F11 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.
- *Basic concepts of deductive logic*. You will be responsible for entailment, tautologousness and absurdity, and the relations between pairs of sentences (i.e., implication, equivalence, exclusiveness, joint exhaustiveness, and contradictoriness). You should be able to define any of these ideas in terms of truth values and possible worlds (see appendix A.1, 1.2.8, and 1.4.1 for samples of such definitions), and you should be ready to answer questions about these concepts and explain your answers in a way that uses the definitions.
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer in a way that uses the definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Synthesis*. Be able to synthesize an English sentence that has a logical form that I specify (as in the second part of the homework on 2.1).
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV (the rules for ⊤ and ⊥) in addition to Ext, Cnj, and QED; but derivations that require EFQ or ENV are much less likely than ones that require only Ext, Cnj, and QED.

F11 test 1 questions

1. Define the idea of sentences φ and ψ being contradictory by completing the following with a definition *in terms of truth values and possible worlds:*

    φ and ψ are contradictory (i.e., φ ⋈ ψ) if and only if …

2. Consider the following two items of information about sentences φ, ψ, and χ:
    (i) both φ ⊨ χ and ψ ⊨ χ
    (ii) φ, ψ ⊨ χ

    Which of (i) and (ii) indicates stronger logical relations among these sentences by ruling out more patterns of truth values for the three? And which further patterns does it rule out?

3. Suppose you know that φ implicates ψ and that ψ implies (i.e., entails) χ. Can you tell whether φ implicates χ? (Be sure to explain you answer by reference to the definitions of implication and implicature.)

4. Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and (and present the same analysis also using English notation—i.e., using both … and … to indicate conjunction). Be sure that the unanalyzed components of your answer are complete and independent sentences, and give a key to the abbreviations you use for them; also try to respect any grouping in the English.

    The first baseman went after the ball but fumbled it, and the runner went on to second.

5. Synthesize an English sentence that has the analysis below. Choose a simple and natural sentence whose organization reflects the grouping of the logical form.

    N ∧ (P ∧ B)

    N: Tom flew to New York; P: Tom drove to Philadelphia; B: Tom drove to Baltimore

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that is a counterexample lurking in an open gap. (Your table should indicate the value of any compound component by writing this value under the main connective of the component.) *Do not use the rule Adj from §2.4.*

6. A ∧ G, F ∧ C ⊨ F ∧ G
7. (A ∧ B) ∧ C ⊨ (B ∧ C) ∧ D

F11 test 1 answers

1. φ and ψ are contradictory (φ ⋈ ψ) if and only if there is no possible world in which φ and ψ share the same truth value (*or:* if and only if, in each possible world, φ has a truth value different from the one ψ has).

2. Both (i) and (ii) rule out certain values for φ and ψ when χ is false. While (ii) says that φ and ψ cannot be both true in this sort of case, (i) says more. It says of each of φ and ψ that it cannot be true when χ is false, so it says, on top of what (i) says, that neither the pattern φ true and ψ false nor the pattern φ false and ψ true can occur when χ is false. So (i) rules out three of the eight possible patterns of truth values for three sentences while (ii) rules out only one of the eight.

3. The basic idea is that ψ says part of what φ suggests, and χ says part of what ψ says. So χ says part of what φ suggests and is therefore an impli-

cature of φ. This can be explained in terms of the definitions as follows. To say that φ implicates ψ is to say that ψ is true whenever φ is appropriate. But if ψ implies χ, then χ must be true whenever ψ is, so it must be true whenever φ is appropriate. Therefore, φ does implicate χ.

4. the first baseman went after the ball but fumbled it, and the runner went on to second

    the first baseman went after the ball but fumbled it ∧ the runner went on to second

    (the first baseman went after the ball ∧ the first baseman fumbled the ball) ∧ the runner went on to second

    (B ∧ F) ∧ S

    both both B and F and S

    B: the first baseman went after the ball; F: the first baseman fumbled the ball; S: the runner went on to second

5. Tom flew to New York ∧ (Tom drove to Philadelphia ∧ Tom drove to Baltimore)

    Tom flew to New York ∧ Tom drove to Philadelphia and Baltimore

    Tom flew to New York, and [*or:* but] he drove to Philadelphia and Baltimore

6.

	A ∧ G	1
	F ∧ C	2
1 Ext	A	
1 Ext	G	(5)
2 Ext	F	(4)
2 Ext	C	
	●	
4 QED	F	3
	●	
5 QED	G	3
3 Cnj	F ∧ G	

7.

	(A ∧ B) ∧ C	1
1 Ext	A ∧ B	2
1 Ext	C	(6)
2 Ext	A	
2 Ext	B	(5)
	●	
5 QED	B	4
	●	
6 QED	C	4
4 Cnj	B ∧ C	3
	○	A, B, C ⊭ D
	D	3
3 Cnj	(B ∧ C) ∧ D	

A	B	C	D	(A ∧ B) ∧ C	/	(B ∧ C) ∧ D
T	T	T	F	T ⊤		T ⑰

F10 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Basic concepts of deductive logic.* You will be responsible for entailment, tautologousness and absurdity, and the relations between pairs of sentences (i.e., implication, equivalence, exclusiveness, joint exhaustiveness, and contradictoriness). You should be able to define any of these ideas in terms of truth values and possible worlds (see appendix A.1 and 1.2.8 for samples of such definitions), and you should be ready to answer questions about these concepts and explain your answers in a way that uses the definitions.
- *Implicature.* Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer in a way that uses the definition.
- *Analysis.* Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Synthesis.* Be able to synthesize an English sentence that has a given logical form.
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV (the rules for ⊤ and ⊥) in addition to Ext, Cnj, and QED; but derivations requiring EFQ and ENV are only a possibility while you are certain to run into derivations requiring the latter three rules.

F10 test 1 questions

1. Define entailment by completing the following with a definition in terms of truth values and possible worlds:

   a set Γ of sentences entails a sentence φ (i.e., Γ ⊨ φ) if and only if …

2. Suppose that φ and ψ are contradictory (i.e., that φ ⋈ ψ) and also that ψ implies χ (i.e., that ψ ⊨ χ). Can you conclude anything about the deductive relations holding between φ and χ? That is, does the information given allow you to rule out one or more of the four conceivable patterns of truth values for two sentences (i.e., TT, TF, FT, FF) in the case of φ and χ, or is it consistent with what you are told that φ and χ be logically independent? You should justify your answer in a way that shows you know the definitions of contradictoriness and implication (but you need not provide the name of the relation between φ and χ if you conclude that they are related).

3. (i) Present a sentence that, when used in a certain context, has an implicature that suggests something beyond what the sentence says literally, and (ii) briefly explain why the sentence has that implicature in the context you describe. In addressing part (i), be sure to show that the implicature is not part of what the sentence says by describing a way that the implicature could be false while what the sentence says literally is true.

4. Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and (and present the same analysis also using English notation—i.e., using both … and … to indicate conjunction). Be sure that the unanalyzed components of your answer are complete and independent sentences, and give a key to the abbreviations you use for them; also try to respect any grouping in the English.

   Ann posed the problem, and Bill and Carol each solved it

Synthesize an English sentence that has the analysis below. Choose a simple and natural sentence whose organization reflects the grouping of the logical form.

5. $(F \land O) \land (C \land W)$

   C: Tom gathered up the contents of the package; F: Sam found the package; O: Sam opened the package; W: Tom gathered up the wrapping of the package

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that is a counterexample lurking in an open gap. (Your table should indicate the value of any compound component by writing this value under the main connective of the component.) *Do not use the rule Adj*.

