Appendices

Appendix A. Reference

A.0. Overview

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A.1. Basic concepts

Concept	*Negative definition*	*Positive definition*
φ is *entailed* by Γ: $\Gamma \vDash \varphi$	There is no logically possible world in which φ is false while all members of Γ are true.	φ is true in every logically possible world in which all members of Γ are true.
φ and ψ are *(logically) equivalent* : $\varphi \simeq \psi$	There is no logically possible world in which φ and ψ have different truth values.	φ and ψ have the same truth value as each other in every logically possible world.
φ is a *tautology*: $\vDash \varphi$ (*or* $\top \vDash \varphi$)	There is no logically possible world in which φ is false.	φ is true in every logically possible world.
φ is *inconsistent with* Γ: $\Gamma, \varphi \vDash$ (*or* $\Gamma, \varphi \vDash \bot$)	There is no logically possible world in which φ is true while all members of Γ are true.	φ is false in every logically possible world in which all members of Γ are true.
Γ is *inconsistent*: $\Gamma \vDash$ (*or* $\Gamma \vDash \bot$)	There is no logically possible world in which all members of Γ are true.	In every logically possible world, at least one member of Γ is false.
φ is *absurd*: $\varphi \vDash$ (*or* $\varphi \vDash \bot$)	There is no logically possible world in which φ is true.	φ is false in every logically possible world.
Σ is *rendered exhaustive* by Γ: $\Gamma \vDash \Sigma$	There is no logically possible world in which all members of Σ are false while all members of Γ are true.	At least one member of Σ is true in each logically possible world in which all members of Γ are true

A.2. Logical forms

Forms for which there is symbolic notation

	Symbolic notation	English notation	*or* English reading
Negation	¬ φ	not φ	
Conjunction	φ ∧ ψ	both φ and ψ	(φ and ψ)
Disjunction	φ ∨ ψ	either φ or ψ	(φ or ψ)
The conditional	φ → ψ ψ ← φ	if φ then ψ yes ψ if φ	(φ implies ψ) (ψ if φ)
Identity	τ = υ	τ is υ	
Predication	$\theta\tau_1\ldots\tau_n$	θ fits $\tau_1, \ldots, \tau_n$	A series of terms $\tau_1, \ldots, \tau_n$ can be read (series) $\tau_1, \ldots,$ ən τ_n (using the expression ən to distinguish this use of and from its use in conjunction and adding series when necessary to avoid ambiguity)
Compound term	$\gamma\tau_1\ldots\tau_n$	γ of $\tau_1, \ldots, \tau_n$ γ applied to $\tau_1, \ldots, \tau_n$	
Predicate abstract	$[\varphi]_{x_1\ldots x_n}$	what φ says of $x_1\ldots x_n$	
Functor abstract	$[\tau]_{x_1\ldots x_n}$	τ for $x_1\ldots x_n$	
Universal quantification	∀x θx	forall x θx everything, x, is such that θx	
Restricted universal	(∀x: ρx) θx	forall x st ρx θx everything, x, such that ρx is such that θx	
Existential quantification	∃x θx	forsome x θx something, x, is such that θx	
Restricted existential	(∃x: ρx) θx	forsome x st ρx θx something, x, such that ρx is such that θx	
Definite description	℩x ρx	the x st ρx the thing, x, such that ρx	

