.3), the distinction between **every** and **any** (§§7.3 and 7.4), how to represent multiple quantifier phrases with overlapping scope (§7.4). Be able restate you analysis using unrestricted quantifiers, but you will not need to present it in English notation.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. In derivations involving restricted universals you will have the option using the rules RUG, SB, SC, and MRC or instead using RUP and RUC along with rules for unrestricted universals and conditionals. You will *not* be responsible for the rules introduced in §7.8.

### F04 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Restate 2 using an unrestricted quantifier.*

1. **Sam checked every lock**
2. **No one who was in the office answered the call**  
[Remember to restate your answer in 2 using an unrestricted quantifier.]
3. **Ralph got the joke if anyone did**
4. **Only bestsellers were on every list**

Use derivations to show that the following arguments are valid. You may use any rules.

5. 
$$\frac{\forall x Fx \quad \forall x \neg Gx}{\forall x (Fx \wedge \neg Gx)}$$
6. 
$$\frac{\forall x (Rxa \rightarrow \forall y Txy)}{\forall x \forall y (Rya \rightarrow Tyx)}$$

Use a derivation to show that the following argument is not valid and present a counterexample by describing a structure that is a counterexample lurking an

open gap. (You may describe the structure either by depicting it in a diagram, as answers in the text usually do, or by giving tables.)

7. 
$$\frac{\forall x Rax}{\forall x (Rxa \rightarrow Rxx)}$$

### F04 test 4 answers

1. **Sam checked every lock**  
**Every lock is such that (Sam checked it)**  
( $\forall x: x \text{ is a lock}$ ) Sam checked x  
( $\forall x: Lx$ ) Csx  
C: [ checked ]; L: [ is a lock ]; s: **Sam**
2. **No one who was in the office answered the call**  
**No one who was in the office is such that (he or she answered the call)**  
( $\forall x: x \text{ is a person who was in the office}$ )  $\neg$  x answered the call  
( $\forall x: x \text{ is a person} \wedge x \text{ was in the office}$ )  $\neg$  Axc  
( $\forall x: Px \wedge Nxo$ )  $\neg$  Axc  
 $\forall x ((Px \wedge Nxo) \rightarrow \neg Axc)$   
A: [ answered ]; P: [ is a person ]; N: [ was in ]; c: **the call**; o: **the office**
3. **Ralph got the joke if anyone did**  
**Everyone is such that (Ralph got the joke if he or she did)**  
( $\forall x: x \text{ is a person}$ ) **Ralph got the joke if** x did  
( $\forall x: Px$ ) (Ralph got the joke  $\leftarrow$  x got the joke)  
( $\forall x: Px$ ) (Grj  $\leftarrow$  Gxj)  
( $\forall x: Px$ ) (Gxj  $\rightarrow$  Grj)  
P: [ is a person ]; G: [ got ]; j: **the joke**
4. **Only bestsellers were on every list**  
**Only bestsellers are such that (they were on every list)**  
( $\forall x: \neg x \text{ is a bestseller}$ )  $\neg$  x was on every list  
( $\forall x: \neg Bx$ )  $\neg$  **every list is such that (x was on it)**  
( $\forall x: \neg Bx$ )  $\neg$  ( $\forall y: y \text{ is a list}$ ) x was on y  
( $\forall x: \neg Bx$ )  $\neg$  ( $\forall y: Ly$ ) Nxy  
B: [ is a bestseller ]; L: [ is a list ]; N: [ was on ]

Phi 270 F03 test 4

F03 test 4 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Analysis.* Be ready to handle any of the key issues discussed in class--for example, the proper analysis of **every**, **no**, and **only** (§7.2), how to incorporate bounds and exceptions (§7.2), ways of handling compound quantifier phrases (such as **only cats and dogs**, §7.3), the distinction between **every** and **any** (§§7.3 and 7.4), how to represent multiple quantifier phrases with overlapping scope (§7.4). Be able restate you analysis using unrestricted quantifiers, but you will not need to present it in English notation.
- *Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. (This sort of question is less likely to appear than a question about analysis and there would certainly be substantially fewer such questions.)
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail (derivations that hold are more likely). I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rules introduced in §7.8.

F03 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Restate 2 using an unrestricted quantifier.*

1. **No one called the new number**
2. **Sam asked everyone he could think of** [Remember to restate this one using an unrestricted quantifier.]
3. **If any door was opened, the alarm sounded**
4. **Only people who'd read everything the author had written were asked to review the book**

Use derivations to show that the following arguments are valid. You may use any rules.

5.  $\forall x (Fx \wedge Gx)$   
 $\forall x Gx$
6.  $\forall x (Fx \rightarrow Gx)$   
 $\forall x \forall y (Gy \rightarrow Rxy)$   
 $\forall x \forall y (Fy \rightarrow Rxy)$

Use a derivation to show that the following argument is not valid and describe

5.  $\forall x Fx$  a: 3  
 $\forall x \neg Gx$  a: 5

3 UI (a)  $Fa$  (4)  
 4 QED  $Fa$  2

5 UI  $\neg Ga$  (6)  
 6 QED  $\neg Ga$  2

2 Cnj  $Fa \wedge \neg Ga$  1

1 UG  $\forall x (Fx \wedge \neg Gx)$

6.  $\forall x (Rxa \rightarrow \forall y Txy)$  c:4

4 UI (b) (c)  $Rca$  (5)  
 5 MPP  $Rca \rightarrow \forall y Tcy$  5  
 6 UI  $\forall y Tcy$  b: 6  
 $Tcb$  (7)

7 QED  $Tcb$  3

3 CP  $Rca \rightarrow Tcb$  2

2 UG  $\forall y (Rya \rightarrow Tyb)$  1

1 UG  $\forall x \forall y (Rya \rightarrow Tyx)$

7.  $\forall x Rax$  a:4, b:5

4 UI (b)  $Rba$   
 5 UI  $\neg Rbb$

$Raa, Rab$   
 $\circ$   $Rba, \neg Rbb, Raa, Rab \neq \perp$

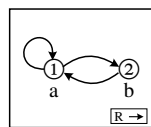
$\perp$  3

3 IP  $Rbb$  2

2 CP  $Rba \rightarrow Rbb$  1

1 UG  $\forall x (Rxa \rightarrow Rxx)$

Counterexample presented by a diagram Counterexample presented by tables



range: 1, 2

| a | b | R | 1 | 2 |
|---|---|---|---|---|
| 1 | 2 | 1 | T | T |
| 2 | 1 | 2 | T | F |

a structure (by using either a diagram or tables) that is a counterexample lurking an open gap.

$$7. \frac{\forall x (Fx \rightarrow Rxa)}{Fa \rightarrow \forall x Rxx}$$

**F03 test 4 answers**

1. No one called the new number

No one is such that (he or she called the new number)

$(\forall x: x \text{ is a person}) \neg x \text{ called the new number}$

$$(\forall x: Px) \neg Cxn$$

C: [ \_ called \_ ]; P: [ \_ is a person ]; n: the new number

2. Sam asked everyone he could think of

everyone Sam could think of is such that (Sam asked him or her)

$(\forall x: x \text{ is a person Sam could think of}) \text{ Sam asked } x$

$(\forall x: x \text{ is a person} \wedge \text{ Sam could think of } x) \text{ Asx}$

$$(\forall x: Px \wedge Tsx) \text{ Asx}$$

$$\forall x ((Px \wedge Tsx) \rightarrow \text{Asx})$$

A: [ \_ asked \_ ]; P: [ \_ is a person ]; T: [ \_ could think of \_ ]; s: Sam

3. If any door was opened, the alarm sounded

every door is such that (if it was opened, the alarm sounded)

