Overview

Basic system

Exploitation and planning rules		
sentence	*as a resource*	*as a goal*
atomic sentence	*none*	IP
negation $\neg \varphi$	CR (if φ not atomic & goal is $\bot$)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \rightarrow \psi$	RC (if goal is $\bot$)	CP
universal $\forall x\, \theta x$	UI	UG
existential $\exists x\, \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and $\neg \varphi$	$\bot$	Nc
	any	$\top$	ENV
	$\bot$	*any*	EFQ
τ—υ	*any*	$\tau = \upsilon$	EC
τ—υ	$\neg\, \tau = \upsilon$	$\bot$	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$	$P\upsilon_1 \ldots \upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$ $\neg P\upsilon_1 \ldots \upsilon_n$	$\bot$	Nc=

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
$\neg (\varphi \wedge \psi)$	φ or ψ	MPT
$\varphi \vee \psi$	$\neg^{\pm} \varphi$ or $\neg^{\pm} \psi$	MTP
$\varphi \rightarrow \psi$	φ $\neg^{\pm} \psi$	MPP MTT

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

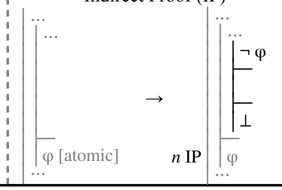
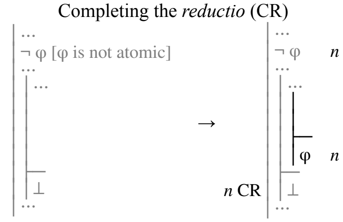
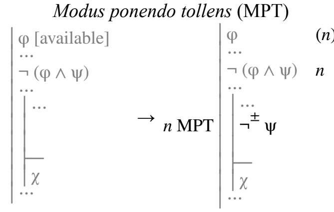
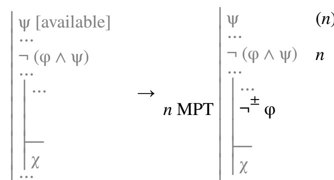
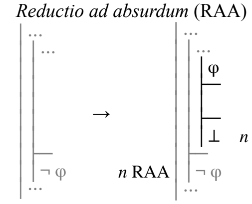
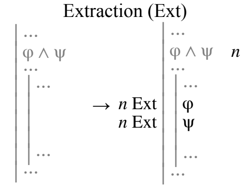
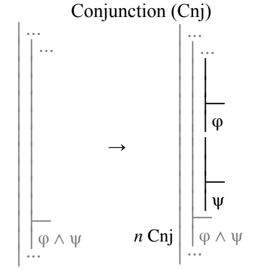
Additional rules

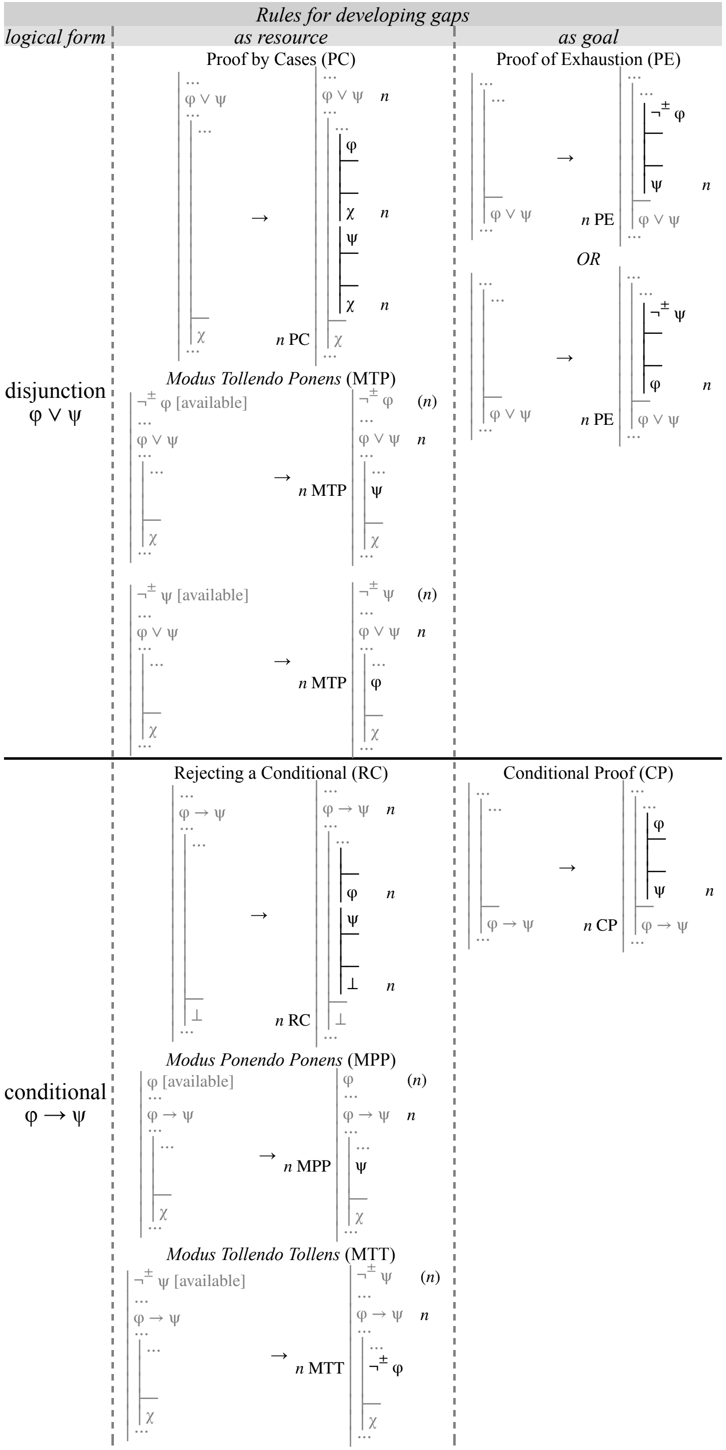
Attachment rules	
resource to be added	*rule*
$\varphi \wedge \psi$	Adj
$\neg (\varphi \wedge \psi)$	Wk
$\varphi \vee \psi$	Wk
$\varphi \rightarrow \psi$	Wk
$\tau = \upsilon$	CE
$\theta \upsilon_1 \ldots \upsilon_n$	Cng
$\exists x\, \theta x$	EG