Some paraphrases of other forms

	Truth-functional compounds
neither φ nor ψ	$\neg(\varphi \lor \psi)$ $\neg\varphi \land \neg\psi$
ψ only if φ	$\neg\psi \leftarrow \neg\varphi$
ψ unless φ	$\psi \leftarrow \neg\varphi$
	Generalizations
All Cs are such that (… they …)	$(\forall x: x \text{ is a } C) \ldots x \ldots$
No Cs are such that (… they …)	$(\forall x: x \text{ is a } C)\, \neg \ldots x \ldots$
Only Cs are such that (… they …)	$(\forall x: \neg\, x \text{ is a } C)\, \neg \ldots x \ldots$
with: among Bs	add to the restriction: $x \text{ is a } B$
except Es	$\neg\, x \text{ is an } E$
other than τ	$\neg\, x = \tau$
	Numerical quantifier phrases
At least 1 C is such that (… it …)	$(\exists x: x \text{ is a } C) \ldots x \ldots$
At least 2 Cs are such that (… they …)	$(\exists x: x \text{ is a } C)\, (\exists y: y \text{ is a } C \land \neg\, y = x)\, (\ldots x \ldots \land \ldots y \ldots)$
Exactly 1 C is such that (… it …)	$(\exists x: x \text{ is a } C)\, (\ldots x \ldots \land (\forall y: y \text{ is a } C \land \neg\, y = x)\, \neg \ldots y \ldots)$ *or* $(\exists x: x \text{ is a } C)\, (\ldots x \ldots \land (\forall y: y \text{ is a } C \land \ldots y \ldots)\, x = y)$
	Definite descriptions (on Russell's analysis)
The C is such that (… it …)	$(\exists x: x \text{ is a } C \land (\forall y: \neg\, y = x)\, \neg\, y \text{ is a } C) \ldots x \ldots$ *or* $(\exists x: x \text{ is a } C \land (\forall y: y \text{ is a } C)\, x = y) \ldots x \ldots$

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A.3. Truth tables

Tautology

$\top$
T

Absurdity

$\bot$
F

Negation

φ	$\neg\ \varphi$
T	F
F	T

Conjunction

φ	ψ	$\varphi \land \psi$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

φ	ψ	$\varphi \lor \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional

φ	ψ	$\varphi \to \psi$
T	T	T
T	F	F
F	T	T
F	F	T

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A.4. Derivation rules

Basic system

Rules for developing gaps		
logical form	*as a resource*	*as a goal*
atomic sentence		IP
negation $\neg\varphi$	CR (if φ not atomic & goal is $\bot$)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \rightarrow \psi$	RC (if goal is $\bot$)	CP
universal $\forall x\, \theta x$	UI	UG
existential $\exists x\, \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and $\neg\varphi$	$\bot$	Nc
		$\top$	ENV
	$\bot$		EFQ
τ—υ		$\tau = \upsilon$	EC
τ—υ	$\neg\, \tau = \upsilon$	$\bot$	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$	$P\upsilon_1 \ldots \upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$ $\neg P\upsilon_1 \ldots \upsilon_n$	$\bot$	Nc=

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
$\varphi \rightarrow \psi$	φ	MPP
	$\neg^{\pm}\psi$	MTT
$\varphi \vee \psi$	$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$	MTP
$\neg(\varphi \wedge \psi)$	φ or ψ	MPT

Additional rules (not guaranteed to be progressive)

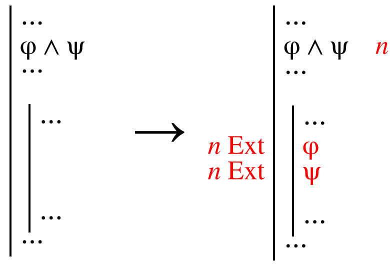
Attachment rules	
added resource	*rule*
$\varphi \wedge \psi$	Adj
$\varphi \rightarrow \psi$	Wk
$\varphi \vee \psi$	Wk
$\neg(\varphi \wedge \psi)$	Wk
$\tau = \upsilon$	CE
$\theta\upsilon_1 \ldots \upsilon_n$	Cng
$\exists x\, \theta x$	EG