$(\forall x: x \text{ is a door}) \text{ if } x \text{ was opened, the alarm sounded}$

$(\forall x: Dx) (x \text{ was opened} \rightarrow \text{the alarm sounded})$

$$(\forall x: Dx) (Ox \rightarrow Sa)$$

D: [ \_ is a door ]; O: [ \_ was opened ]; S: [ \_ sounded ]; a: the alarm

4. Only people who'd read everything the author had written were asked to review the book

Only people who'd read everything the author had written are such that (they were asked to review the book)

$(\forall x: \neg x \text{ is a person who'd read everything the author had written})$

$\neg x \text{ was asked to review the book}$

$(\forall x: \neg (x \text{ is a person} \wedge x \text{ had read everything the author had written})) \neg \text{Asx}$

$(\forall x: \neg (x \text{ is a person} \wedge \text{everything the author had written is such that } (x \text{ had read it}))) \neg \text{Asx}$

$(\forall x: \neg (Px \wedge (\forall y: y \text{ is a thing the author had written}) x \text{ had read } y)) \neg \text{Asx}$

$(\forall x: \neg (Px \wedge (\forall y: \text{the author had written } y) Rxy)) \neg \text{Asx}$

$$(\forall x: \neg (Px \wedge (\forall y: \text{Way}) Rxy)) \neg \text{Asx}$$

A: [ \_ was asked to review \_ ]; P: [ \_ is a person ]; R: [ \_ had read \_ ];

R: [ \_ had written \_ ]; a: the author; b: the book

5.

|  |                            |                  |
|--|----------------------------|------------------|
|  | $\forall x (Fx \wedge Gx)$ | a: 2             |
|  | 2 UI                       | $Fa \wedge Ga$ 3 |
|  | 3 Ext                      | Fa               |
|  | 3 Ext                      | Ga (4)           |
|  |                            | ●                |
|  | 4 QED                      | Ga 1             |
|  | 1 UG                       | $\forall x Gx$   |

6.

|  |  |  |
|--|--|--|
|  | $\forall x (Fx \rightarrow Gx)$            | b:4  |
|  | $\forall x \forall y (Gy \rightarrow Rxy)$ | a:6  |
|  |  | (a)  |
|  |  | (b)  |
|  |  | Fb (5)                                     |
|  | 4 UI                                       | $Fb \rightarrow Gb$ 5                      |
|  | 5 MPP                                      | Gb (8)                                     |
|  | 6 UI                                       | $\forall y (Gy \rightarrow Ray)$ b:7       |
|  | 7 UI                                       | $Gb \rightarrow Rab$ 8                     |
|  | 8 MPP                                      | Rab (9)                                    |
|  |  | ●  |
|  | 9 QED                                      | Rab 3                                      |
|  | 3 CP                                       | $Fb \rightarrow Rab$ 2                     |
|  | 2 UG                                       | $\forall y (Fy \rightarrow Ray)$ 1         |
|  | 1 UG                                       | $\forall x \forall y (Fy \rightarrow Rxy)$ |

7.

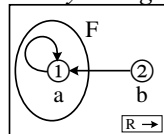
|       |                                  |   |
|-------|----------------------------------|---|
|       | $\forall x (Fx \rightarrow Rxa)$ | a:2, b:5                                      |
| 2 UI  | Fa                               | (3)   |
| 3 MPP | Fa $\rightarrow$ Raa             | 3   |
| 5 UI  | Raa                              |   |
| 5 UI  | Fb $\rightarrow$ Rba             | 7   |
|       | $\neg$ Rbb                       |   |
|       | $\neg$ Fb                        |   |
|       | O                                | Fa, Raa, $\neg$ Rbb, $\neg$ Fb $\neq$ $\perp$ |
|       | $\perp$                          | 8   |
| 8 IP  | Fb                               | 7   |
|       | Rba                              |   |
|       | O                                | Fa, Raa, $\neg$ Rbb, Rba $\neq$ $\perp$       |
|       | $\perp$                          | 7   |
| 7 RC  | $\perp$                          | 6   |
| 6 IP  | Rbb                              | 4   |
| 4 UG  | $\forall x Rxx$                  | 1   |
| 1 CP  | Fa $\rightarrow \forall x Rxx$   |   |

Counterexample presented by tables  
range: 1, 2

|   |   |        |          |   |   |   |
|---|---|--------|----------|---|---|---|
| a | b | $\tau$ | F $\tau$ | R | 1 | 2 |
| 1 | 2 | 1      | T        | 1 | T | F |
| 2 | F | 2      | F        | 2 | T | F |

(This counterexample lurks in both gaps; the value of F2 is needed only for the 1st and the value of R21 only for the 2nd.)

Counterexample presented by a diagram



### Phi 270 F02 test 4

#### F02 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

1. **Only bears performed.**
2. **If everyone cheered, the elephant bowed.** [In this case, restate your answer using an unrestricted quantifier.]
3. **No one laughed at any performers except clowns.**

Synthesize an English sentence with the following logical form:

4.  $(\forall x: Px \wedge Cxt) Ctx$

C: [   called   ]; P: [   is a person   ]; t: **Tom**

Use derivations to establish the validity of the following arguments. You may use attachment rules.

5. 
$$\frac{\forall x Fx \quad \forall x \neg (Fx \wedge Gx)}{\forall x \neg Gx}$$

6. 
$$\frac{\forall x \forall y (Fy \rightarrow Rxy)}{\forall x (Fx \rightarrow \forall y Ryx)}$$

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that is a counterexample lurking in one of the derivation's open gaps.

7. 
$$\frac{\forall x Rax \quad \forall x (Rbx \rightarrow \neg Rxa)}{\forall x \neg Rbx}$$

#### F02 test 4 answers

1. **Only bears performed**  
 $(\forall x: \neg x \text{ is a bear}) \neg x \text{ performed}$   
 $(\forall x: \neg Bx) \neg Px$
2. **If everyone cheered, the elephant bowed**  
**everyone cheered  $\rightarrow$  the elephant bowed**  
 $(\forall x: x \text{ is a person}) x \text{ cheered} \rightarrow \text{the elephant bowed}$

$$(\forall x: Px) Cx \rightarrow Be$$

$$\forall x (Px \rightarrow Cx) \rightarrow Be$$

B: x **bowed**; C: x **cheered**; P: x **is a person**; e: **the elephant**

*Incorrect:*  $(\forall x: Px) (Cx \rightarrow Be)$  or:  $\forall x (Px \rightarrow (Cx \rightarrow Be))$   
*these say: If anyone cheered, the elephant bowed*

3. No one laughed at any performers except clowns  
all performers except clowns are such that (no one laughed at them)

$(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown}) \neg \text{no one laughed at } x$   
 $(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown}) (\forall y: y \text{ is a person}) \neg y \text{ laughed at } x$

$$(\forall x: Fx \wedge \neg Cx) (\forall y: Py) \neg Lyx$$

C: [ \_ is a clown]; F: [ \_ is a performer]; P: [ \_ is a person]; L: [ \_ laughed at \_ ]

Incorrect:  $(\forall y: Py) \neg (\forall x: Fx \wedge \neg Cx) Lyx$   
 says: No one laughed at all performers who weren't clowns

