Overview

Basic system

Exploitation and planning rules		
sentence	*as a resource*	*as a goal*
atomic sentence	*none*	IP
negation $\neg\varphi$	CR (if φ not atomic & goal is ⊥)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \rightarrow \psi$	RC (if goal is ⊥)	CP
universal $\forall x\, \theta x$	UI	UG
existential $\exists x\, \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and ¬ φ	⊥	Nc
	any	⊤	ENV
	⊥	*any*	EFQ
τ—υ	*any*	τ = υ	EC
τ—υ	¬ τ = υ	⊥	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1\ldots\tau_n$	$P\upsilon_1\ldots\upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1\ldots\tau_n$, $\neg P\upsilon_1\ldots\upsilon_n$	⊥	Nc=

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
¬ (φ ∧ ψ)	φ or ψ	MPT
φ ∨ ψ	$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$	MTP
φ → ψ	φ	MPP
	$\neg^{\pm}\psi$	MTT

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

Additional rules

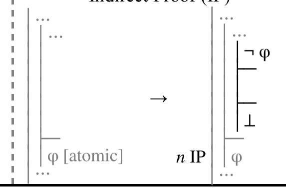
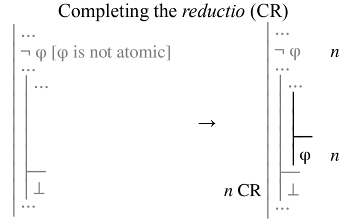
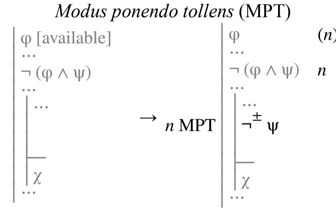
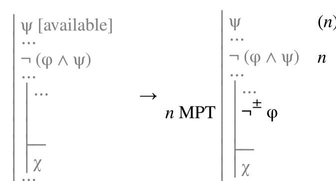
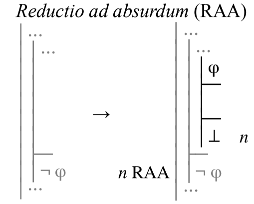
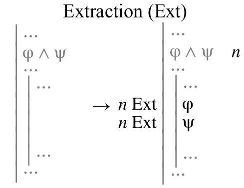
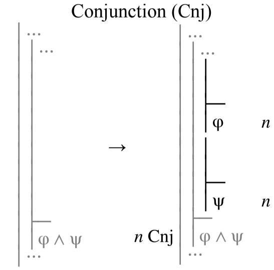
Attachment rules	
resource to be added	*rule*
φ ∧ ψ	Adj
¬ (φ ∧ ψ)	Wk
φ ∨ ψ	Wk
φ → ψ	Wk
τ = υ	CE
$\theta\upsilon_1\ldots\upsilon_n$	Cng
∃x θx	EG

Rule for lemmas	
prerequisite	*rule*
the goal is ⊥	LFR

Derivation rules

Basic system

Rules for developing gaps

logical form	*as resource*	*as goal*
atomic sentence	*no rule*	Indirect Proof (IP) 
negation ¬ φ	Completing the *reductio* (CR)  *Modus ponendo tollens* (MPT)  	*Reductio ad absurdum* (RAA) 
conjunction φ ∧ ψ	Extraction (Ext) 	Conjunction (Cnj) 

