7. Generalizations

7.1. Generalizations in English

7.1.0. Overview

Although quantifier phrases are grammatically similar to individual terms, their logical function is quite different; indeed, they are to predicates what predicates are to individual terms.

7.1.1. Theories of quantifier phrases
  In the history of logic, it took a couple millennia for the contemporary understanding of quantifier phrases to appear.

7.1.2. Pronouns and quantifier phrases
  Pronouns with individual phrases as antencedents can be replaced by their antecedents but those with quantifier phrases as their antencedents are often ineliminable.

7.1.3. Finding quantifier phrases
  Quantifier phrases are like definite descriptions in that they often include adjectival modifiers like prepositional phrases and relative clauses.

7.1.4. Kinds of generalizations
  Quantifier phrases are like conditionals in that they come in a variety of forms. This means that one of the key steps in analyzing a generalization will be to classify it.

7.1.5. Bounds and exceptions
  While the domain of objects about which a generalization is asserted will usually be determined by a quantifier phrase itself, there are some phrases which modify this domain from outside the quantifier phrase.

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7.1.1. Theories of quantifier phrases

In 6.1.6 individual terms like Spot and the dog were distinguished from other singular noun phrases like every dog, no dog, and a dog because the semantic function of the latter phrases is not to make definite references to single objects. We call phrases of the latter sort *quantifier phrases* and we will call words like every, no, and a that are used to form them *quantifier words*. The term *quantifier* here is intended to suggest the semantic function these phrases do have: they serve to say how many objects of a certain kind have a certain property. Thus Every dog likes bones says that all objects of the kind indicated by the term dog have the property expressed by the predicate [_ likes bones] while No dog climbs trees says that 0 objects of this kind have the property of climbing trees.

Sentences with an indefinite article—for example, A dog has been digging in the garden—are particularly instructive. The claim made by a sentence like this is that at least one dog has the property of having been digging in the garden. (Or else that there *was* at least one dog that *had* the property of digging in the garden. The difference between the two interpretations is important but it is tied to the logical properties of tenses, which is something we cannot explore in this course.) Although the truth of a sentence A dog … may be traced to the activities of an individual dog in a particular case, it is not a claim about any individual dog but rather about the class of dogs as a whole because any dog with the right properties could make it true. That is why No dog has been digging in the garden—which is the denial of A dog has been digging in the garden—does not give even the appearance of concerning an individual dog.

The study of quantifier phrases is one of the oldest parts of deductive logic, but it is also the part that was last to be fully developed. The study of sentences like the examples above was the heart of Aristotle's work in logic and it is the one part of logic that was preserved in periods (such as the Renaissance and early modern era) when interest in logic dwindled and much of truth-functional logic was ignored or forgotten. However, Aristotle's account of these sentences was not fully satisfactory, and the problems they raised were not solved completely until the work of Frege.

Aristotle's treatment of quantifier phrases is the basis of the traditional theory of *syllogisms*. That theory was principally concerned with the sentences that contained quantifier phrases, and it applied directly only to sentences that could be restated with a single quantifier phrase in subject position. Because of this, it did not account for the validity of arguments that depended on the pres-

ence and interaction of several quantifier phrases in the same sentence. The validity of the argument

$$\frac{\text{Every horse is a mammal}}{\text{Any head of a horse is a head of a mammal}}$$

turns on the function of the two quantifier phrases a horse and a mammal in the conclusion, but the theory of syllogisms did not provide for the analysis of the phrases head of a horse and head of an animal.

Although the examples like the one above were popularized by De Morgan in the 19th century, the issue was recognized in the middle ages, and medieval logicians began to develop an account of quantifier phrases that looked at individual phrases rather than at a limited variety of sentences containing them. The idea was to look at quantifier phrases, like individual terms, as serving to refer—with individual terms and the various quantifier phrases referring in different ways. The sort of reference, or *supposition*, would depend, in the case of quantifier phrases, on the quantifier word used and also on the place of the quantifier phrase in the sentence. Thus we might speak, as indeed we have, of Spot and the dog as making a definite reference. And, in a similar way, we might speak of every dog and no dog as both making a general reference to dogs (differing in whether they say they all have or all lack a given property) and of a dog as making an indefinite reference to a dog (since no more than one dog need be in question but no particular dog is crucial). This, by itself, does not settle the problem of multiple quantifier phrases but it does provide an approach to quantifier phrases that could be applied to each of several phrases in the same sentence.