4.  $(\forall x: x \text{ is a person} \wedge x \text{ called Tom}) \text{Tom called } x$   
 $(\forall x: x \text{ is a person who called Tom}) \text{Tom called } x$   
 everyone who called Tom is such that (Tom called him or her)  
 Tom called everyone who called him

|       |                                 |     |
|-------|---------------------------------|-----|
|       | $\forall x Fx$                  | a:2 |
|       | $\forall x \neg (Fx \wedge Gx)$ | a:3 |
|       | ⓐ                               |     |
| 2 UI  | Fa                              | (4) |
| 3 UI  | $\neg (Fa \wedge Ga)$           | 4   |
| 4 MPT | $\neg Ga$                       | (5) |
|       | ●                               |     |
| 5 QED | $\neg Ga$                       | 1   |
| 1 UG  | $\forall x \neg Gx$             |     |

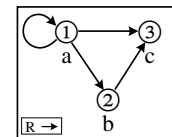
|       |  |     |
|-------|--|-----|
|       | $\forall x \forall y (Fy \rightarrow Rxy)$ | b:4 |
|       | ⓐ  |     |
|       | Fa   | (6) |
|       | ⓑ  |     |
| 4 UI  | $\forall y (Fy \rightarrow Rby)$           | a:5 |
| 5 UI  | Fa $\rightarrow$ Rba                       | 6   |
| 6 MPP | Rba  | (7) |
|       | ●  |     |
| 7 QED | Rba  | 3   |
| 3 UG  | $\forall y Rya$                            | 2   |
| 2 CP  | Fa $\rightarrow \forall y Rya$             | 1   |
| 1 UG  | $\forall x (Fx \rightarrow \forall y Ryx)$ |     |

|       |  |   |
|-------|--|---|
|       | $\forall x Rax$                        | a:3,b:4,c:5   |
|       | $\forall x (Rbx \rightarrow \neg Rxa)$ | c:6,a:8,b:10  |
|       | ⓐ                                      |   |
|       | Rbc                                    | (7)   |
| 3 UI  | Raa                                    | (9)   |
| 4 UI  | Rab                                    |   |
| 5 UI  | Rac                                    |   |
| 6 UI  | Rbc $\rightarrow \neg Rca$             | 7   |
| 7 MPP | $\neg Rca$                             |   |
| 8 UI  | Rba $\rightarrow \neg Raa$             | 9   |
| 9 MTT | $\neg Rba$                             |   |
| 10 UI | Rbb $\rightarrow \neg Rba$             | 11  |
|       | ⓑ                                      |   |
|       | $\neg Rbb$                             |   |
|       | ○                                      | Raa,Rab,Rac, $\neg Rba$ ,<br>$\neg Rbb,Rbc,\neg Rca \neq \perp$ |
|       | ⊥                                      | 12  |
| 12 IP | Rbb                                    | 11  |
|       | ⓐ                                      |   |
|       | $\neg Rba$                             |   |
|       | ○                                      | Rbc,Raa,Rab,Rac, $\neg Rca,\neg Rba \neq \perp$                 |
|       | ⊥                                      | 11  |
| 11 RC | ⊥                                      | 2   |
| 2 RAA | $\neg Rbc$                             | 1   |
| 1 UG  | $\forall x \neg Rbx$                   |   |

Counterexample presented by tables

|                |   |   |   |   |   |   |   |
|----------------|---|---|---|---|---|---|---|
| range: 1, 2, 3 | a | b | c | R | 1 | 2 | 3 |
|                | 1 | 2 | 3 | 1 | T | T | T |
|                |   |   |   | 2 | F | F | T |
|                |   |   |   | 3 | F | F | F |

Counterexample presented by a diagram



Grayed values are not required for either gap, and the value for R22 is not required for the 2nd gap



## Phi 270 F00 test 4

### F00 test 4 questions

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

1. **Only necessary projects were funded.** [Different interpretations of the scope of *only* are possible here; any of them will do.]
2. **Tom can solve the puzzle if anyone can.** [In this case, restate your answer using an unrestricted quantifier.]
3. **No one received every vote**

Use derivations to establish the validity of the following arguments. You may use attachment rules. English interpretations are suggested but remember that they play no role in derivations, and don't hesitate to ignore them if they don't help you think about the derivations.

$$4. \quad \frac{\forall x (Dx \rightarrow Mx) \quad \forall x (\neg Ax \rightarrow \neg Mx)}{\forall x (Dx \rightarrow Ax)}$$

A: [ \_ is an animal]; D: [ \_ is dog]; M: [ \_ is a mammal]

$$5. \quad \frac{\forall x \forall y ((Py \wedge Byx) \rightarrow Dyx)}{\forall x (Px \rightarrow \forall y (Bxy \rightarrow Dxy))}$$

**Everyone who has built anything is proud of it / Everyone is proud of everything he or she has built**

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that is a counterexample lurking in one of the derivation's open gaps.

$$6. \quad \frac{\forall x (Rxx \rightarrow \neg Fx) \quad \forall x Rxc}{\forall x \forall y (Fy \rightarrow \neg Rxy)}$$

## F00 test 4 answers

1. **Only necessary projects were funded**

$(\forall x: \neg x \text{ was a necessary project}) \neg x \text{ was funded}$

$(\forall x: \neg (x \text{ was a project} \wedge x \text{ was necessary})) \neg x \text{ was funded}$

$(\forall x: \neg (Px \wedge Nx)) \neg Fx$

F: [ \_ was funded]; N: [ \_ was necessary]; P: [ \_ was a project]

$(\forall x: Px \wedge \neg Nx) \neg Fx$ —i.e., **No unnecessary projects were funded**—and  $(\forall x: Nx \wedge \neg Px) \neg Fx$ —i.e., **Among the necessities only projects were funded**—are not equivalent but are possible interpretations that would be marked by emphasis on **necessary** and **projects**, respectively.

2. **Tom can solve the puzzle if anyone can**

$(\forall x: x \text{ is a person}) \text{ Tom can solve the puzzle if } x \text{ can}$

$(\forall x: Px) (\text{Tom can solve the puzzle} \leftarrow x \text{ can solve the puzzle})$

$(\forall x: Px) (S \text{ Tom the puzzle} \leftarrow S x \text{ the puzzle})$

$(\forall x: Px) (\text{Stp} \leftarrow \text{Sxp})$  [or:  $(\forall x: Px) (\text{Sxp} \rightarrow \text{Stp})$ ]

$\forall x (Px \rightarrow (\text{Stp} \leftarrow \text{Sxp}))$  [or:  $\forall x (Px \rightarrow (\text{Sxp} \rightarrow \text{Stp}))$ ]

P: [ \_ is a person]; S: [ \_ can solve \_ ]; p: the puzzle; t: Tom

3. **No one received every vote**

$(\forall x: x \text{ is a person}) \neg x \text{ received every vote}$

$(\forall x: Px) \neg x \text{ received every vote}$

$(\forall x: Px) \neg (\forall y: y \text{ is a vote}) x \text{ received } y$

$(\forall x: Px) \neg (\forall y: Vy) Rxy$

P: [ \_ is a person]; R: [ \_ received \_ ]; V: [ \_ is a vote]

*Incorrect answers:*

$(\forall x: Px) (\forall y: Vy) \neg Rxy$  says **No one received any vote**

$\neg (\forall x: Px) (\forall y: Vy) Rxy$  says **Not everyone received every vote**