6. $E \land (A \land K) \vDash K \land E$
7. $D \land E, R \land S \vDash R \land (D \land T)$

F10 test 1 answers

1. a set Γ of sentences entails a sentence φ (i.e., Γ ⊨ φ) if and only if there is no possible world in which every member of Γ is true and φ is false (*or:* if and only if φ is true in each possible world in which every member of Γ is true)

2. We can conclude that φ and χ cannot be both false. Since φ and ψ are contradictory, we know that they cannot be both true or both false—i.e., their

truth values must be different—so ψ must be true if φ is false. And, since ψ implies χ, we know that χ cannot be false when ψ is true. Therefore, χ cannot be false when φ is false.

   [Although it is not part of what you were asked, note that we cannot conclude anything else about the truth values of φ and χ. In some cases they might exhibit the patterns TF and FT because φ and ψ might exhibit both, and χ might in fact be equivalent to ψ. And they might exhibit the pattern TT since the given information is consistent with this pattern in a case where ψ is false; for example, suppose φ says The glass is not empty, ψ says The glass is empty, and χ says The glass is not full, and consider a case where the glass is half full.]

3. Here's a sample answer. (i) If I say, "I have a lot of work to do" in answer to the question "Are you going to the movie?," the person asking would in most circumstances conclude that I am not going; but it would be perfectly possible for me to have a lot work to do (so what I have said is true) and nevertheless go to the movie (in which case what I implicated would be false). (ii) The implicature arises because my response to the question is appropriate only if it is taken as an answer; and, unless I am known to be someone makes a point of going to the movies when I have a lot of work to do, the answer made most likely by my response is that I am not going.

4. Ann posed the problem, and Bill and Carol each solved it
   Ann posed the problem ∧ Bill and Carol each solved the problem
   Ann posed the problem ∧ (Bill solved the problem ∧ Carol solved the problem)

   $$A \land (B \land C)$$
   both A and both B and C

   A: Ann posed the problem; B: Bill solved the problem; C: Carol solved the problem

   [The function of each in this sentence is to pointedly leave open the possibility that Bill and Carol worked independently; a sentence that said instead that they worked together could not be analyzed as a conjunction of the components B and C above.]

5. (Sam found the package ∧ Sam opened the package) ∧ (Tom gathered up the contents of the package ∧ Tom gathered up the wrapping of the package)
   Sam found the package and opened it ∧ Tom gathered up both its contents and its wrapping
   Sam found the package and opened it, and Tom gathered up both its contents and its wrapping

   [If the switch from Sam to Tom was unexpected, you'd probably express the main conjunction by using but (or a synonym of it) instead of and.]

6.

	E ∧ (A ∧ K)	1
1 Ext	E	(5)
1 Ext	A ∧ K	2
2 Ext	A	
2 Ext	K	(4)
	●	
4 QED	K	3
	●	
5 QED	E	3
3 Cnj	K ∧ E	

7.

	D ∧ E	1
	R ∧ S	2
1 Ext	D	(6)
1 Ext	E	
2 Ext	R	(4)
2 Ext	S	
	●	
4 QED	R	3
	●	
6 QED	D	5
	○	D, E, R, S ⊭ T
	T	5
5 Cnj	D ∧ T	3
3 Cnj	R ∧ (D ∧ T)	

D E R S	D ∧ E, R ∧ S / R ∧ (D ∧ T)			
T T T F	⊤	⊤	⒡	F

F09 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Basic concepts of deductive logic*. You will be responsible for entailment, tautologousness and absurdity, and the relations between pairs of sentences (i.e., implication, equivalence, exclusiveness, joint exhaustiveness, and contradictoriness). You should be able to define any of these ideas in terms of truth values and possible worlds (see appendix A.1 and 1.2.8 for samples of such definitions), and you should be ready to answer questions about these concepts and explain your answers in a way that uses the definitions.
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer in a way that uses the definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Synthesis*. Be able to synthesize an English sentence that has a given logical form.
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4.3 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV (the rules for ⊤ and ⊥) as well as Ext, Cnj, and QED; but derivations involving the latter three are much more likely.

F09 test 1 questions

1. Define the idea of a sentence φ implying a sentence ψ by completing the following with a definition in terms of truth values and possible worlds:

   φ implies ψ (i.e., φ ⊨ ψ) if and only if …

2. Suppose that φ and ψ are mutually exclusive (i.e., that φ ⚊ ψ) and also that χ and ψ are mutually exclusive (i.e., that χ ⚊ ψ). Can you conclude that φ and χ are equivalent (i.e., that φ ≃ χ)? Say why or why not in a way that makes use of the definitions of mutual exclusiveness and equivalence.

3. In the right circumstances, a tautology like Dave is Dave can convey genuine information—i.e., can convey information that rules out some possible worlds. (i) Use the definition of tautology to explain why this information cannot come from what is said—i.e., from the proposition expressed by the tautology. And (ii) use the definition of implicature to explain how such information might be conveyed as a suggestion.

4. Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and also English notation (i.e., using both … and …). Be sure that the unanalyzed components of your answer are complete and independent sentences (and give a key to the abbreviations you use for them); also try to respect any grouping in the English.

   Al saw the meteor go by and Bill did, too; but Cal actually saw it land

5. Synthesize an English sentence that has the analysis below. Choose a simple and natural sentence whose organization reflects the grouping of the logical form.

   (L ∧ S) ∧ (V ∧ F)

   F: Al told Hugh about Florence; L: Al told Lew about Paris; S: Al told Sue about Paris; V: Al told Hugh about Venice

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that is a counterexample lurking in an open gap. (Your table should indicate the value of any compound component by writing this value under the main connective of the component.) *Do not use the rule Adj*.

6. (C ∧ D) ∧ F ⊨ F ∧ G
7. A ∧ (B ∧ D), C ∧ E ⊨ (B ∧ C) ∧ D

F09 test 1 answers

1. φ implies ψ if and only if there is no possible world in which φ is true and ψ is false (*or*: if and only if ψ is true in each possible world in which φ is true)

2. No. It does not follow that φ ≃ χ; that is, there are sentences φ and χ as described that are not equivalent. While φ and χ are each false in any possible world in which ψ is true, this still allows them to have different truth values in a possible world in which ψ is false. And, if they do have different truth values in any possible world, they are not equivalent. (For example, It's very hot and It's very cold each exclude The temperature is moderate and thus each forms a mutually exclusive pair when taken together with the latter sentence, but the first two sentences are clearly not equivalent.)

3. (i) A tautology cannot convey genuine information by way of the proposi-

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tion it expresses because there is no possible world in which it is false, so the proposition expressed by a tautology rules out no possibilities. (ii) However, if the tautology is appropriate in only certain circumstances, using the tautology will suggest that such circumstances do occur. This suggestion will be false if the circumstances do not in fact occur, so it rules out some possibilities and provides genuine information.

[For example, if someone asked the question Why did Dave do that?, the answer Dave is Dave would tend to suggest that Dave might be expected to do that sort of thing. This may be because the tautology could not do much that is relevant to answering the question apart from calling attention to features of Dave—such as his personality, character, or habits—so it would be appropriate as an answer only if such things sufficed to explain Dave's action.]

4. Al saw the meteor go by and Bill did, too; but Cal actually saw it land

   Al saw the meteor go by and Bill did, too ∧ Cal actually saw the meteor land

   (Al saw the meteor go by ∧ Bill saw the meteor go by) ∧ Cal saw the meteor land

$$(A \land B) \land C$$

   both both A and B and C

   A: Al saw the meteor go by; B: Bill saw the meteor go by; C: Cal saw the meteor land

   [Although there would be nothing really wrong with leaving the word actually in C, this term serves merely to emphasize the contrast between this sentence on the one hand and A and B on the other, so it loses its point once C is considered independently.]