Rules for developing gaps		
logical form	*as resource*	*as goal*
disjunction $\varphi \vee \psi$	**Proof by Cases (PC)** … $\varphi \vee \psi$ … | … | χ → *n* PC … $\varphi \vee \psi$ *n* … | … || φ || χ *n* || ψ || χ *n* | χ	**Proof of Exhaustion (PE)** … | … | $\varphi \vee \psi$ … → *n* PE … | … || $\neg^{\pm}\varphi$ || ψ *n* | $\varphi \vee \psi$ … *OR* … | … | $\varphi \vee \psi$ … → *n* PE … | … || $\neg^{\pm}\psi$ || φ *n* | $\varphi \vee \psi$ …
	***Modus Tollendo Ponens* (MTP)** $\neg^{\pm}\varphi$ [available] … $\varphi \vee \psi$ … | … | χ … → *n* MTP $\neg^{\pm}\varphi$ (*n*) … $\varphi \vee \psi$ *n* … | … | ψ | χ … $\neg^{\pm}\psi$ [available] … $\varphi \vee \psi$ … | … | χ … → *n* MTP $\neg^{\pm}\psi$ (*n*) … $\varphi \vee \psi$ *n* … | … | φ | χ …	
conditional $\varphi \rightarrow \psi$	**Rejecting a Conditional (RC)** … $\varphi \rightarrow \psi$ … | … | $\bot$ … → *n* RC … $\varphi \rightarrow \psi$ *n* … | … || φ *n* || ψ || $\bot$ *n* | $\bot$ …	**Conditional Proof (CP)** … | … | $\varphi \rightarrow \psi$ … → *n* CP … | … || φ || ψ *n* | $\varphi \rightarrow \psi$ …
	***Modus Ponendo Ponens* (MPP)** φ [available] … $\varphi \rightarrow \psi$ … | … | χ … → *n* MPP φ (*n*) … $\varphi \rightarrow \psi$ *n* … | … | ψ | χ …	
	***Modus Tollendo Tollens* (MTT)** $\neg^{\pm}\psi$ [available] … $\varphi \rightarrow \psi$ … | … | χ … → *n* MTT $\neg^{\pm}\psi$ (*n*) … $\varphi \rightarrow \psi$ *n* … | … | $\neg^{\pm}\varphi$ | χ …	

Attachment rules		
what is required	*added resource*	*rule*
τ and υ are co-aliases	$\tau = \upsilon$	**Co-alias Equation (CE)** … [τ and υ are co-aliases] … | … | φ … → *n* CE … [τ and υ are co-aliases] … | … | $\tau = \upsilon$ X | φ …
have co-alias relations $\tau_1—\upsilon_1, \cdots, \tau_n—\upsilon_n$ and $\theta\tau_1\cdots\tau_n$ is available	$\theta\upsilon_1\cdots\upsilon_n$	**Congruence (Cng)** … [have co-alias relations: $\tau_1—\upsilon_1, \cdots, \tau_n—\upsilon_n$] … $\theta\tau_1\cdots\tau_n$ … | … | φ … → *n* Cng … [have co-alias relations: $\tau_1—\upsilon_1, \cdots, \tau_n—\upsilon_n$] … $\theta\tau_1\cdots\tau_n$ (*n*) … | … | $\theta\upsilon_1\cdots\upsilon_n$ X | φ …
$\theta\tau$ is available	$\exists x\ \theta x$	**Existential Generalization (EG)** … $\theta\tau$ … | … | φ … → *n* EG … $\theta\tau$ (*n*) … | … | $\exists x\ \theta x$ X | φ …

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	**Lemma for *Reductio* (LFR)** … | … | $\bot$ … → *n* LFR … | … || φ *n* || φ || $\bot$ *n* | $\bot$ …

Additional rules (not guaranteed to be progressive)

Attachment rules		
what is required	*added resource*	*rule*
φ and ψ are both available	$\varphi \wedge \psi$	Adjunction (Adj) … φ [available] … ψ [available] … | … | | χ … → n Adj … φ (n) … ψ (n) … | … | $\varphi \wedge \psi$ X | χ …
$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$ is available	$\neg(\varphi \wedge \psi)$	Weakening (Wk) … $\neg^{\pm}\varphi$ [available] … | … | χ … → n Wk … $\neg^{\pm}\varphi$ (n) … | … | $\neg(\varphi \wedge \psi)$ X | χ … … $\neg^{\pm}\psi$ [available] … | … | χ … → n Wk … $\neg^{\pm}\psi$ (n) … | … | $\neg(\varphi \wedge \psi)$ X | χ …
φ or ψ is available	$\varphi \vee \psi$	… φ [available] … | … | χ … → n Wk … φ (n) … | … | $\varphi \vee \psi$ X | χ … … ψ [available] … | … | χ … → n Wk … ψ (n) … | … | $\varphi \vee \psi$ X | χ …
$\neg^{\pm}\varphi$ or ψ is available	$\varphi \to \psi$	$\neg^{\pm}\varphi$ [available] … | … | χ … → n Wk $\neg^{\pm}\varphi$ (n) … | … | $\varphi \to \psi$ X | χ … ψ [available] … | … | χ … → n Wk ψ (n) … | … | $\varphi \to \psi$ X | χ …