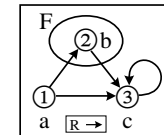
$(\forall y: Vy) \neg (\forall x: Px) Rxy$  says **No vote is such that everyone received it**

|       |   |     |
|-------|---|-----|
| 4.    | $\forall x (Dx \rightarrow Mx)$           | a:3 |
|       | $\forall x (\neg Ax \rightarrow \neg Mx)$ | a:5 |
|       | (a)                                       |     |
|       | Da  | (4) |
| 3 UI  | Da $\rightarrow$ Ma                       | 4   |
| 4 MPP | Ma  | (6) |
| 5 UI  | $\neg$ Aa $\rightarrow$ $\neg$ Ma         | 6   |
| 6 MTT | Aa  | (7) |
|       | ●   |     |
| 7 QED | Aa  | 2   |
| 2 CP  | Da $\rightarrow$ Aa                       | 1   |
| 1 UG  | $\forall x (Dx \rightarrow Ax)$           |     |

|       |  |      |
|-------|--|------|
| 5.    | $\forall x \forall y ((Py \wedge Byx) \rightarrow Dyx)$      | b:5  |
|       | (a)  |      |
|       | Pa   | (9)  |
|       | (b)  |      |
|       | Bab  | (10) |
| 5 UI  | $\forall y ((Py \wedge Byb) \rightarrow Dyb)$                | a:6  |
| 6 UI  | $(Pa \wedge Bab) \rightarrow Dab$                            | 8    |
|       | $\neg$ Dab   | (8)  |
| 8 MTT | $\neg (Pa \wedge Bab)$                                       | 9    |
| 9 MPT | $\neg$ Bab   | (10) |
|       | $\perp$  | 7    |
| 10 Nc | $\perp$  | 7    |
| 7 IP  | Dab  | 4    |
| 4 CP  | Bab $\rightarrow$ Dab  | 3    |
| 3 UG  | $\forall y (Bay \rightarrow Day)$                            | 2    |
| 2 CP  | Pa $\rightarrow \forall y (Bay \rightarrow Day)$             | 1    |
| 1 UG  | $\forall x (Px \rightarrow \forall y (Bxy \rightarrow Dxy))$ |      |

[This can be done without the *reductio* argument begun at stage 7 by using Adj to derive  $Pa \wedge Bab$  in order to exploit  $(Pa \wedge Bab) \rightarrow Dab$  for a]

|        |   |   |
|--------|---|---|
| 6.     | $\forall x (Rxx \rightarrow \neg Fx)$           | b:4, c:9, a:11  |
|        | $\forall x Rxc$                                 | a:6, b:7, c:8   |
|        | (a)   |   |
|        | (b)   |   |
|        | Fb  | (5)   |
| 4 UI   | Rbb $\rightarrow \neg$ Fb                       | 5   |
| 5 MTT  | $\neg$ Rbb                                      |   |
| 6 UI   | Rac   |   |
| 7 UI   | Rbc   |   |
| 8 UI   | Rcc   | (10)  |
| 9 UI   | Rcc $\rightarrow \neg$ Fc                       | 10  |
| 10 MPP | $\neg$ Fc                                       |   |
| 11 UI  | Raa $\rightarrow \neg$ Fa                       | 13  |
|        | Rab   |   |
|        | $\neg$ Raa                                      |   |
|        | ○   | Fb, $\neg$ Fc, $\neg$ Raa, Rab, Rac,<br>$\neg$ Rbb, Rbc, Rcc $\neq \perp$ |
|        | $\perp$   | 14  |
| 14 IP  | Raa   | 13  |
|        | $\neg$ Fa                                       |   |
|        | ○   | $\neg$ Fa, Fb, $\neg$ Fc, Rab, Rac,<br>$\neg$ Rbb, Rbc, Rcc $\neq \perp$  |
|        | $\perp$   | 13  |
| 13 RC  | $\perp$   | 12  |
| 12 RAA | $\neg$ Rab                                      | 3   |
| 3 CP   | Fb $\rightarrow \neg$ Rab                       | 2   |
| 2 UG   | $\forall y (Fy \rightarrow \neg$ Ray)           | 1   |
| 1 UG   | $\forall x \forall y (Fy \rightarrow \neg$ Rxy) |   |



lurks in both open gaps

Phi 270 F99 test 4

F99 test 4 questions

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

- Sam invited every vertebrate to the party, but only people accepted his invitation
- Tom didn't send anything to the printer
- No game that every child liked was complete

Synthesize an English sentence whose analysis would yield the following form.

- $(\forall x: Px) (\forall y: Ry \wedge Txy) Sy$   
P: [ \_ is a person ]; R: [ \_ is a room ]; S: [ \_ was reserved ]; T: [ \_ thought of \_ ]

Use derivations to establish the validity of the following arguments. You may use attachment rules.

- $$\frac{\forall x (Fx \rightarrow Gx)}{\forall x Fx \rightarrow \forall x Gx}$$
- $$\frac{\forall x \forall y (Fyx \rightarrow \neg Py)}{\forall x (Px \rightarrow \forall y \neg Fxy)}$$

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that is a counterexample lurking one of the derivation's open gaps.

- $$\frac{\forall x \forall y (Fy \rightarrow \neg Rxy) \quad \forall x Rxx}{\forall x \forall y \neg Rxy}$$

F99 test 4 answers

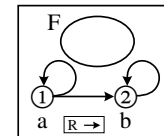
- Sam invited every vertebrate to the party, but only people accepted his invitation  
Sam invited every vertebrate to the party  $\wedge$  only people accepted Sam's invitation  
every vertebrate is such that (Sam invited it to the party)  $\wedge$  only people are such that (they accepted Sam's invitation)  
 $(\forall x: x \text{ is a vertebrate})$  Sam invited x to the party  $\wedge$   $(\forall x: \neg x \text{ is a person}) \rightarrow x$  accepted Sam's invitation  
 $(\forall x: \forall x) I_{xsp} \wedge (\forall x: \neg Px) \rightarrow Ax(\text{Sam's invitation})$   
 $(\forall x: \forall x) I_{xsp} \wedge (\forall x: \neg Px) \rightarrow Ax(\text{is})$   
A: [ \_ accepted \_ ]; I: [ \_ invited \_ to \_ ]; P: [ \_ is a person ]; V: [ \_ is a vertebrate ]; i: [ \_ 's invitation ]; p: the party; s: Sam
- Tom didn't send anything to the printer  
everything is such that (Tom didn't send it to the printer)  
 $\forall x$  Tom didn't send x to the printer  
 $\forall x \neg$  Tom sent x to the printer  
 $\forall x \neg Stxp$   
S: [ \_ sent \_ to \_ ]; p: the printer; t: Tom
- No game that every child liked was complete  
No game that every child liked is such that (it was complete)  
 $(\forall x: x \text{ was a game that every child liked}) \rightarrow x$  was complete  
 $(\forall x: x \text{ was a game} \wedge \text{every child liked } x) \rightarrow Cx$   
 $(\forall x: x \text{ was a game} \wedge \text{every child is such that (he or she liked } x)) \rightarrow \neg Cx$   
 $(\forall x: Gx \wedge (\forall y: y \text{ was a child}) y \text{ liked } x) \rightarrow Cx$   
 $(\forall x: Gx \wedge (\forall y: Dy) Lyx) \rightarrow Cx$   
C: [ \_ was complete ]; D: [ \_ was a child ]; G: [ \_ was a game ]; L: [ \_ liked \_ ]
- $(\forall x: x \text{ is a person}) (\forall y: y \text{ is a room} \wedge x \text{ thought of } y) y$  was reserved  
 $(\forall x: x \text{ is a person}) (\forall y: y \text{ is a room } x \text{ thought of}) y$  was reserved  
 $(\forall x: x \text{ is a person})$  every room x thought of was such that (it was reserved)  
 $(\forall x: x \text{ is a person})$  every room x thought of was reserved  
everyone is such that (every room he or she thought of was reserved)  
every room anyone thought of was reserved