The problem with a theory like this lies in the possibility that the kind of reference a quantifier phrase exhibits depends not only on the quantifier word it contains and its place in a sentence but also on its relation to other quantifier phrases. The ambiguity of sentences like the following one shows that the interaction of quantifier phrases can be important:

A reporter interviewed each juror.

This could be understood to assert either the weak claim that each juror was interviewed or the stronger claim that there was a single reporter who interviewed them all. The difference between these two claims can be brought out by restating the sentence to make one or the other of the quantifier phrases the subject followed by the phrase is such that—that is, by using the device of expansion that we used for a different purpose in 6.2 .

weak reading: **Each juror** is such that (**a reporter** interviewed him or her)

strong reading: **A reporter** is such that (he or she interviewed **each juror**)

If we were to account for this ambiguity by something like a theory of supposition, we would have to find some sort of ambiguity in the quantifier phrases. The most natural way of doing this would be to say that a phrase like a reporter can have two sorts of reference, either a fixed indefinite reference or a variably indefinite reference (giving rise to the strong and weak readings, respectively); and this is roughly the account given by medieval logicians working on the theory of supposition.

The difficulty is that this sort of ambiguity is not the only one that can arise. For example, consider the following:

Every reporter asked a question of each juror

This is four ways ambiguous, with the interpretations indicated in the following table:

weakest reading: **Every reporter** and **juror** are such that the former asked the latter **a question**

intermediate reading 1: For **every reporter**, **some question** is such that he or she asked it of **each juror**

intermediate reading 2: For **each juror**, **some question** is such that **every reporter** asked it of him or her

strongest reading: **A question** is such that **every reporter** asked it of **each juror**

The strongest reading entails all the rest and all entail the weakest reading, but neither of the two intermediate readings entails the other.

The problem here is not simply that the different meanings of a question have begun to mount up implausibly but that the differences cannot be described without reference to other quantifier phrases. That is, a question could be said to exhibit a variably indefinite reference not only in the weakest reading but also in the two intermediate readings. These three readings differ because the variation in reference is allowed to depend on different factors: on both the reporter and the juror (the weakest), on the reporter only (the first intermediate reading), or on the juror only (the second intermediate reading). And this makes it seem that the differences lie not in the way a single quantifier phrase refers but in the way the sentence as a whole is put together.

One way of diagnosing the problem is to note that, while the theory of supposition provides the resources for a more subtle description of the ways sentences can be put together than is available in Aristotelian logic, it does not

provide a way of describing the structure of sentences in varying levels of detail comparable to the gradually developing analyses that we can offer using connectives. There were hints of another approach in the Middle Ages but nothing was worked out fully until Frege's *Begriffschrift* (or 'Concept notation') of 1879. And it is hard to see how anything like Frege's solution to the problem of quantifier phrases could have been developed much earlier; for it depended on his understanding of predicates as expressing truth-valued functions of objects. This idea was a considerable extension of the concept of a function current in his day, and this had already been extended far beyond anything that might have occurred to a mathematician in the 18th century, to say nothing of a philosopher in the Middle Ages.

Still the medieval hints and Frege's final approach handled the problem we have been considering in a similar way. In both views, predicates come first and quantifier phrases are added to them—and added to them in a certain order. The different readings we may give to the resulting sentence then depend on the order in which the quantifier phrases were added. In the case of Every reporter asked a question of each juror, the strongest reading comes when a question is added last, for then we are saying that at least one question has the property expressed by [every reporter asked _ of every juror]. The weakest reading comes when we add this quantifier phrase first, for then we say only that the predicate [_ asked a question of _]—roughly, [_ questioned _]—is true of every reporter and juror. The other two possibilities come when a question has been added to the sentence before one but not the other of the phrases every reporter and each juror.

What distinguished Frege's approach was that he went beyond the metaphor of quantifier phrases lining up to be added to a sentence. Instead, he spoke of quantifier phrases as signs for operations that apply to predicates just as predicates are operations that apply to individual terms. Thus, on this view, the grammatical similarity between individual terms and quantifier phrases disguises the semantic difference that is shown in Figure 7.1.1-1:
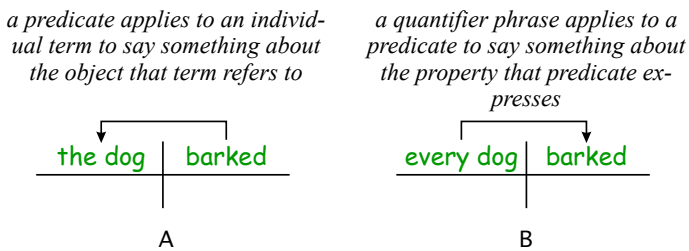

Fig. 7.1.1-1. The semantic roles of an individual term (A) and a quantifier phrase (B).