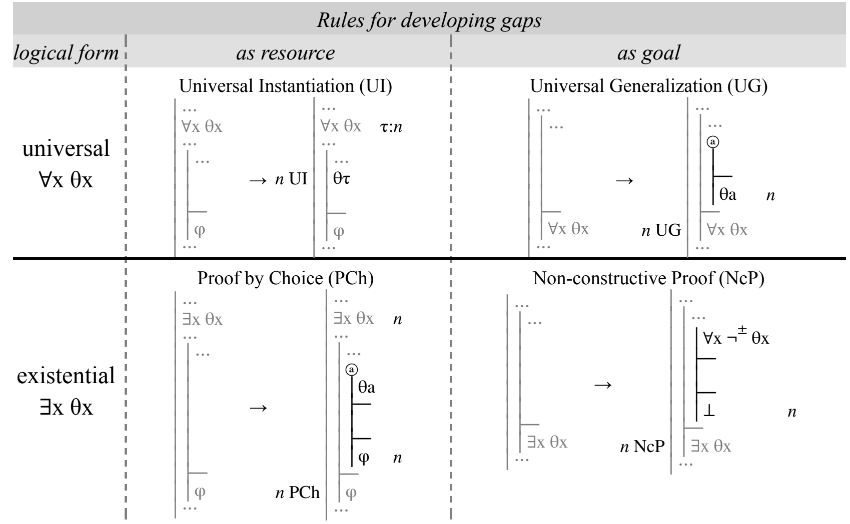
Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	LFR

Derivation rules

Basic system

Rules for developing gaps		
logical form	*as resource*	*as goal*
atomic sentence	*no rule*	Indirect Proof (IP) 
negation $\neg \varphi$	Completing the *reductio* (CR)  *Modus ponendo tollens* (MPT)  	*Reductio ad absurdum* (RAA) 
conjunction $\varphi \wedge \psi$	Extraction (Ext) 	Conjunction (Cnj) 



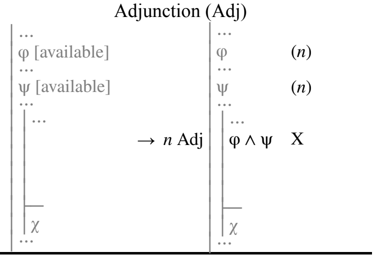
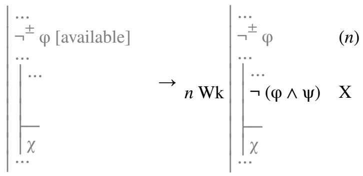
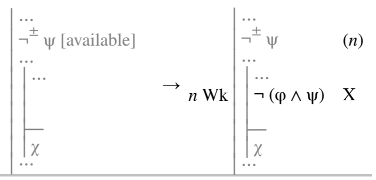
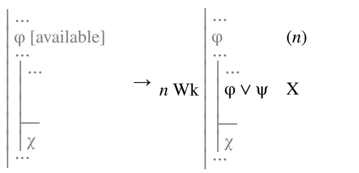
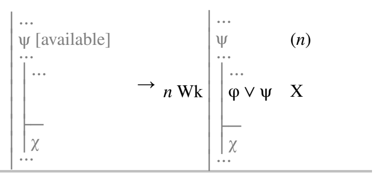
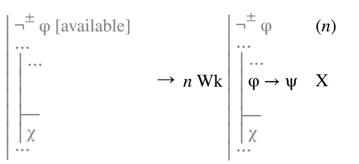
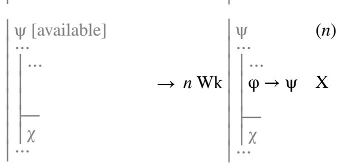