|       |  |      |                     |   |      |      |     |       |      |     |  |   |  |  |
|-------|--|------|---------------------|---|------|------|-----|-------|------|-----|--|---|--|--|
| 5.    | $\forall x (Fx \rightarrow Gx)$  | a:3  |                     |   |      |      |     |       |      |     |  |   |  |  |
|       | $\forall x Fx$   | a:4  |                     |   |      |      |     |       |      |     |  |   |  |  |
|       | <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">3 UI</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>Fa \rightarrow Ga</math></td> <td style="padding-left: 10px;">5</td> </tr> <tr> <td style="padding-right: 5px;">4 UI</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>Fa</math></td> <td style="padding-left: 10px;">(5)</td> </tr> <tr> <td style="padding-right: 5px;">5 MPP</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>Ga</math></td> <td style="padding-left: 10px;">(6)</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px; text-align: center;">●</td> <td></td> </tr> </table> | 3 UI | $Fa \rightarrow Ga$ | 5 | 4 UI | $Fa$ | (5) | 5 MPP | $Ga$ | (6) |  | ● |  |  |
| 3 UI  | $Fa \rightarrow Ga$  | 5    |                     |   |      |      |     |       |      |     |  |   |  |  |
| 4 UI  | $Fa$   | (5)  |                     |   |      |      |     |       |      |     |  |   |  |  |
| 5 MPP | $Ga$   | (6)  |                     |   |      |      |     |       |      |     |  |   |  |  |
|       | ●  |      |                     |   |      |      |     |       |      |     |  |   |  |  |
|       | $Ga$   | 2    |                     |   |      |      |     |       |      |     |  |   |  |  |
|       | $\forall x Gx$   | 1    |                     |   |      |      |     |       |      |     |  |   |  |  |
|       | $\forall x Fx \rightarrow \forall x Gx$  |      |                     |   |      |      |     |       |      |     |  |   |  |  |

|       |   |      |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|-------|---|------|---------------------------------------|-----|-------|--|------|---------------------------------------|-----------|-------|---------------------------|---|--|-----------|-----|--|---|--|--|--|---------|---|--|------------|---|--|----------------------|---|--|-------------------------------------|---|--|---|--|
| 6.    | $\forall x \forall y (Fyx \rightarrow \neg Py)$   | b:5  |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
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| 5 UI  | $Pa$  | (8)  |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
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| 6 UI  | $\forall y (Fyb \rightarrow \neg Py)$   | a:6  |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
| 7 MPP | $Fab \rightarrow \neg Pa$   | 7    |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | $\neg Pa$   | (8)  |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | ●   |      |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | $\perp$   | 4    |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | $\neg Fab$  | 3    |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | $\forall y \neg Fay$  | 2    |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | $Pa \rightarrow \forall y \neg Fay$   | 1    |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |
|       | $\forall x (Px \rightarrow \forall y \neg Fxy)$   |      |                                       |     |       |  |      |                                       |           |       |                           |   |  |           |     |  |   |  |  |  |         |   |  |            |   |  |                      |   |  |                                     |   |  |   |  |

|        |   |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|--------|---|--|---------------------------------------|----------|------|---------------------------------------|--|------|---------|------|------|---------|------|------|--|-------|------------|---------------------------|----|--------|--|--|---------|-----------|--|---------|---------------------------|----|-------|---------------------------|----|--------|----------------------|---|--|--|-------|-----------|--|--|---|--|--|---------|----|--|------|----|--|--|-------|------------|--|--|---|--|--|---------|----|--|---------|---|--|--|------------|---|--|----------------------|---|--|--------------------------------|--|
| 7.     | $\forall x \forall y (Fy \rightarrow \neg Rxy)$   | a:4, b:5   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\forall x Rxx$   | a:6, b:7   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
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| 4 UI   | $\forall y (Fy \rightarrow \neg Ray)$   | a:8, b:9   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 5 UI   | $\forall y (Fy \rightarrow \neg Rby)$   | a:12, b:13   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 6 UI   | $Raa$   | (10)   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 7 UI   | $Rbb$   | (14)   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 8 UI   | $Fa \rightarrow \neg Raa$   | 10   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 9 UI   | $Fb \rightarrow \neg Rab$   | 11   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 10 MTT | $\neg Fa$   |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 11 MTT | $\neg Fb$   |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 12 UI  | $Fa \rightarrow \neg Rba$   | 15   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 13 UI  | $Fb \rightarrow \neg Rbb$   | 14   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 14 MTT | $\neg Fb$   |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">16 IP</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\neg Fa</math></td> <td></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px; text-align: center;">○</td> <td style="padding-left: 10px;"><math>\neg Fa, \neg Fb, Rab, Raa, Rbb \neq \perp</math></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">16</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>Fa</math></td> <td style="padding-left: 10px;">15</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">15 RC</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\neg Rba</math></td> <td></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px; text-align: center;">○</td> <td style="padding-left: 10px;"><math>\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \neq \perp</math></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">15</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">3</td> </tr> </table> </td> <td></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\neg Rab</math></td> <td style="padding-left: 10px;">2</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\forall y \neg Ray</math></td> <td style="padding-left: 10px;">1</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\forall x \forall y \neg Rxy</math></td> <td></td> </tr> </table>  | 16 IP  | $\neg Fa$                             |          |      | ○                                     | $\neg Fa, \neg Fb, Rab, Raa, Rbb \neq \perp$           |      | $\perp$ | 16   |      | $Fa$    | 15   |      | <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">15 RC</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\neg Rba</math></td> <td></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px; text-align: center;">○</td> <td style="padding-left: 10px;"><math>\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \neq \perp</math></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">15</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">3</td> </tr> </table> | 15 RC | $\neg Rba$ |                           |    | ○      | $\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \neq \perp$ |  | $\perp$ | 15        |  | $\perp$ | 3                         |    |       | $\neg Rab$                | 2  |        | $\forall y \neg Ray$ | 1 |  | $\forall x \forall y \neg Rxy$   |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 16 IP  | $\neg Fa$   |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | ○   | $\neg Fa, \neg Fb, Rab, Raa, Rbb \neq \perp$           |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\perp$   | 16   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $Fa$  | 15   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 5px;">15 RC</td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\neg Rba</math></td> <td></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px; text-align: center;">○</td> <td style="padding-left: 10px;"><math>\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \neq \perp</math></td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">15</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; padding-left: 5px;"><math>\perp</math></td> <td style="padding-left: 10px;">3</td> </tr> </table>  | 15 RC  | $\neg Rba$                            |          |      | ○                                     | $\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \neq \perp$ |      | $\perp$ | 15   |      | $\perp$ | 3    |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
| 15 RC  | $\neg Rba$  |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | ○   | $\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \neq \perp$ |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\perp$   | 15   |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\perp$   | 3  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\neg Rab$  | 2  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\forall y \neg Ray$  | 1  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |
|        | $\forall x \forall y \neg Rxy$  |  |                                       |          |      |                                       |  |      |         |      |      |         |      |      |  |       |            |                           |    |        |  |  |         |           |  |         |                           |    |       |                           |    |        |                      |   |  |  |       |           |  |  |   |  |  |         |    |  |      |    |  |  |       |            |  |  |   |  |  |         |    |  |         |   |  |  |            |   |  |                      |   |  |                                |  |

The counterexample below lurks in both gaps:



Phi 270 F98 test 4

F98 test 4 questions

(Questions 1-2 are from quiz 4 and 3-8 are from quiz 5 out of 6 quizzes—these two quizzes addressed the part of the course your test is designed to cover.)