Rules for developing gaps		
logical form	*as resource*	*as goal*
universal $\forall x\, \theta x$	Universal Instantiation (UI) … $\forall x\, \theta x$ … | … | φ … → n UI … $\forall x\, \theta x$ $\tau{:}n$ … | … | $\theta\tau$ | φ …	Universal Generalization (UG) … | … | $\forall x\, \theta x$ … → … | … | ⓐ | θa n n UG | $\forall x\, \theta x$ …
existential $\exists x\, \theta x$	Proof by Choice (PCh) … $\exists x\, \theta x$ … | … | φ … → … $\exists x\, \theta x$ n … | … | ⓐ | θa | | φ n n PCh | φ …	Non-constructive Proof (NcP) … | … | $\exists x\, \theta x$ … → … | … | | $\forall x\, \neg^{\pm}\theta x$ | | $\bot$ n n NcP | $\exists x\, \theta x$ …

The parameter a used in UG and PCh should be new to the derivation; that is, it should appear only to the right of the scope line it labels

Rules for closing gaps (truth-functional logic)		
when to close		*rule*
resources	*goal*	*Quod Erat Demonstrandum* (QED)
φ	φ	… φ [available] … │ … │ φ … → *n* QED … φ (*n*) … │ ● │ φ …
		Non-contradiction (Nc)
φ and ¬ φ	⊥	… ¬ φ [available] … φ [available] … │ … │ ⊥ … → *n* Nc … ¬ φ (*n*) … φ (*n*) … │ ● │ ⊥ …
		Ex Nihilo Verum (ENV)
any	⊤	… │ … │ ⊤ … → *n* ENV … │ ● │ ⊤ …
		Ex Falso Quodlibet (EFQ)
⊥	*any*	… ⊥ … │ … │ φ … → *n* EFQ … ⊥ (*n*) … │ ● │ φ …

Rules for closing gaps (equations)

In addition to the following rules for closing gaps, if the conditions for applying any rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding “=” to its label; QED= and Nc= below are examples of this in the case of rules for closing gaps.

when to close			*rule*
co-aliases	*resources*	*goal*	Equated Co-aliases (EC)
τ—υ	*any*	τ = υ	… [τ and υ are co-aliases] … │ … │ τ = υ … → *n* EC … [τ and υ are co-aliases] … │ ● │ τ = υ …
			Distinguished Co-aliases (DC)
τ—υ	¬ τ = υ	⊥	… [τ and υ are co-aliases] … ¬ τ = υ … │ … │ ⊥ … → *n* DC … [τ and υ are co-aliases] … ¬ τ = υ (*n*) … │ ● │ ⊥ …
			QED given equations (QED=)
τ_1—υ_1, ⋯, τ_n—υ_n	$P\tau_1\cdots\tau_n$	$P\upsilon_1\cdots\upsilon_n$	… [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1\cdots\tau_n$ … │ … │ $P\upsilon_1\cdots\upsilon_n$ … → *n* QED= … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1\cdots\tau_n$ (*n*) … │ ● │ $P\upsilon_1\cdots\upsilon_n$ …
			Non-contradiction given equations (Nc=)
τ_1—υ_1, ⋯, τ_n—υ_n	$P\tau_1\cdots\tau_n$ $\neg P\upsilon_1\cdots\upsilon_n$	⊥	… [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1\cdots\tau_n$ … $\neg P\upsilon_1\cdots\upsilon_n$ … │ … │ ⊥ … → *n* Nc= … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1\cdots\tau_n$ (*n*) … $\neg P\upsilon_1\cdots\upsilon_n$ (*n*) … │ ● │ ⊥ …