5. (Al told Lew about Paris ∧ Al told Sue about Paris) ∧ (Al told Hugh about Venice ∧ Al told Hugh about Florence)

   Al told Lew and Sue about Paris ∧ Al told Hugh about Venice and Florence

   Al told Lew and Sue about Paris, and he told Hugh about Venice and Florence

   [Or you might say … but he told Hugh … if there was reason to emphasize the fact that Al told different people about different places or different numbers of places.]

6.

	(C ∧ D) ∧ F	1
1 Ext	C ∧ D	2
1 Ext	F	(4)
2 Ext	C	
2 Ext	D	
	●	
4 QED	F	3
	○	C, D, F ⊭ G
	G	3
3 Cnj	F ∧ G	

C	D	F	G	(C ∧ D) ∧ F	/	F ∧ G
T	T	T	F	⊤ T		Ⓕ

7.

	A ∧ (B ∧ D)	1
	C ∧ E	2
1 Ext	A	
1 Ext	B ∧ D	3
2 Ext	C	(7)
2 Ext	E	
3 Ext	B	(6)
3 Ext	D	(8)
	●	
6 QED	B	5
	●	
7 QED	C	5
5 Cnj	B ∧ C	4
	●	
8 QED	D	4
4 Cnj	(B ∧ C) ∧ D	

F08 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.
- *Basic concepts of deductive logic.* You will be responsible for entailment, tautologousness and absurdity, and the relations between pairs of sentences (i.e., implication, equivalence, exclusiveness, joint exhaustiveness, and contradictoriness). You should be able to define any of these ideas in terms of truth values and possible worlds (see appendix A.1 and 1.2.3 for samples of such definitions), and you should be ready to answer questions about the concepts using such definitions.
- *Implicature.* Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer by reference to its definition.
- *Analysis.* Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Synthesis.* Be able to synthesize an English sentence that has a given logical form.
- *Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV (the rules for ⊤ and ⊥) as well as Ext, Cnj, and QED; but derivations involving the latter three are much more likely.

F08 test 1 questions

1. Define the idea of two sentences being mutually exclusive by completing the following with a definition in terms of truth values and possible worlds:

   φ and ψ are mutually exclusive if and only if …

2. Suppose you are told about some sentences φ and ψ and the tautology ⊤ that φ ⊨ ⊤ and ⊤ ⊨ ψ. (i) What does this tell you about the possible truth values of φ? And (ii) what does it tell you about the possible truth values of ψ? In each case, explain your answer by reference to the definitions of a tautology and of implication (i.e., entailment).

3. Give an example of three sentences where the first implies (i.e., entails) the second, the second implicates the third (i.e., has the third as an implicature), but the first does not implicate the third. (It may be easiest to choose the second and third sentences as an example of an implicature—and it doesn't have to be a new example—and then look for a sentence that implies one without implicating the other.) Be sure to say enough about the context of your sentences for me to be able to see that what you claim about them is so.

4. Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and also English notation (i.e., using both … and). Be sure that the unanalyzed components of your answer are complete and independent sentences (and give a key to the abbreviations you use for them); also try to respect any grouping in the English.

   Although Al took the first turn, he missed the second; but he found his way to the meeting.

5. Synthesize an English sentence that has the analysis below. Choose a simple and natural sentence whose organization reflects the grouping of the logical form.

   B ∧ (C ∧ D)

   B: Al wrote to Bob; C: Al spoke to Carol; D: Al spoke to Dave

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that is a counterexample lurking in an open gap. (Your table should indicate the value of any compound component by writing this value under the main connective of the component.) *Do not use the rule Adj in the first derivation*, but you *may* use it in the second.

   6. A ∧ (B ∧ C), D ⊨ C ∧ D
   7. A ∧ B, B ∧ D ⊨ A ∧ (C ∧ D)

F08 test 1 answers

1. φ and ψ are mutually exclusive if and only if there is no possible world in which φ and ψ are both true (*or:* if and only if, in each possible world, at least one of φ and ψ is false)

2. (i) It tells you nothing about the possible truth values of φ. The fact that φ ⊨ ⊤ tells you that φ is not true in any world in which ⊤ is false; but, since ⊤ is a tautology, there are no worlds in which it is false, so you are told nothing about the truth value of φ in any possible world.

   (ii) It tells you that ψ is true in every possible world—i.e., it, too, is a tautology. The fact that ⊤ ⊨ ψ tells you that ψ is true in any world in which ⊤ is true; but, since ⊤ is a tautology, it is true in every possible world.

3. Although it isn't the only way to find an example, an easy way of doing

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so is to think about a yes-but answer in the case of a true sentence with a false implicature. Such an answer will imply the sentence (since it says it is true) but not have the implicature (because it explicitly rejects it). For example, in a context where There's a cooler in the trunk implicates There's beer in the cooler, the sentence There's a cooler in the trunk but there's no beer in it will imply the first but won't implicate the second.

4. Although Al took the first turn, he missed the second; but he found his way to the meeting

   Although Al took the first turn, he missed the second ∧ Al found his way to the meeting

   (Al took the first turn ∧ Al missed the second turn) ∧ Al found his way to the meeting

   (F ∧ S) ∧ W

   both both F and S and W

   F: Al took the first turn; S: Al missed the second turn; W: Al found his way to the meeting

5. Al wrote to Bob ∧ (Al spoke to Carol ∧ Al spoke to Dave)

   Al wrote to Bob ∧ Al spoke to Carol and Dave

   Al wrote to Bob, and [*or:* but] he Al spoke to Carol and Dave

   [While reporting the same facts, you might choose and if you were asked *whether* Al reached these people and but if you were asked *how* he reached them and were asked this in a context where the difference between writing and speech was important.]

6.

	A ∧ (B ∧ C)	1
	D	(5)
1 Ext	A	
1 Ext	B ∧ C	2
2 Ext	B	
2 Ext	C	(4)
	●	
4 QED	C	3
	●	
5 QED	D	3
3 Cnj	C ∧ D	

7.

	A ∧ B	1
	B ∧ D	2
1 Ext	A	(4)
1 Ext	B	
2 Ext	B	
2 Ext	D	(6)
	●	
4 QED	A	3
	○	A, B, D ⊭ C
	C	5
	●	
6 QED	D	5
5 Cnj	C ∧ D	3
3 Cnj	A ∧ (C ∧ D)	

A B C D	A ∧ B	B ∧ D	/	A ∧ (C ∧ D)
T T F T	⊤	⊤		Ⓕ F

F06 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Basic concepts of deductive logic*. You will be responsible for the concepts listed in appendix A.1. You should be able to define each in terms of possible worlds and truth values, and you should be prepared to answer questions about them, justifying your answer by reference to the definitions. Appendix A.1 includes relative exhaustiveness and all the concepts you've seen are special cases of it, so in principle, all are fair game. However, if I ask about something like mutual exclusiveness I'll tell you how it is expressed using relative exhaustiveness. That does mean that you need to be able to understand how relative exhaustiveness is defined in special cases where there are no assumptions on the left of the arrow or no alternatives on the right; that means you need to know what it means to say, for example, that "φ, ψ ⊨."
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer by reference to its definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Synthesis*. Be able to synthesize an English sentence that has a given logical form.
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV as well as Ext, Cnj, and QED; but derivations involving the latter three are much more likely.