Identify individual terms and quantifier phrases in the following sentence and indicate links between pronouns and their antecedents. (You can do this by marking up an English sentence; you are *not* being asked to provide a symbolic analysis.)

1. Sam ordered a book, but instead of it he received a book he didn't want.

Analyze the following generalization in as much detail as possible. Provide a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

2. No one saw the book that was lying on the table.

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

3. No one except numismatists understood the joke
4. The movie delighted all boys and girls
5. If anyone relayed the message to everyone, then no one understood every part of it

Use derivations to establish the validity of the following arguments. You may use attachment rules.

6. 
$$\frac{\forall x (Fx \vee Gx)}{\forall x \neg Gx}$$
  
$$\frac{}{\forall x Fx}$$
7. 
$$\frac{\forall x (Fx \rightarrow \forall y (Pxy \rightarrow Rxy))}{\forall y \forall x ((Fx \wedge Pxy) \rightarrow Rxy)}$$

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in one of the derivation's open gaps.

8. 
$$\frac{\forall x (Fx \rightarrow \neg Rxx)}{\forall x \forall y (Fy \rightarrow \neg Rxy)}$$

F98 test 4 answers

1. Sam ordered a book, but instead of it he received a book he didn't want  
T Q Q

2. No one saw the book that was lying on the table.  
No one is such that (he or she saw the book that was lying on the table)

$$(\forall x: x \text{ is a person}) \neg x \text{ saw } \underline{\text{the book that was lying on the table}}$$

$$(\forall x: Px) \neg Sx(\underline{\text{the book that was lying on the table}})$$

$$(\forall x: Px) \neg Sx(bt)$$

P: [ \_ is a person]; S: [ \_ saw \_ ]; b: [the book that was lying on \_ ]; t: the table

3. No one except numismatists understood the joke  
( $\forall x: x \text{ is a person} \wedge \neg x \text{ is a numismatist}) \neg x \text{ understood } \underline{\text{the joke}}$ )

$$(\forall x: Px \wedge \neg Nx) \neg Uxj$$

N: [ \_ is a person]; P: [ \_ is a numismatist]; U: [ \_ understood \_ ]; j: the joke

4. The movie delighted all boys and girls  
all boys and girls are such that (the movie delighted them)

$$(\forall x: x \text{ is a boy or girl}) \text{ the movie delighted } x$$

$$(\forall x: x \text{ is a boy} \vee x \text{ is a girl}) \underline{\text{the movie}} \text{ delighted } x$$

$$(\forall x: Bx \vee Gx) Dmx$$

B: [ \_ is a boy]; D: [ \_ delighted \_ ]; G: [ \_ is a girl]; m: the movie

5. If anyone relayed the message to everyone, then no one understood every part of it

$$(\forall x: x \text{ is a person}) \text{ if } x \text{ relayed the message to everyone, then no one understood every part of it}$$

$$(\forall x: Px) (x \text{ relayed the message to everyone} \rightarrow \text{no one understood every part of the message})$$

$$(\forall x: Px) ((\forall y: y \text{ is a person}) x \text{ relayed the message to } y \rightarrow (\forall z: z \text{ is a person}) \neg z \text{ understood every part of the message})$$

$$(\forall x: Px) ((\forall y: Py) x \text{ relayed } \underline{\text{the message}} \text{ to } y \rightarrow (\forall z: Pz) \neg (\forall w: w \text{ is a part of the message}) z \text{ understood } w)$$

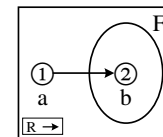
$$(\forall x: Px) ((\forall y: Py) Rxy \rightarrow (\forall z: Pz) \neg (\forall w: Twm) Uzw)$$

P: [ \_ is a person]; R: [ \_ relayed \_ to \_ ]; T: [ \_ is a part of \_ ]; U: [ \_ understood \_ ]; m: the message

|       |   |  |
|-------|---|--|
| 6.    | $\forall x (Fx \vee Gx)$ a:2<br>$\forall x \neg Gx$ a:3   |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;">           (a)         </div>   |  |
| 2 UI  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa \vee Ga</math> 4           </div>                                     |  |
| 3 UI  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Ga</math> (4)           </div>                                      |  |
| 4 MTP | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa</math> (5)           </div>   |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\bullet</math> </div>  |  |
| 5 QED | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa</math> 1           </div>   |  |
| 1 UG  | $\forall x Fx$  |  |
| 7.    | $\forall x (Fx \rightarrow \forall y (Pxy \rightarrow Rxy))$ b:5  |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;">           (a)         </div>   |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;">           (b)         </div>   |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fb \wedge Pba</math> 4           </div>                                  |  |
| 4 Ext | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fb</math> (6)           </div>   |  |
| 4 Ext | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Pba</math> (8)           </div>  |  |
| 5 UI  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fb \rightarrow \forall y (Pby \rightarrow Rby)</math> 6           </div> |  |
| 6 MPP | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y (Pby \rightarrow Rby)</math> a:7           </div>              |  |
| 7 UI  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Pba \rightarrow Rba</math> 8           </div>                            |  |
| 8 MPP | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rba</math> (9)           </div>  |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\bullet</math> </div>  |  |
| 9 QED | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rba</math> 3           </div>  |  |
| 3 CP  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>(Fb \wedge Pba) \rightarrow Rba</math> 2           </div>                |  |
| 2 UG  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall x ((Fx \wedge Pxa) \rightarrow Rxa)</math> 1           </div>    |  |
| 1 UG  | $\forall y \forall x ((Fx \wedge Pxy) \rightarrow Rxy)$   |  |

|       |  |  |
|-------|--|--|
| 8.    | $\forall x (Fx \rightarrow \neg Rxx)$ b:5, a:7   |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;">           (a)         </div>                            |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;">           (b)         </div>                            |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fb</math> (6)           </div>                    |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab</math> </div>                                 |  |
| 5 UI  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fb \rightarrow \neg Rbb</math> 6           </div> |  |
| 6 MPP | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Rbb</math> </div>                            |  |
| 7 UI  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa \rightarrow \neg Raa</math> 8           </div> |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Fa</math> </div>                             |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\circ</math> </div>                               | $Fb, Rab, \neg Rbb, \neg Fa \neq \perp$  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div>                               | 9  |
| 9 IP  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa</math> 8           </div>                      |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Raa</math> </div>                            |  |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\circ</math> </div>                               | $Fb, Rab, \neg Rbb, \neg Raa \neq \perp$ |
|       | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div>                               | 8  |
| 8 RC  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div>                               | 4  |
| 4 RAA | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Rab</math> </div>                            | 3  |
| 3 CP  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fb \rightarrow \neg Rab</math> </div>             | 2  |
| 2 UG  | <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y (Fy \rightarrow \neg Ray)</math> </div> | 1  |
| 1 UG  | $\forall x \forall y (Fy \rightarrow \neg Rxy)$  |  |

This counterexample lurks in both gaps:



Phi 270 F97 test 4

F97 test 4 questions

(Questions 1-3 are from quiz 4 and 4-9 are from quiz 5 out of 6 quizzes—these two quizzes addressed the part of the course your test is designed to cover.)