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	LFR

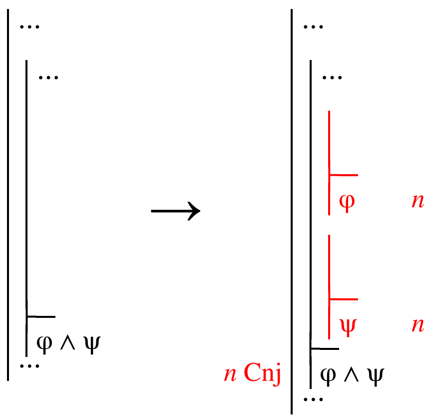
Diagrams

Rules from chapter 2

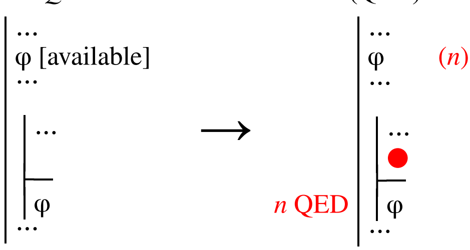
Extraction (Ext)



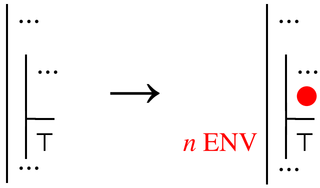
Conjunction (Cnj)



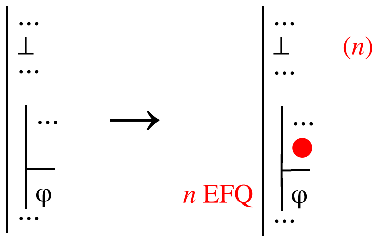
Quod Erat Demonstrandum (QED)



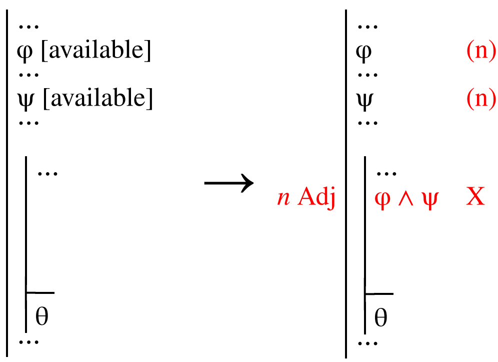
Ex Nihilo Verum (ENV)



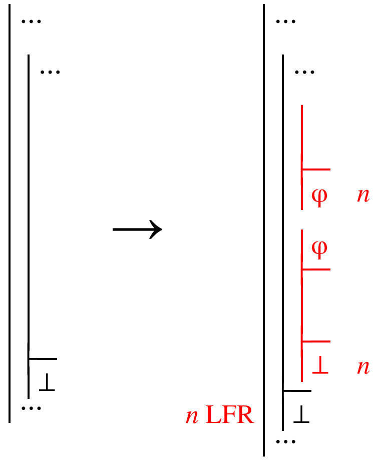
Ex Falso Quodlibet (EFQ)



Adjunction (Adj)

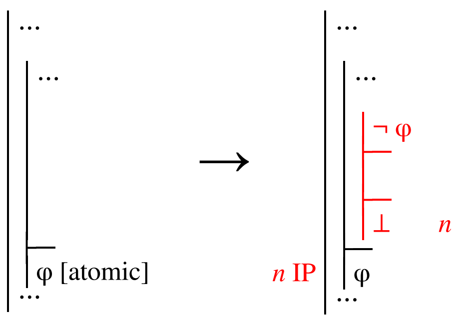


Lemma for *Reductio* (LFR)

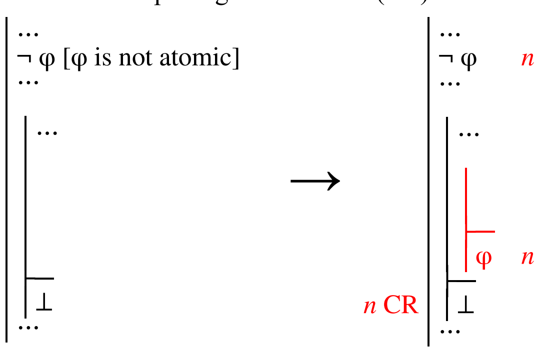


Rules from chapter 3

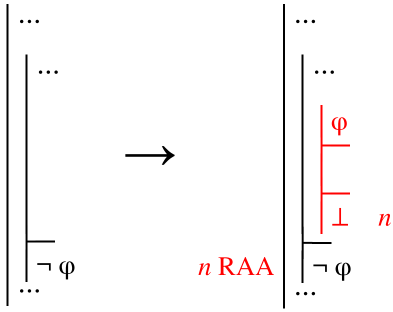
Indirect Proof (IP)