F06 test 1 questions

1. Define tautologousness by completing the following with a definition in terms of truth values and possible worlds:

   φ is a tautology if and only if …

2. Explain what truth values are possible for sentences φ and ψ that are both mutually exclusive (i.e., φ, ψ ⊨) and jointly exhaustive (i.e., ⊨ φ, ψ).

The nursery rhyme "Jack and Jill" contains the line

   Jack fell down and broke his crown

Even when this is taken out of context, it is natural to suppose that Jack broke his crown as a result of falling down rather than that falling down and the injury were simply two things that happened to him. I would claim that this tie between the two events is an implicature rather than an implication of the sentence.

3. Explain what I mean when I make that claim in a way that shows you understand the definition of implicature. (You need not support or reject my claim; I'm asking you only to explain what it means.)

4. If the line did imply (rather than merely implicate) that Jack's broken crown was the result of the fall, the sentence would not be a conjunction of Jack fell down and Jack broke his crown. Explain why this is so in a way that shows you understanding the meaning of implication and the conditions under which conjunctions are true.

Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and also English notation (i.e., using both … and). Be sure that the unanalyzed components of your answer are complete and independent sentences (and give a key to the abbreviations you use for them); also try to respect any grouping in the English.

5. The building was completed on time and with no cost overruns, but not everyone was satisfied with it.

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that is a counterexample lurking in an open gap. Your table should show the value of any compound component of the premises and conclusion by writing this value under the main connective of the component. *Do not use the rule Adj in the first derivation*, but you *may* use it in the second.

6. A ∧ C, B ∧ D ⊨ B ∧ C
7. A ∧ (D ∧ E), B ∧ F ⊨ (A ∧ B) ∧ C

F06 test 1 answers

1. φ is a tautology if and only if there is no possible world in which φ is false (*or:* if and only if φ is true in every possible world)

2. Sentences that are both mutually exclusive and jointly exhaustive can only have different truth values because mutually exclusive sentences cannot be both true and jointly exhaustive sentences cannot be both false.

3. To say that Jack fell down and broke his crown implicates but does not imply that breaking his crown is the result of falling down is to say that the sentence would be inappropriate if the two events were not connected

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in this way but the sentence doesn't actually say that they were so connected.

4. The conjunction Jack fell down ∧ Jack broke his crown is true whenever both components are true whether or not one led to the other, so Jack fell down ∧ Jack broke his crown does not imply that one led to the other because it can be true even if the sentence Jack's broken crown is the result of his fall is false. So, if Jack fell down and broke his crown did imply this connection, it would not say the same thing as the conjunction.

5. The building was completed on time and with no cost overruns, but not everyone was satisfied with it

   The building was completed on time and with no cost overruns ∧ not everyone was satisfied with the building

   (the building was completed on time ∧ the building was completed with no cost overruns) ∧ not everyone was satisfied with the building

   (T ∧ C) ∧ S

   both both T and C and S

   C: the building was completed with no cost overruns; S: not everyone was satisfied with the building; T: the building was completed on time

6.

	A ∧ C	1
	B ∧ D	2
1 Ext	A	
1 Ext	C	(5)
2 Ext	B	(4)
2 Ext	D	
	●	
4 QED	B	3
	●	
5 QED	C	3
3 Cnj	B ∧ C	

7.

	A ∧ (D ∧ E)	1
	B ∧ F	3
1 Ext	A	(6)
1 Ext	D ∧ E	2
2 Ext	D	
2 Ext	E	
3 Ext	B	(7)
3 Ext	F	
	●	
6 QED	A	5
	●	
7 QED	B	5
5 Cnj	A ∧ B	4
	○	A, B, D, E, F ⊭ C
	C	4
4 Cnj	(A ∧ B) ∧ C	

A B C D E F	A ∧ (D∧E)	B ∧ F	/	(A ∧ B) ∧ C
T T F T T T	Ⓣ	T	Ⓣ	T Ⓕ

F05 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Basic concepts of deductive logic*. You will be responsible for entailment (or validity) and implication, equivalence, tautologousness, absurdity, and inconsistency. You should be able to define each in terms of possible worlds and truth values, and you should be prepared to answer questions about them, justifying your answer by reference to the definitions. (You can find the definitions in 1.4 and also in Appendix A.1.)
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer by reference to its definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the rule Adj introduced in 2.4 would be convenient to use; but it is never necessary. You should be ready to use EFQ and ENV as well as Ext, Cnj, and QED; but derivations involving the latter three are much more likely.

F05 test 1 questions

1. Define entailment by completing the following: Γ entails φ (i.e., Γ ⊨ φ) if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of the ideas of truth values and possible worlds. Remember that Γ is a set, not a sentence, so it does not have a truth value; but any members of it are sentences and have truth values.)

2. Suppose you know that (i) the set containing φ and ψ is inconsistent (i.e., φ, ψ ⊨) and (ii) the set containing ψ and χ is inconsistent (i.e., ψ, χ ⊨). What, if anything, can you conclude about the consistency or inconsistency of the set containing φ and χ? That is, what can you conclude about the truth of a claim that φ, χ ⊨? Be sure to explain your answers in terms of the definition of inconsistency.

3. Consider the following exchange:

   Al: I'm going to the restaurant Chuck told us about.
   Bob: I was there yesterday. Do you have health insurance?

   Bob could be said to convey information about the restaurant not only through his assertion but also through the question that follows it. Use the idea of *implicature* to explain how this might work. (Just what information you think might be conveyed by the question is less important than your explanation of how that information would be conveyed.)

Analyze the sentences below in as much detail as possible, presenting the result in both symbolic and English notation (i.e., using both … and). Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

4. The water was cool and clear.

5. Adam found Barb's number and called her, but she was out; nevertheless, he went to the party.

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that is a counterexample lurking in an open gap. *Do not use the rule Adj in the first derivation*, but you may use it in the second.

6. (A ∧ B) ∧ C ⊨ C ∧ A

7. F ∧ C, A ∧ (D ∧ E) ⊨ E ∧ (B ∧ C)

F05 test 1 answers

1. Γ is entails φ (i.e., Γ ⊨ φ) if and only if there is no possible world in which all members of Γ are true but φ is false. (*Or:* … if and only if, in every possible world in all members of Γ are true, φ is true as well.)
2. You know, from (i), that φ and ψ cannot be true together and, from (ii), that ψ and χ cannot be true together. But it may still be possible for φ and χ to be true together (provided ψ is false). Therefore, (i) and (ii) do not provide enough information for you to be able to conclude that the set containing φ and χ is inconsistent.
3. The specific information conveyed will depend on the circumstances; but Bob's question fits into the conversation—i.e., is appropriate—only if eating at the restaurant makes having health insurance somehow relevant. So it would be an implicature of the question that eating at the restaurant

makes whether you have health insurance in some way significant. (In most contexts, the specific suggestion would probably be that eating at the restaurant puts one's health in danger; but, in certain contexts, the suggestion might be something different—for example, that the restaurant is very expensive.)

4. The water was cool and clear
   The water was cool ∧ the water was clear

$$C \land R$$

   both C and R

   C: the water was cool; R: the water was clear

5. Adam found Barb's number and called her, but she was out; nevertheless, he went to the party
   Adam found Barb's number and called her, but she was out ∧ Adam went to the party
   (Adam found Barb's number and called her ∧ Barb was out) ∧ Adam went to the party
   ((Adam found Barb's number ∧ Adam called Barb) ∧ Barb was out) ∧ Adam went to the party

$$((F \land C) \land O) \land W$$

   both both both F and C and O and W

   C: Adam called Barb; F: Adam found Barb's number; O: Barb was out; W: Adam went to the party

6.