Identify individual terms and quantifier phrases in the following sentence and indicate links between pronouns and their antecedents. (You can do this by marking up an English sentence; you are *not* being asked to provide a symbolic analysis.)

1. **Everyone who Carol lent the book to spoke to her at length about it.**  
Analyze the following generalizations in as much detail as possible. Provide a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer *and restate the result using an unrestricted quantifier.*

2. **Bob called no one.**

3. **Among contestants, only professionals were finalists.**  
Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

4. **Bob doesn't own any map showing Dafter.**

5. **Nothing anyone said bothered Dave.**  
Use derivations to establish the validity of the following arguments. You may use attachment rules.

6. 
$$\frac{\forall x (Fx \wedge Gx)}{\forall x Fx}$$

7. 
$$\frac{\forall x (Rxa \rightarrow \forall y Rxy)}{\forall x (\forall y Rxy \rightarrow Rxb)}$$

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in one of the derivation's open gaps. (You will *not* need the rules UG+ and ST of 7.8 that were designed to avoid unending derivations.)

8. 
$$\frac{\forall x (Fx \rightarrow Rax)}{\forall x (Fx \rightarrow Rxa)}$$

You will receive credit for *one* of the following (but you may attempt both):  
Synthesize an English sentence whose analysis would yield the following form.

9a.  $(\forall x: Dx) (Okx \rightarrow (\forall y: Dy) Oky)$

D: [ is a door ]; O: [ opens ]; k: **the key**

Use derivations to establish the validity of the following argument. You may use attachment rules.

9b. 
$$\frac{\forall x \forall y (Rxy \rightarrow \neg Fy)}{\forall x (Fx \rightarrow Rxx)}$$
  
$$\frac{\quad}{\forall x \neg Fx}$$

F97 test 4 answers

1. **Everyone who Carol lent the book to spoke to her at length about it**  
Q T T

2. **Bob called no one**  
no one is such that (Bob called him or her)

$(\forall x: x \text{ is an person}) \neg \text{Bob called } x$

$(\forall x: Px) \neg Cbx$

$\forall x (Px \rightarrow \neg Cbx)$

C: [  called ]; P: [  is person ]; b: **Bob**

3. **Among contestants, only professionals were finalists**  
**Among contestants, only professionals are such that (they were finalists)**

$(\forall x: x \text{ was a contestant} \wedge \neg x \text{ was a professional}) \neg x \text{ was a finalist}$

$(\forall x: Cx \wedge \neg Px) \neg Fx$

$\forall x ((Cx \wedge \neg Px) \rightarrow \neg Fx)$

C: [  was a contestant ]; F: [  was a finalist ]; P: [  was a professional ]

4. **Bob doesn't own any map showing Dafter**  
**every map showing Dafter is such that (Bob doesn't own it)**

$(\forall x: x \text{ is a map showing Dafter}) \neg \text{Bob owns } x$

$(\forall x: x \text{ is a map} \wedge x \text{ shows Dafter}) \neg \text{Obx}$

$(\forall x: Mx \wedge Sxd) \neg \text{Obx}$

M: [  is a map ]; O: [  owns ]; S: [  shows ]; b: **Bob**; d: **Dafter**

5. **Nothing anyone said bothered Dave**  
**everyone is such that (nothing he or she said bothered Dave)**

$(\forall x: x \text{ is a person}) \text{ nothing } x \text{ said bothered Dave}$

$(\forall x: Px) \text{ nothing } x \text{ said is such that (it bothered Dave)}$

$(\forall x: Px) (\forall y: y \text{ is a thing } x \text{ said}) \neg y \text{ bothered Dave}$

$(\forall x: Px) (\forall y: x \text{ said } y) \neg \text{Byd}$

$(\forall x: Px) (\forall y: Sxy) \neg \text{Byd}$

B: [  bothered ]; P: [  is a person ]; S: [  said ]; d: **Dave**



6.

|       |                            |     |
|-------|----------------------------|-----|
|       | $\forall x (Fx \wedge Gx)$ | a:2 |
| 2 UI  | ⓐ                          |     |
| 3 Ext | Fa $\wedge$ Ga             | 3   |
| 3 Ext | Fa                         |     |
| 3 Ext | Ga                         | (4) |
|       | ●                          |     |
| 4 QED | Fa                         | 1   |
| 1 UG  | $\forall x Fx$             |     |

7.

|       |   |     |
|-------|---|-----|
|       | $\forall x (Rxa \rightarrow \forall y Rxy)$ |     |
|       | ⓐ   |     |
|       | $\forall y Rcy$                             | a:3 |
| 3 UI  | Rcb   | (4) |
|       | ●   |     |
| 4 QED | Rcb   | 2   |
| 2 CP  | $\forall y Rcy \rightarrow Rcb$             | 1   |
| 1 UG  | $\forall x (\forall y Rxy \rightarrow Rxb)$ |     |

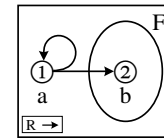
[The first premise is never used in the derivation for this question (shown above). The fact that it was not needed was a slip on my part in making up the question. Below is a derivation for a different conclusion, one that makes for the sort of argument I probably intended.]

|       |   |     |
|-------|---|-----|
|       | $\forall x (Rxa \rightarrow \forall y Ryx)$ | c:4 |
|       | ⓐ   |     |
|       | $\forall y Rcy$                             | a:3 |
| 3 UI  | Rca   | (5) |
| 4 UI  | $Rca \rightarrow \forall y Ryc$             | 5   |
| 5 MPP | $\forall y Ryc$                             | b:6 |
| 6 UI  | Rbc   | (7) |
|       | ●   |     |
| 7 QED | Rbc   | 2   |
| 2 CP  | $\forall y Rcy \rightarrow Rbc$             | 1   |
| 1 UG  | $\forall x (\forall y Rxy \rightarrow Rbx)$ |     |

8.

|       |                                  |  |
|-------|----------------------------------|--|
|       | $\forall x (Fx \rightarrow Rax)$ | b:3, a:5                                   |
|       | ⓐ                                |  |
|       | Fb                               | (4)  |
| 3 UI  | $Fb \rightarrow Rab$             | 4  |
| 4 MPP | Rab                              |  |
| 5 UI  | $Fa \rightarrow Raa$             | 7  |
|       |                                  |  |
|       | $\neg Rba$                       |  |
|       |                                  |  |
|       | $\neg Fa$                        |  |
|       | ○                                | Fb, Rab, $\neg Rba$ , $\neg Fa \neq \perp$ |
|       | $\perp$                          | 8  |
| 8 IP  | Fa                               | 7  |
|       |                                  |  |
|       | Raa                              |  |
|       |                                  |  |
|       | ○                                | Fb, Rab, $\neg Rba$ , Raa $\neq \perp$     |
|       | $\perp$                          | 7  |
| 7 RC  | $\perp$                          | 6  |
| 6 IP  | Rba                              | 2  |
| 2 CP  | $Fb \rightarrow Rba$             | 1  |
| 1 UG  | $\forall x (Fx \rightarrow Rxa)$ |  |

The counterexample below lurks in both gaps. The arrow from 1 to itself is not needed for the first gap; and it would continue to be a counterexample lurking in the second gap if the extension of F were enlarged to include both objects.