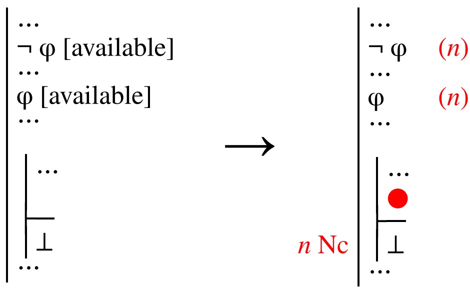
Completing the *Reductio* (CR)



Reductio ad Absurdum (RAA)

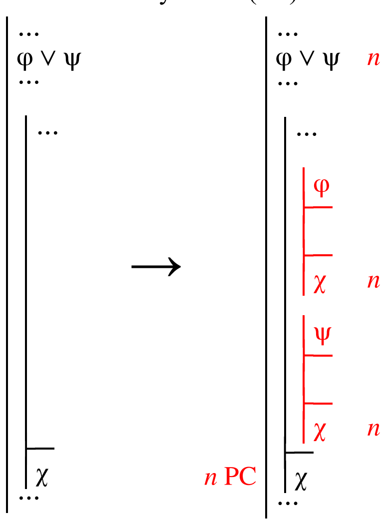


Non-contradiction (Nc)

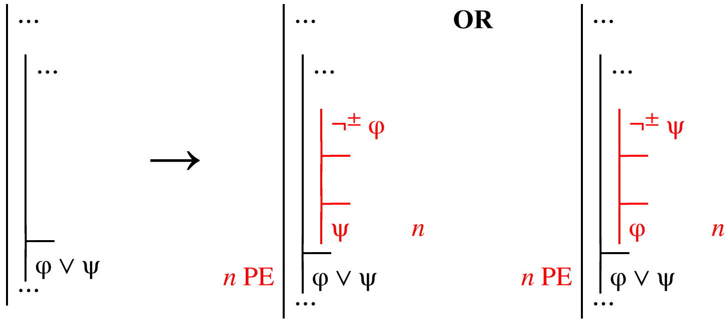


Rules from chapter 4

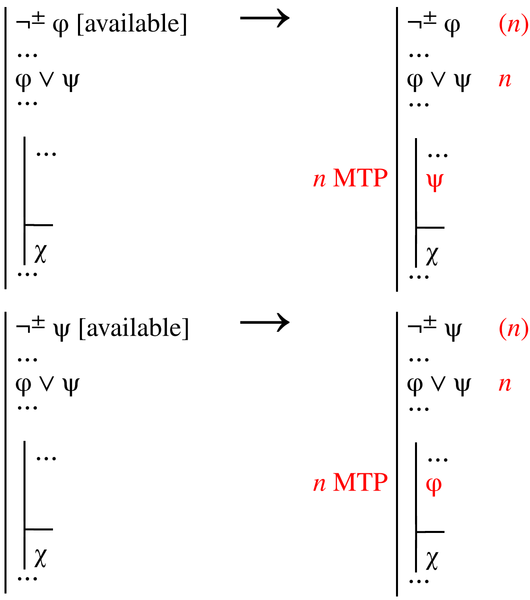
Proof by Cases (PC)



Proof of Exhaustion (PE)



Modus Tollendo Ponens (MTP)



Modus Ponendo Tollens (MPT)

```
| φ [available]      ⟶              | φ            (n)
| …                                  | …
| ¬ (φ ∧ ψ)                          | ¬ (φ ∧ ψ)    n
| …                                  | …
|  | …                               |  | …
|  |                         n MPT   |  | ¬± ψ
|  |___                              |  |___
|  | θ                               |  | θ
| …                                  | …
```