	(A ∧ B) ∧ C	1
1 Ext	A ∧ B	2
1 Ext	C	(4)
2 Ext	A	(5)
2 Ext	B	
	●	
4 QED	C	3
	●	
5 QED	A	3
3 Cnj	C ∧ A	

7.

	F ∧ C	1
	A ∧ (D ∧ E)	2
1 Ext	F	
1 Ext	C	(7)
2 Ext	A	
2 Ext	D ∧ E	3
3 Ext	D	
3 Ext	E	(5)
	●	
5 QED	E	4
	○	A, C, D, E, F ⊭ B
	B	6
	●	
7 QED	C	6
6 Cnj	B ∧ C	4
4 Cnj	E ∧ (B ∧ C)	

A B C D E F	F ∧ C	A ∧ (D ∧ E)	/	E ∧ (B ∧ C)
T F T T T T	Ⓣ	Ⓣ T		Ⓕ F

F04 test 1 topics

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.
- *Basic concepts of deductive logic*. You will be responsible for entailment (or validity) and implication, equivalence, tautologousness, absurdity, and inconsistency. You should be able to define each in terms of possible worlds and truth values, and you should be prepared to answer questions about them, justifying your answer by reference to the definitions.
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer by reference to its definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the logical-and symbol ∧ and with the both … and … way of expressing forms).
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the new rule Adj would be convenient to use but, of course, it is never necessary. You should be ready to use EFQ and ENV as well as Ext, Cnj, and QED; but derivations involving the latter three are much more likely.

F04 test 1 questions

1. Define inconsistency by completing the following: Γ is inconsistent (i.e., Γ ⊨) if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define inconsistency in terms of the ideas of truth values and possible worlds. Remember that Γ is a set, not a sentence, so it does not have a truth value; but any members of it are sentences and have truth values.)

2. Define equivalence by completing the following: φ ≃ ψ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of the ideas of truth values and possible worlds.)

3. Suppose you know that (i) φ ⊨ ψ (i.e., φ entails ψ), (ii) ψ ⊨ χ (i.e., ψ entails χ), and (iii) ψ is true (in the actual world). What, if anything, can you conclude about the truth values of φ and χ (in the actual world)? Be sure to say what can be known about each of φ and χ and be sure to explain your answers in terms of the definition of entailment.

4. Suppose that φ implies ψ and also that φ implicates χ. Which of the following patterns of truth values are ruled out and which are permitted by the cited relations among the three sentences? Explain your answer using the definitions of implication and implicature.

	φ	ψ	χ
(a)	T	T	F
(b)	T	F	T
(c)	T	F	F

Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation (i.e., using both … and). Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

5. Ed tried the door, but it was locked; however, the window was open, and he climbed through it

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) that is a counterexample lurking in an open gap. *Do not use the rule Adj in the first derivation, but you may use it in the second.*

6. A ∧ C, B ∧ D ⊨ B ∧ (C ∧ D)
7. A ∧ (B ∧ C) ⊨ (A ∧ B) ∧ (C ∧ D)

F04 test 1 answers

1. Γ is inconsistent (i.e., Γ ⊨) if and only if there is no possible world in which all members of Γ are true. (Or: … if and only if, in each possible world, at least one member of Γ is false.)

2. φ ≃ ψ if and only if there is no possible world in which φ and ψ have different truth values. (Or: … if and only if, in each possible world, φ has the same truth value as ψ.)

3. You know that χ is true because you know that ψ is true and also that χ must be true in every possible world in which ψ is true (because ψ ⊨ χ). However, you know nothing about the truth value of φ because, while you know that ψ is true if φ is (because φ ⊨ ψ), it may be that ψ is true also in some cases in which φ is false.

4. Patterns (b) and (c) are ruled out but (a) is not. Since φ implies ψ, it cannot be true when ψ is false; that rules out (b) and (c) but not (a). And, while a sentence with a false implicature is inappropriate and misleading, it may be true; therefore, the fact that φ implicates χ rules out no pattern of truth values for the two.

5. Ed tried the door, but it was locked; however, the window was

open, and he climbed through it

Ed tried the door, but it was locked ∧ the window was open, and Ed climbed through it

(Ed tried the door ∧ the door was locked) ∧ (the window was open ∧ Ed climbed through the window)

(T ∧ L) ∧ (O ∧ C)

both both T and L and both O and C

C: Ed climbed through the window; L: the door was locked; O: the window was open; T: Ed tried the door

6.
1 Ext	A ∧ C	1
	B ∧ D	2
1 Ext	A	
1 Ext	C	(6)
2 Ext	B	(5)
2 Ext	D	(7)

●
| 5 QED | B | 3 |

●
| 6 QED | C | 4 |

●
7 QED	D	4
4 Cnj	C ∧ D	3
3 Cnj	B ∧ (C ∧ D)	

7.
	A ∧ (B ∧ C)	1
1 Ext	A	(5)
1 Ext	B ∧ C	2
2 Ext	B	(6)
2 Ext	C	(8)

●
| 5 QED | A | 4 |

●
| 6 QED | B | 4 |
| 4 Cnj | A ∧ B | 3 |

●
8 QED	C	7
	O	A, B, C ⊭ D
	D	7
7 Cnj	C ∧ D	3
3 Cnj	(A ∧ B) ∧ (C ∧ D)	

A	B	C	D	A ∧ (B ∧ C)	/	(A ∧ B) ∧ (C ∧ D)
T	T	T	F	Ⓣ T	T	T Ⓕ F

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

- *Basic concepts of deductive logic*. You will be responsible for entailment (or validity), equivalence, tautologousness, absurdity, and inconsistency. You should be able to define each in terms of possible worlds and truth values, and you should be prepared to answer questions about them, justifying your answer by reference to the definitions.
- *Implicature*. Be able to define it and distinguish it from implication. Be able to give examples and explain them. Be ready to answer questions about it, justifying your answer by reference to its definition.
- *Analysis*. Be able to analyze the logical form of a sentence as fully as possible using conjunction and present the form in both symbolic and English notation (that is, with the symbol ∧ and with the both … and … way of expressing forms).
- *Derivations*. Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. There may be some derivations where the new rule Adj would be convenient to use but, of course, it is never necessary (but you should be ready to use EFQ and ENV as well as Ext, Cnj, and QED).

F03 test 1 questions

1. Define entailment by completing the following: Γ ⊨ φ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds. Remember that Γ is a set, not a sentence, though its members are sentences.)

2. Define absurdity by completing the following: φ is absurd if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define absurdity in terms of truth values and possible worlds.)

3. Is it possible for there to be a pair of sentences φ and ψ where (i) φ ≃ ψ (i.e., φ and ψ are equivalent) and (ii) φ and ψ together form an inconsistent set (i.e., the set {φ, ψ} is inconsistent)? If it is possible for both (i) and (ii) to be true of a pair of sentences φ and ψ, describe (in terms of the possibilities for truth values) what φ and ψ must be like. If it is not possible, explain why in terms of possibilities for truth values. (Hint: this is not a trick question but it may trip you up if you try to answer it intuitively; you'll do better to just think through the consequences of the definitions of equivalence and inconsistency.)

4. Give an example of implicature, presenting a sentence and describing situations in which it (i) is true and not misleading, (ii) is true but misleading, and (iii) is false. Explain your answer.

5. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation (i.e., using both … and). Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

   The road was completed and opened to traffic, but it was closed for repairs and has not been re-opened

Use derivations to check whether each of the entailments below holds. If one fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which is a counterexample lurking in an open gap.