- 9a.  $(\forall x: x \text{ is a door}) ( \text{the key opens } x \rightarrow (\forall y: y \text{ is a door}) \text{ the key opens } y )$   
 $(\forall x: x \text{ is a door}) ( \text{the key opens } x \rightarrow \text{every door is such that (the key opens it)} )$   
 $(\forall x: x \text{ is a door}) ( \text{the key opens } x \rightarrow \text{the key opens every door} )$   
 $(\forall x: x \text{ is a door}) \text{ if the key opens } x, \text{ then it opens every door}$   
 every door is such that (if the key opens it, then it opens every door)  
 If the key opens any door, then it opens every door

|       |   |            |
|-------|---|------------|
| 9b.   | $\forall x \forall y (Rxy \rightarrow \neg Fy)$<br>$\forall x (Fx \rightarrow Rxx)$ | a:2<br>a:4 |
| 2 UI  | $\forall y (Ray \rightarrow \neg Fy)$   | a:6        |
|       | $Fa$  | (5), (8)   |
| 4 UI  | $Fa \rightarrow Raa$  | 5          |
| 5 MPP | $Raa$   | (7)        |
| 6 UI  | $Raa \rightarrow \neg Fa$   | 7          |
| 7 MPP | $\neg Fa$   | (8)        |
|       | $\bullet$   |            |
| 8 Nc  | $\perp$   | 3          |
| 3 RAA | $\neg Fa$   | 1          |
| 1 UG  | $\forall x \neg Fx$   |            |

**Phi 270 F96 test 4**

**F96 test 4 questions**

(Questions 1-3 are from quiz 4 and 4-9 are from quiz 5 out of 6 quizzes—these two quizzes addressed the part of the course your test is designed to cover.)

Identify individual terms and quantifier phrases in the following sentence and indicate links between pronouns and their antecedents. (You can do this by marking up an English sentence; you are *not* being asked to provide a symbolic analysis.)

1. **Al called everyone who left him a message concerning the accident and told them he had seen it.**

Analyze the following generalizations in as much detail as possible. Provide a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer *and restate the result using an unrestricted quantifier.*

2. **Every employee received the letter.**
3. **Among bystanders, Sam interviewed only soldiers.**

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

4. **If anyone guessed the number, the prize was awarded.**
5. **Everyone who worked on any part of the project was honored.**

Synthesize an English sentence whose analysis would yield the following form.

6.  $(\forall x: Px) \neg \forall y Axy$   
A: [ ate ]; P: [ is a person ]

Use derivations to establish the validity of the following arguments. You may use attachment rules.

- |  |   |
|--|---|
| $\frac{\forall x Fx \quad \forall x Gx}{\forall x (Fx \wedge Gx)}$ | $\frac{\forall x (Fx \rightarrow Rxa) \quad \forall x (Rxa \rightarrow \forall y Ryx)}{\forall x \forall y (Fy \rightarrow Rxy)}$ |
|--|---|

Use a derivation to show that the following argument is not valid and describe a counterexample lurking in one of the derivation's open gaps. (You will *not* need the rules UG+ and ST introduced in §7.8 that are designed to avoid unending gaps.)

9. 
$$\frac{\forall x Rxx}{Rab \rightarrow \forall x Rxa}$$

### F96 test 4 answers

1.  $\overbrace{\underbrace{\underbrace{\text{Al called everyone who left him a message concerning the accident}}_Q \text{ and told them he had seen it}}_T}$

[it could instead have a message concerning the accident as its antecedent]

2. **Every employee received the letter**

Every employee is such that (he or she received the letter)

$(\forall x: x \text{ is an employee}) x \text{ received the letter}$

$(\forall x: Ex) Rx1$

$\forall x (Ex \rightarrow Rx1)$

E: [ \_ is an employee]; R: [ \_ received \_ ]; l: the letter

3. **Among bystanders, Sam interviewed only soldiers**

Among bystanders, only soldiers are such that (Sam interviewed them)

$(\forall x: x \text{ was a bystander} \wedge \neg x \text{ was a soldier}) \neg \text{Sam interviewed } x$

$(\forall x: Bx \wedge \neg Sx) \neg Isx$

$\forall x ((Bx \wedge \neg Sx) \rightarrow \neg Isx)$

B: [ \_ was a bystander]; I: [ \_ interviewed \_ ]; S: [ \_ was a soldier]; s: Sam

4. **If anyone guessed the number, the prize was awarded**

Everyone is such that (if he or she guessed the number, the prize was awarded)

$(\forall x: x \text{ is a person}) (\text{if } x \text{ guessed the number, the prize was awarded})$

$(\forall x: Px) (x \text{ guessed the number} \rightarrow \text{the prize was awarded})$

$(\forall x: Px) (Gxn \rightarrow Ap)$

P: [ \_ is a person]; G: [ \_ guessed \_ ]; n: the number

5. **Everyone who worked on any part of the project was honored**

Every part of the project is such that (everyone who worked on it was honored)

$(\forall x: x \text{ is a part of the project})$  everyone who worked on x was honored

$(\forall x: Rxj) (\forall y: y \text{ is a person who worked on } x) y \text{ was honored}$

$(\forall x: Rxj) (\forall y: y \text{ is a person} \wedge y \text{ worked on } x) Hy$

$(\forall x: Rxj) (\forall y: Py \wedge Wyx) Hy$

H: [ \_ was honored]; P: [ \_ is a person]; R: [ \_ is a part of \_ ]; W: [ \_ worked on \_ ]; j: the project

6.  $(\forall x: x \text{ is a person}) \neg \forall y x \text{ ate } y$

$(\forall x: x \text{ is a person}) \neg x \text{ ate everything}$

No one is such that (he or she ate everything)

No one ate everything

|    |                |                              |
|----|----------------|------------------------------|
| 7. | $\forall x Fx$ | a:2                          |
|    | $\forall x Gx$ | a:3                          |
|    | 2 UI           | Fa (5)                       |
|    | 3 UI           | Ga (6)                       |
|    | 5 QED          | ●<br>  Fa 4                  |
|    | 6 QED          | ●<br>  Ga 4                  |
|    | 4 Cnj          | Fa ∧ Ga 1                    |
|    | 1 UG           | $\forall x (Fx \wedge Gx) 1$ |

|    |   |  |
|----|---|--|
| 8. | $\forall x (Fx \rightarrow Rxa)$            | c:4  |
|    | $\forall x (Rxa \rightarrow \forall y Ryx)$ | c:6  |
|    | 4 UI  | Fc (5)                                     |
|    | 5 MPP                                       | Fc → Rca 5                                 |
|    | 6 UI  | Rca (7)                                    |
|    | 7 MPP                                       | Rca → $\forall y Ryc$ 7                    |
|    | 8 UI  | $\forall y Ryc$ b:8                        |
|    | 8 UI  | Rbc (9)                                    |
|    | 9 QED                                       | ●<br>  Rbc 3                               |
|    | 3 CP  | Fc → Rbc 2                                 |
|    | 2 UG  | $\forall y (Fy \rightarrow Rby)$ 1         |
|    | 1 UG  | $\forall x \forall y (Fy \rightarrow Rxy)$ |

9.  $\forall x Rxx$       a:1,b:2,c:5

1 UI

Raa

2 UI

Rbb

Rab

5 UI

$\odot$  Rcc

$\neg Rca$

$\circ$

Raa,Rab,Rbb,Rcc, $\neg Rca \neq \perp$

$\perp$

6

6 IP

Rca

4

4 UG

$\forall x Rxa$

3

3 CP

Rab  $\rightarrow \forall x Rxa$