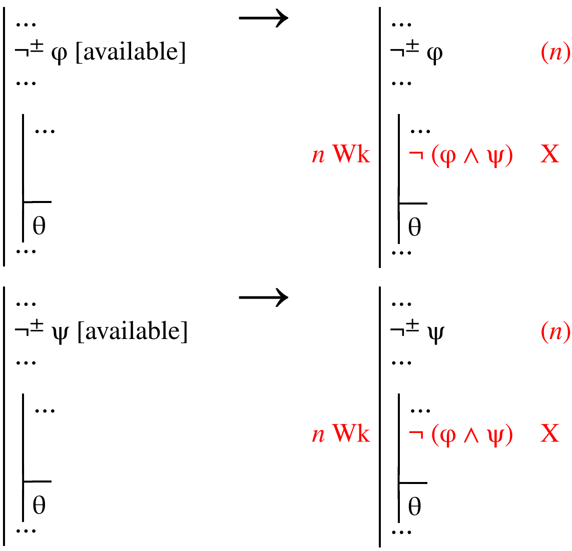
```
| ψ [available]      ⟶              | ψ            (n)
| …                                  | …
| ¬ (φ ∧ ψ)                          | ¬ (φ ∧ ψ)    n
| …                                  | …
|  | …                               |  | …
|  |                         n MPT   |  | ¬± φ
|  |___                              |  |___
|  | θ                               |  | θ
| …                                  | …
```

Weakening (Wk)

```
| …                  ⟶              | …
| φ [available]                      | φ            (n)
| …                                  | …
|  | …                               |  | …
|  |                         n Wk    |  | φ ∨ ψ     X
|  |___                              |  |___
|  | θ                               |  | θ
| …                                  | …
```

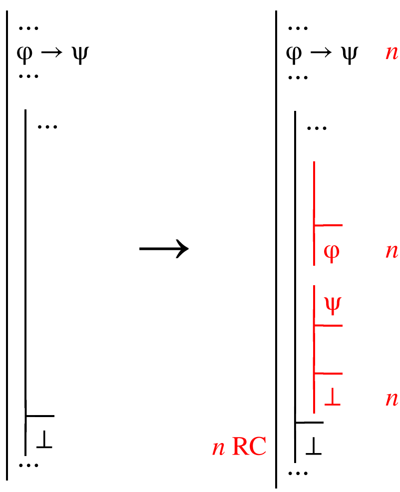
```
| …                  ⟶              | …
| ψ [available]                      | ψ            (n)
| …                                  | …
|  | …                               |  | …
|  |                         n Wk    |  | φ ∨ ψ     X
|  |___                              |  |___
|  | θ                               |  | θ
| …                                  | …
```

Weakening (Wk)

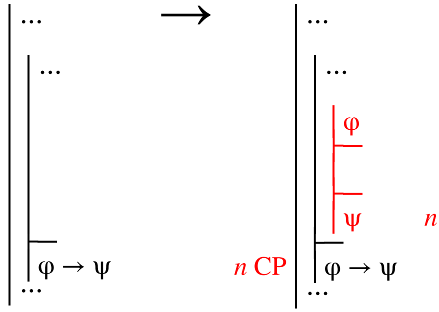


Rules from chapter 5

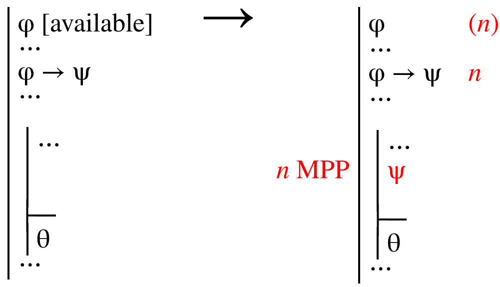
Rejecting a Conditional (RC)