6. A ∧ B, C ∧ (D ∧ E) ⊨ B ∧ D
7. A ∧ B, D ∧ E ⊨ (A ∧ C) ∧ D

F03 test 1 answers

1. Γ ⊨ φ if and only if there is no possible world in which φ is false while all members of Γ are true

2. φ is absurd if and only if there is no possible world in which φ is true

3. If φ and ψ are equivalent they must have the same truth value as each other in every possible world. If they form an inconsistent set, there is no possible world in which both are true. Both of these are possible if there is no possible world in which either φ or ψ is true; that is, it is possible if both are absurd.

4. To re-use an example from class, if I say "My class was taught this morning" to someone, my statement (i) would be true and not misleading if someone else taught my class, (ii) would be true but misleading if I taught it, and (iii) would be false if it wasn't taught at all. (i) and (ii) differ regarding the truth of an implicature of the sentence, which arises because saying "My class was taught" instead of "I taught my class" would usually be inappropriate unless I didn't teach my class.

5. The road was completed and opened to traffic, but it was closed for repairs and has not been re-opened

   The road was completed and opened to traffic ∧ the road was closed for repairs and has not been re-opened

   (the road was completed ∧ the road was opened to traffic) ∧ (the

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road was closed for repairs ∧ the road has not been re-opened)

(C ∧ O) ∧ (R ∧ N)
both both C and O and both R and N

C: the road was completed; N: the road has not been re-opened; O: the road was opened to traffic; R: the road was closed for repairs

6.

	A ∧ B	1
	C ∧ (D ∧ E)	2
1 Ext	A	
1 Ext	B	(5)
2 Ext	C	
2 Ext	D ∧ E	3
3 Ext	D	(6)
3 Ext	E	
5 QED	● B	4
6 QED	● D	4
4 Cnj	B ∧ D	

7.

	A ∧ B	1
	D ∧ E	2
1 Ext	A	(5)
1 Ext	B	
2 Ext	D	(6)
2 Ext	E	
5 QED	● A	4
	○	A, B, D, E ⊭ C
	C	4
4 Cnj	A ∧ C	3
6 QED	● D	3
3 Cnj	(A ∧ C) ∧ D	

This derivation could be ended after the first dead-end open gap appears at stage 4.

A	B	C	D	E	A ∧ B	D ∧ E	(A ∧ C) ∧ D
T	T	F	T	T	⊤	⊤	F

A ∧ B, D ∧ E / (A ∧ C) ∧ D

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1. Define entailment for the special case of three premises by completing the following: φ, ψ, χ ⊨ θ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds.)

2. Suppose that each of φ and ψ is a tautology (i.e., ⊨ φ and also ⊨ ψ). What, if anything can you conclude about the equivalence of φ and ψ? That is, do you have enough information to conclude that they are equivalent (i.e., that φ ≃ ψ)? to conclude that they aren't equivalent?—or can't you say for sure? Explain your answer by reference to the definitions of tautologousness and equivalence, using the concepts of possible worlds and truth values that appear in those definitions.

3. Does a statement always entail everything it implicates? If you answer yes, explain why using the definitions of entailment and implicature? If you answer no, give an example of a case of implicature that isn't a case of entailment.

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

   Jack saw the book and told Jill about it, but she had already read it

Use derivations to check whether each of the entailments below holds. If one fails, confirm a counterexample—that is, provide a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values which is a counterexample lurking in an open gap.

5. (A ∧ B) ∧ (C ∧ D) ⊨ A ∧ D
6. A ∧ D ⊨ (A ∧ B) ∧ (C ∧ D)
7. [This question was on a topic not covered this year]
8. Why is a single dead-end open gap enough to show that an entailment does not hold? That is, why must all gaps close for a derivation to show that an entailment *does* hold?

F02 test 1 answers

1. φ, ψ, χ ⊨ θ if and only if there is no possible world in which φ, ψ, and χ are true but θ is false

2. φ and ψ are equivalent. Because each is a tautology, there is no possible

world in which either is false, so there is no possible world in which they have different truth values.

3. No. Any true sentence with a false implicature provides an example of an implicature that isn't an entailment because no true sentence can entail a false one, so you need only provide an example of a true sentence with a false implicature.

4. Jack saw the book and told Jill about it, but she had already read it

   Jack saw the book and told Jill about it ∧ Jill had already read the book

   (Jack saw the book ∧ Jack told Jill about the book) ∧ Jill had already read the book

   $$(S \land T) \land R$$
   both both S and T and R

   R: Jill had already read the book; S: Jack saw the book; T: Jack told Jill about the book

5.
```
        (A ∧ B) ∧ (C ∧ D)   1
1 Ext   A ∧ B               2
1 Ext   C ∧ D               3
2 Ext   A                   (5)
2 Ext   B
3 Ext   C
3 Ext   D                   (6)

          ●
5 QED   │ A                 4

          ●
6 QED   │ D                 4
4 Cnj   A ∧ D
```

6.
```
        A ∧ D               1
1 Ext   A                   (5)
1 Ext   D                   (6)

            ●
5 QED   │ │ A               3
            ○               A, D ⊭ B
            B               3
3 Cnj   │ A ∧ B             2
            ○               A, D ⊭ C
            C               4
            ●
6 QED   │ │ D               4
4 Cnj   │ C ∧ D             2
2 Cnj   (A ∧ B) ∧ (C ∧ D)
```

This derivation could be ended after the first dead-end open gap appears at stage 3, and each of the first two counterexamples below lurks in that gap. The second and third both lurk the second dead-end open gap. So, to confirm a counterexample for the derivation above, it would be enough to provide a table showing one of these three rows.

A	B	C	D	A ∧ D / (A ∧ B) ∧ (C ∧ D)
T	F	T	T	Ⓣ F Ⓕ T
T	F	F	T	Ⓣ F Ⓕ F
T	T	F	T	Ⓣ T Ⓕ F

7. [This question was on a topic not covered this year]

8. A dead-end open gap is always has a lurking counterexample that also makes the initial resources of the derivation true and its initial goal false. So any dead-end open gap shows that the derivation for which the derivation is constructed can have true premises and a false conclusion.

F00 test 1 questions

1. Define (logical) equivalence by completing the following: φ ≃ ψ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of truth values and possible worlds.)

2. Suppose you know that φ entails ψ (i.e., φ ⊨ ψ) and that φ is false. What, if anything can you conclude about the truth value of ψ? Explain your answer by reference to the definitions of entailment and equivalence, making explicit reference to the possibilities of truth and falsity mentioned in these definitions.

3. Suppose everything implicated (and thus everything implied) by a sentence φ is actually implied by a sentence ψ (so ψ explicitly says everything that φ suggests) and suppose that ψ has no further implicatures. Could ψ be true but inappropriate (and thus true but misleading)? Explain your answer using the definitions of implicature and implication.

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

   Sam got a red jellybean but wanted a green one; and Tom got a green one even though he wanted a red one
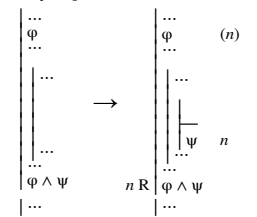
Use derivations to check whether each of the entailments below holds. If one fails, confirm a counterexample. That is, provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which is a counterexample lurking in an open gap.