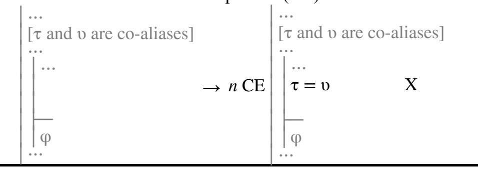
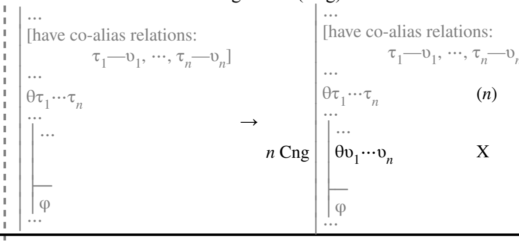
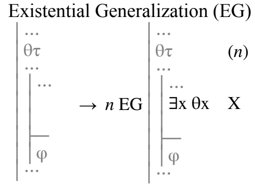
The parameter a used in UG and PCh should be new to the derivation; that is, it should appear only to the right of the scope line it labels

Rules for closing gaps (truth-functional logic)		
when to close		*rule*
resources	*goal*	*Quod Erat Demonstrandum* (QED) … φ [available] … … φ … → … φ (n) … ● φ … n QED
φ	φ	
		Non-contradiction (Nc) … ¬ φ [available] … φ [available] … … ⊥ … → … ¬ φ (n) … φ (n) … ● ⊥ … n Nc
φ and ¬ φ	⊥	
		Ex Nihilo Verum (ENV) … … ⊤ … → … ● ⊤ … n ENV
any	⊤	
		Ex Falso Quodlibet (EFQ) … ⊥ … … φ … → … ⊥ (n) … ● φ … n EFQ
⊥	*any*	

Rules for closing gaps (equations)			
In addition to the following rules for closing gaps, if the conditions for applying any rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding “=” to its label; QED= and Nc= below are examples of this in the case of rules for closing gaps.			
when to close			*rule*
co-aliases	*resources*	*goal*	Equated Co-aliases (EC) … [τ and υ are co-aliases] … … τ = υ … → … [τ and υ are co-aliases] … ● τ = υ … n EC
τ—υ	*any*	τ = υ	
			Distinguished Co-aliases (DC) … [τ and υ are co-aliases] … ¬ τ = υ … … ⊥ … → … [τ and υ are co-aliases] … ¬ τ = υ (n) … ● ⊥ … n DC
τ—υ	¬ τ = υ	⊥	
			QED given equations (QED=) … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ … … $P\upsilon_1 \cdots \upsilon_n$ … → … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ (n) … ● $P\upsilon_1 \cdots \upsilon_n$ … n QED=
τ_1—υ_1, ⋯, τ_n—υ_n	$P\tau_1 \cdots \tau_n$	$P\upsilon_1 \cdots \upsilon_n$	
			Non-contradiction given equations (Nc=) … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ … ¬ $P\upsilon_1 \cdots \upsilon_n$ … … ⊥ … → … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ (n) … ¬ $P\upsilon_1 \cdots \upsilon_n$ (n) … ● ⊥ … n Nc=
τ_1—υ_1, ⋯, τ_n—υ_n	$P\tau_1 \cdots \tau_n$ ¬ $P\upsilon_1 \cdots \upsilon_n$	⊥	

Additional rules (not guaranteed to be progressive)

Attachment rules		
what is required	*added resource*	*rule*
φ and ψ are both available	φ ∧ ψ	Adjunction (Adj) 
$\neg^{\pm}$ φ or $\neg^{\pm}$ ψ is available	¬ (φ ∧ ψ)	Weakening (Wk)  
φ or ψ is available	φ ∨ ψ	 
$\neg^{\pm}$ φ or ψ is available	φ → ψ	 

Attachment rules		
what is required	*added resource*	*rule*
τ and υ are co-aliases	τ = υ	Co-alias Equation (CE) 
have co-alias relations τ_1—υ_1, ⋯, τ_n—υ_n and $\theta\tau_1\cdots\tau_n$ is available	$\theta\upsilon_1\cdots\upsilon_n$	Congruence (Cng) 
θτ is available	∃x θx	Existential Generalization (EG) 

Rule for lemmas	
prerequisite	*rule*
the goal is ⊥	Lemma for *Reductio* (LFR)