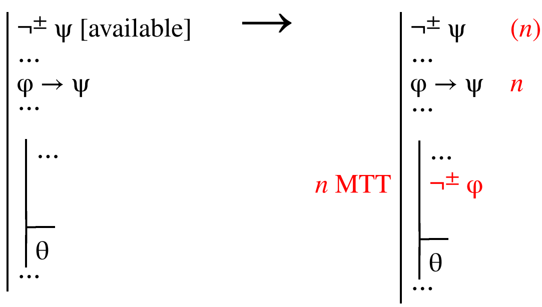
Conditional Proof (CP)



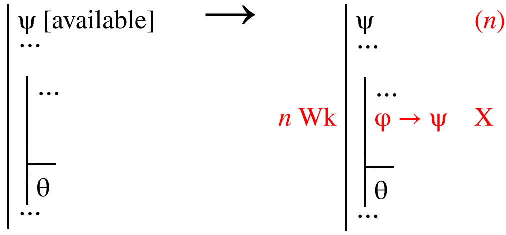
Modus Ponendo Ponens (MPP)



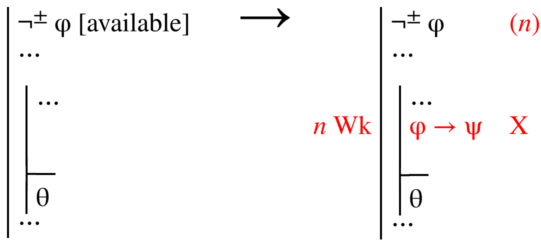
Modus Tollendo Tollens (MTT)



Weakening (Wk)

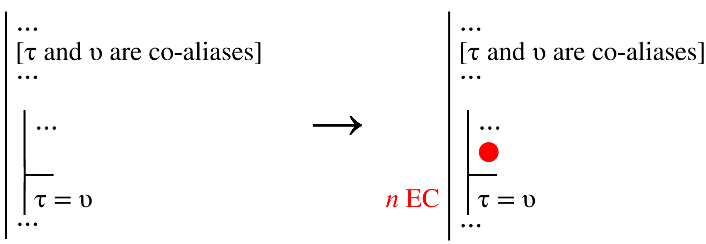


Weakening (Wk)

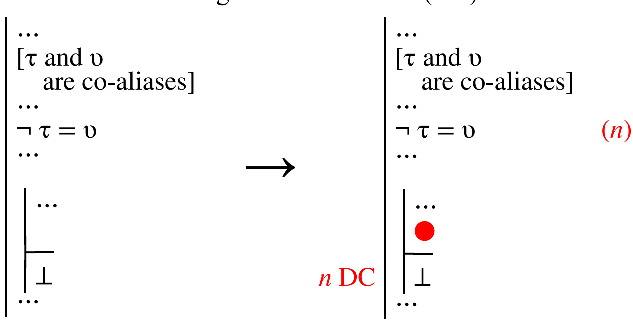


Rules from chapter 6

Equated Co-aliases (EC)



Distinguished Co-aliases (DC)



QED given equations (QED=)

…		…
[$\tau_1 \ldots \tau_n$ and $\upsilon_1 \ldots \upsilon_n$ are co-alias series]		[$\tau_1 \ldots \tau_n$ and $\upsilon_1 \ldots \upsilon_n$ are co-alias series]
…		…
$P\tau_1 \ldots \tau_n$	$\longrightarrow$	$P\tau_1 \ldots \tau_n$ (n)
…		…
	…			…
				●
	$P\upsilon_1 \ldots \upsilon_n$	n QED=		$P\upsilon_1 \ldots \upsilon_n$
…		…

Note: Two series of terms are co-alias series when their corresponding members are co-aliases.