5. A ∧ (B ∧ C) ⊨ (A ∧ B) ∧ D

6. (A ∧ B) ∧ C, D ∧ E ⊨ (D ∧ B) ∧ E

7. [This question was on a topic not covered this year]

8. Explain why the derivation rule R at the right would be a legitimate one. That is, explain why it would be legitimate, when φ appears among the active resources of a gap, to develop the gap by replacing a goal φ ∧ ψ with the new goal ψ. [When evaluating your answer, I'll be less concerned about your mastery of my technical terminology than in your intuitive understanding of how derivations work. Your answer need not be long; two or three sentences would be enough to give an entirely satisfactory answer.]

```
 …              …
 φ              φ          (n)
 …              …

   …              …
                  │
   …              │ ψ    n
 …            →   …
 φ ∧ ψ        n R │ φ ∧ ψ
 …              …
```

F00 test 1 answers

1. φ ≃ ψ if and only if there is no possible world in which φ and ψ have different truth values.

2. You can conclude nothing. Although the definition of entailment requires that ψ be true φ is true, it places no constraints on its value when φ is false; that is, it doesn't rule out either the possibility that φ and ψ are both false or the possibility that ψ is true even though φ isn't.

3. No. Since ψ implicates nothing beyond what φ does and implies all of φ's implicatures, anything required for ψ to be appropriate is required for it to be true.

4. Sam got a red jellybean but wanted a green one; and Tom got a green one even though he wanted a red one

   Sam got a red jellybean but wanted a green one ∧ Tom got a green jellybean even though he wanted a red one

   (Sam got a red jellybean ∧ Sam wanted a green jellybean) ∧ (Tom got a green jellybean ∧ Tom wanted a red jellybean)

   $$(R \land G) \land (D \land N)$$
   both both R and G and both D and N

   D: Tom got a green jellybean; G: Sam wanted a green jellybean; N: Tom wanted a red jellybean; R: Sam got a red jellybean

5.
```
        A ∧ (B ∧ C)         1
1 Ext   A                   (5)
1 Ext   B ∧ C               2
2 Ext   B                   (6)
2 Ext   C

            ●
5 QED   │ │ A               4

            ●
6 QED   │ │ B               4
4 Cnj   │ A ∧ B             3
            ○               A, B, C, ⊭ D
            D               3
3 Cnj   (A ∧ B) ∧ D
```

A	B	C	D	A ∧ (B ∧ C) / (A ∧ B) ∧ D
T	T	T	F	Ⓣ T T Ⓕ

6.

```
        | (A ∧ B) ∧ C   1
        | D ∧ E         2
 ───────┼──────────────────
 1 Ext  | A ∧ B         3
 1 Ext  | C
 2 Ext  | D             (6)
 2 Ext  | E             (8)
 3 Ext  | A
 3 Ext  | B             (7)
        |
        |   | ●
        |   |──
 6 QED  |   | D         5
        |
        |   | ●
        |   |──
 7 QED  |   | B         5
 5 Cnj  |   D ∧ B       4
        |
        |   ●
        |  ──
 8 QED  | E             4
 4 Cnj  | (D ∧ B) ∧ E
```

7. [This question was on a topic not covered this year]

8. A gap with φ ∧ ψ as a goal represents the question whether φ ∧ ψ is entailed by the gap's active resources. In this case, the question is whether φ ∧ ψ is entailed by a group of assumptions including φ. That will be so if and only if each of φ and ψ is entailed and φ is bound to be entailed by a group of assumptions that includes it, so the only question is whether ψ is entailed by these assumptions. And that's the question represented by gap that results when this rule is applied.

Phi 270 F99 test 1

F99 test 1 questions

1. Define tautologousness by completing the following: ⊨ φ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define tautologousness in terms of truth values and possible worlds.)

2. Suppose you know that φ ⊨ χ and that ψ ⊨ χ (i.e., χ is implied, or entailed, by each one of φ and ψ). Can you conclude that φ ≃ ψ (i.e., φ and ψ are equivalent)? Explain why or why not by reference to the definitions of entailment and equivalence, making explicit reference to the possibilities of truth and falsity mentioned in these definitions.

3. Give your own example of a true sentence with a false implicature, using the definition of implicature to explain why it is an example. [The originality of the example counts for something here but your explanation is the more important aspect of the answer.]

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Sam finished the job even though he was tired and it wasn't urgent.

Use the basic system of derivations to check whether each of the entailments below holds. If one fails, confirm a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which is a counterexample lurking in an open gap.

5. (A ∧ B) ∧ C ⊨ C ∧ A

6. A ∧ D, E ∧ A ⊨ (A ∧ B) ∧ C

7. [This question was on a topic not covered this year]

F99 test 1 answers

1. ⊨ φ if and only if there is no possible world in which φ is false.

2. No. The information about entailment tells you that χ is true in every possible world in which φ is true and also in every one where ψ is true. But there could still be possible worlds where one of φ and ψ is true while the other is false, and it is such worlds that would have to be ruled out for us to be sure that φ and ψ are equivalent.

3. [An example of the kind of thing you might say:] Al got into a house

---

would be true when said of someone who entered his own house through an unlocked door. But the sentence would ordinarily implicate that Al entered a house that wasn't his own and faced some difficulty in doing so because such factors would be the most likely reason for it to be appropriate to use a house rather than his house and got into rather than went into.

4. Sam finished the job even though he was tired and it wasn't urgent

Sam finished the job ∧ Sam was tired and the job wasn't urgent

Sam finished the job ∧ (Sam was tired ∧ the job wasn't urgent)

$$F ∧ (T ∧ N)$$

both F and both T and N

F: Sam finished the job; N: the job wasn't urgent; T: Sam was tired

5.

```
        | (A ∧ B) ∧ C   1
 ───────┼──────────────────
 1 Ext  | A ∧ B         2
 1 Ext  | C             (4)
 2 Ext  | A             (5)
 2 Ext  | B
        |
        |   ●
        |  ──
 4 QED  | C             3
        |
        |   ●
        |  ──
 5 QED  | A             3
 3 Cnj  | C ∧ A
```

6.

```
        | A ∧ D         1
        | E ∧ A         2
 ───────┼──────────────────
 1 Ext  | A             (5)
 1 Ext  | D
 2 Ext  | E
 2 Ext  | A
        |
        |   | ●
        |   |──
 5 QED  |   | A         4
        |
        |   | ○         A, D, E ⊭ B
        |   |──
        |   | B         4
 4 Cnj  | A ∧ B         3
        |
        |   | ○         A, D, E ⊭ C
        |   |──
        |   | C         3
 3 Cnj  | (A ∧ B) ∧ C
```

Only one of the following counterexamples need be confirmed. The first two lurk in the first open gap and the last two lurk in the second one.

A	B	C	D	E	A ∧ D	,	E ∧ A	/	(A ∧ B) ∧ C
T	F	T	T	T	Ⓣ		Ⓣ		F	Ⓕ
T	F	F	T	T	Ⓣ		Ⓣ		F	Ⓕ
T	T	F	T	T	Ⓣ		Ⓣ		T	Ⓕ

7. [This question was on a topic not covered this year]

F98 test 1 questions

1. Define entailment by completing the following: Γ ⊨ φ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds. Remember that Γ is a set, not a sentence, so it does not itself have a truth-value.)

2. Suppose you know that ⊨ φ (i.e., φ is a tautology) and that φ ⊨ ψ. Can you conclude that φ ≃ ψ ? Explain why or why not by considering possibilities of truth and falsity.

3. Explain how a sentence can implicate something it doesn't imply. (While you don't need to state the definitions of implication and implicature, you will need to employ the ideas used in them.)

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

   Although Carol called Dave, she didn't reach him; but Ed stopped by and helped her finish the job.