Non-contradiction given equations (Nc=)

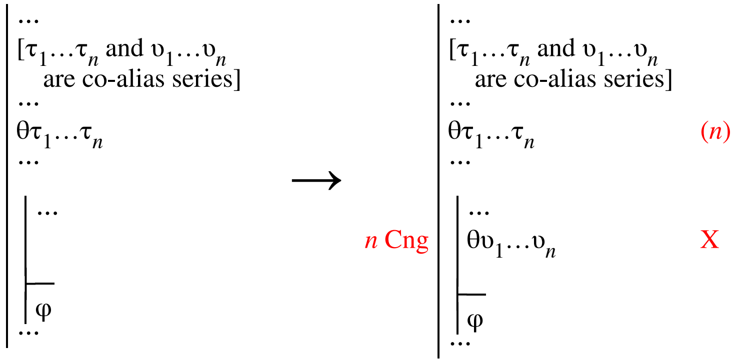
…		…
[$\tau_1 \ldots \tau_n$ and $\upsilon_1 \ldots \upsilon_n$ are co-alias series]		[$\tau_1 \ldots \tau_n$ and $\upsilon_1 \ldots \upsilon_n$ are co-alias series]
…		…
$\neg\, P\tau_1 \ldots \tau_n$		$\neg\, P\tau_1 \ldots \tau_n$ (n)
…		…
$P\upsilon_1 \ldots \upsilon_n$	$\longrightarrow$	$P\upsilon_1 \ldots \upsilon_n$ (n)
…		…
	…			…
				●
	$\bot$	n Nc=		$\bot$
…		…

Note: Two series of terms are co-alias series when their corresponding members are co-aliases.

Co-alias Equation (CE)

…		…
[τ and υ are co-aliases]		[τ and υ are co-aliases]
…		…
	…	$\longrightarrow$		…
		n CE		$\tau = \upsilon$ X
	φ			φ
…		…

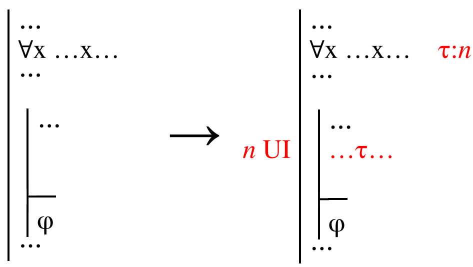
Congruence (Cng)



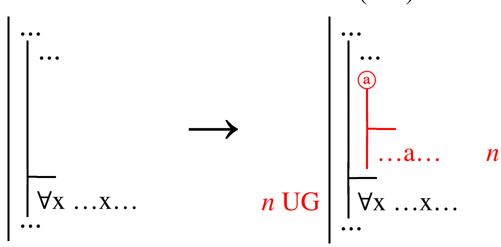
Note: θ can be an abstract, so $\theta\tau_1 \ldots \tau_n$ and $\theta\upsilon_1 \ldots \upsilon_n$ are any formulas that differ only in the occurrence of terms and in which the corresponding terms are co-aliases.

Rules from chapter 7

Universal Instantiation (UI)

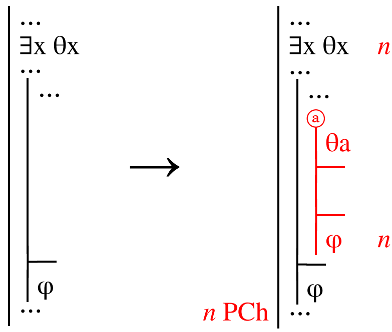


Universal Generalization (UG)

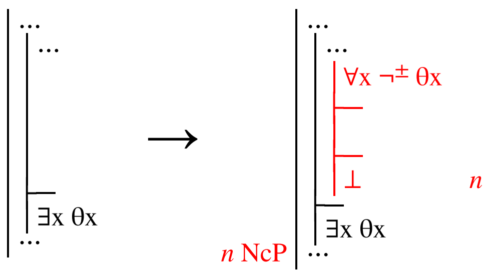


Rules from chapter 8

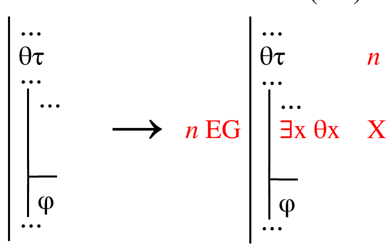
Proof by Choice (PCh)



Non-constructive Proof (NcP)



Existential Generalization (EG)



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