5. Use the basic system of derivations (i.e., no replacement rules) to check whether the entailment below holds. If it fails, provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which makes the premises true and conclusion false.

   A ∧ (B ∧ C), B ∧ D ⊨ (C ∧ D) ∧ E

6. [This question was on a topic not covered this year]

F98 test 1 answers

1. Γ ⊨ φ if and only if there is no possible world in which φ is false while every member of Γ is true.

2. Yes. In every possible world, φ is true (because it is a tautology) so ψ is true too (since φ ⊨ ψ), so there is no possible world in which φ and ψ have different truth values.

3. Since a true sentence can still be inappropriate for other reasons, something can be required for a sentence to be appropriate (and thus be implicated by it) without being required for its truth (and thus without being implied by it).

4. Although Carol called Dave, she didn't reach him; but Ed stopped by and helped her finish the job
   Although Carol called Dave, she didn't reach him ∧ Ed stopped by and helped Carol finish the job
   (Carol called Dave ∧ Carol didn't reach Dave) ∧ (Ed stopped by ∧ Ed helped Carol finish the job)

   (C ∧ D) ∧ (E ∧ F)
   both both C and D and both E and F

   C: Carol called Dave; D: Carol didn't reach Dave; E: Ed stopped by; F: Ed helped Carol finish the job

5.

	A ∧ (B ∧ C)	1
	B ∧ D	2
1 Ext	A	
1 Ext	B ∧ C	3
2 Ext	B	
2 Ext	D	(7)
3 Ext	B	
3 Ext	C	(6)
	●	
6 QED	C	5
	●	
7 QED	D	5
5 Cnj	C ∧ D	4
	○	A, B, C, D ⊭ E
	E	4
4 Cnj	(C ∧ D) ∧ E	

A	B	C	D	E	A ∧ (B ∧ C)	,	B ∧ D	/	(C ∧ D) ∧ E
T	T	T	T	F	Ⓣ		T		Ⓕ

6. [This question was on a topic not covered this year]

F97 test 1 questions

1. Define (a special case of) entailment by completing the following: φ, ψ ⊨ χ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds.)

2. Suppose you know that φ ≃ ψ and that χ ≃ ψ. Can you conclude that φ ≃ χ ? Explain why or why not by considering possibilities of truth and falsity.

3. Provide an example of a true sentence that has a false implicature. (Be sure to state both the sentence and its implicature and to explain why one is true and the other false and also why one implicates the other.)

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

   Ann and Bill helped to plan the campaign, but Carol directed it and reported its results

5. Use the basic system of derivations (i.e., no replacement rules) to establish the following:

   A ∧ B, C ∧ D ⊨ (A ∧ C) ∧ B

6. Use the basic system of derivations (i.e., no replacement rules) to show that the entailment below fails; provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which makes the premises true and conclusion false:

   (A ∧ B) ∧ C ⊨ A ∧ (D ∧ C)

F97 test 1 answers

1. φ, ψ ⊨ χ if and only if there is no possible world in which χ is false while φ and ψ are both true.

2. Yes. In any possible world, each of φ and χ must have the same truth value as ψ, so they must have the same truth value as each other.

3. I pushed the button but the motor shut off is true even if the button is an emergency shutoff button for the motor, but it implicates (at least) The button did not shut off the motor, which is false in this case. The implicature arises because the word *but* is not appropriate in cases where the clause it introduces describes an expected result of the truth of the other clause.

4. Ann and Bill helped to plan the campaign, but Carol directed it and reported its results
   Ann and Bill helped to plan the campaign ∧ Carol directed the campaign and reported its results
   (Ann helped to plan the campaign ∧ Bill helped to plan the campaign) ∧ (Carol directed the campaign ∧ Carol reported the campaign's results)

   (A ∧ B) ∧ (D ∧ R)
   both both A and B and both D and R

   A: Ann helped to plan the campaign; B: Bill helped to plan the campaign; D: Carol directed the campaign; R: Carol reported the campaign's results

5.

	A ∧ B	1
	C ∧ D	2
1 Ext	A	(5)
1 Ext	B	(7)
2 Ext	C	(6)
2 Ext	D	
	●	
5 QED	A	4
	●	
6 QED	C	4
4 Cnj	A ∧ C	3
	●	
7 QED	B	3
3 Cnj	(A ∧ C) ∧ B	

6.

```
          |(A ∧ B) ∧ C    1
1 Ext     |A ∧ B          2
1 Ext     |C              (6)
2 Ext     |A              (5)
2 Ext     |B
          |  ●
5 QED     |  A            3
          |  |○           A, B, C ⊭ D
          |  |D           4
          |  ●
6 QED     |  |C           4
4 Cnj     |  D ∧ C        3
3 Cnj     |A ∧ (D ∧ C)
```

A	B	C	D	(A ∧ B) ∧ C	/	A ∧ (D ∧ C)
T	T	T	F	T ⓣ		Ⓕ F

Phi 270 F96 test 1

F96 test 1 questions

1. Define equivalence by completing the following: φ ≃ ψ if and only if … . (Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of truth values and possible worlds.)

2. Suppose you know that φ, ψ ⊨ χ and that θ ⊨ ψ. Can you conclude that φ, θ ⊨ χ ? Explain why or why not by considering possibilities of truth and falsity.

3. Give an example of a case of implicature that is not a case of implication, referring to defintions of implication and implicature to explain why your example is a case of one but not the other.

Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

4. Although Mikita and Stapleton assisted him, Hull scored the goal

Use the basic system of derivations (i.e., no replacement rules) to establish the following:

5. A, C ∧ D ⊨ C ∧ (A ∧ D)

Use the basic system of derivations (i.e., no replacement rules) to show that the entailment below fails (as a generalization); give an extensional interpretation (i.e., an assignment of truth values) which makes its premises true and conclusion false:

6. (D ∧ C) ∧ B ⊨ A ∧ C

F96 test 1 answers

1. φ ≃ ψ if and only if there is no possible world in which φ and ψ differ in their truth values.

2. Yes. In any possible world in which φ and θ are true, ψ will be true (since θ ⊨ ψ), so χ will be true (since φ, ψ ⊨ χ). So there can be no world in which φ and θ are true while χ is false.

3. For example: A car is in my driveway implicates but does not imply A car other than mine is in my driveway; for, while it would in general be inappropriate to say A car is in my driveway (rather than say, for example, My car is in the driveway) if the car is mine, it would not be false when the driveway contained only my car.

4. Although Mikita and Stapleton assisted him, Hull scored the goal
Mikita and Stapleton assisted Hull ∧ Hull scored the goal

(Mikita assisted Hull ∧ Stapleton assisted Hull) ∧ Hull scored the goal

(M ∧ S) ∧ H
both both M and S and H

H: Hull scored the goal; M: Mikita assisted Hull ; S: Stapleton assisted Hull

5.

```
          |A              (5)
          |C ∧ D          1
1         |C              (3)
1         |D              (6)
          |  |
3 QED     |  C            2
          |  ●
5 QED     |  |A           4
          |  ●
6 QED     |  |D           4
4 Cnj     |  A ∧ D        2
2 Cnj     |C ∧ (A ∧ D)
```

6.

```
          |(D ∧ C) ∧ B    1
1 Ext     |D ∧ C          2
1 Ext     |B
2 Ext     |D
2 Ext     |C              (4)
          |  |○           B, C, D ⊭ A
          |  |A           3
          |  ●
4 QED     |  |C           3
3 Cnj     |A ∧ C
```

A	B	C	D	(D ∧ C) ∧ B	/	A ∧ C
F	T	T	T	T ⓣ		Ⓕ