This kind of analysis can be applied to a sentence several times over by using the device of expansion. For example, the second intermediate reading of the quadruply ambiguous sentence above could be expressed along with its component predicates as follows:

Each juror is such that (a question is such that (every reporter asked it of him or her))

a question is such that (every reporter asked it of _____)

every reporter asked _ of _____

Here each quantifier phrase is seen to apply to a predicate that may contain previously applied quantifier phrases.

Glen Helman 28 Oct 2011

7.1.2. Pronouns and quantifier phrases

When a logically complex predicate is applied to an individual term, the result can be restated as a compound of simple predications. We used this as basis for our analyses in truth-functional logic. The first example of that in §2.1.1 was to analyze That bear is large and edgy into That bear is large and That bear is edgy. However, when a sentence is formed from a complex predicate by applying a quantifier phrase to this predicate, no such simplification need be possible. The sentence Some company is such that it does business in Tokyo and Terre Haute cannot be restated as the conjunction of Some company does business in Tokyo and Some company does business in Terre Haute since, although all these sentences are true, the last two require only that each of these cities has some company operating while the first implies some international business.

The reason for the difference in the two examples is a different relation between the logical form of the complex predicate and the logical form of the sentence as a whole depending on whether the predicate is applied to an individual term or has a quantifier phrase applied to it. The logical form of a complex predicate—as it would be displayed were we to analyze the body of a predicate abstract—reflects the form of any sentence that results from predicating it of a term. So every predication of this predicate has the same sort of structure. For example, the predicate [_ does business in Tokyo and Terre Haute] can be analyzed as

$$[x \text{ does business in Tokyo} \land x \text{ does business in Terre Haute}]_x$$

so, if we apply it to a term—Sony say—

$$[x \text{ does business in Tokyo} \land x \text{ does business in Terre Haute}]_x \text{ Sony}$$

we will be asserting a conjunction because the body of the predicate is a conjunction, so the predication in reduced form

$$\text{Sony does business in Tokyo} \land \text{Sony does business in Terre Haute}$$

will also be a conjunction.

On the other hand, when we apply a quantifier phrase to a complex predicate, we make a claim about the population of the predicate's extension. And there is no reason to think that the form of this claim will reflect the logical form of the predicate's claims about individual objects. In the example at hand, Some company is such that it does business in Tokyo and Terre Haute says that the extension of [_ does business in Tokyo and Terre Haute] contains at least one company. To say this is to say something about the way in which the extensions of [_ does business in Tokyo] and [_ does business in Terre Haute] overlap, and this sort of relation between their extensions

cannot be expressed by a truth functional compound of claims made about the two extensions individually. This is one instance of a general point: few interesting relations between things can be restated as truth-functional compounds of claims made about the things individually. For example, try restating the claim John Stuart Mill was the son of James Mill as a truth-functional compound of claims each of which refers to only one of the two men.

This difference between individual terms and quantifier phrases has an impact on the significance of pronouns whose antecedents are quantifier phrases. A pronoun that has an individual term as an antecedent is a device for avoiding repetition, and it can be eliminated if we are willing to tolerate the repetition. That is why nothing like pronouns was introduced into our symbolic notation for truth-functional logic. We are able to restate Jack built the house and sold it so that the pronoun it is replaced by a second occurrence of the house, giving us a compound of two independent components. When we use abstracts, we are able to represent pronouns could analyze this sentence as [Jack built x and sold x]$_x$ the house. The possibility of replacing pronouns by their antecedents means that this expanded form can be restated in reduced form by replacing the variable x by the term the house. But this sort of restatement or reduction is not possible with Jack built *a* house and sold it—since Jack built a house and sold a house can be true without him having sold a house he built—so we cannot eliminate the pronoun.

Something similar can happen with compounded predicates and other compounded phrases. We can restate the sentence The Titanic ran into an iceberg and sank so that and joins clauses rather than predicates if we repeat the phrase the Titanic. But A ship ran into an iceberg and sank cannot be restated as a conjunction of clauses by repeating the phrase a ship. Since we regard conjunction only as an operation on sentences, a restatement as a conjunction of clauses is necessary if we are to subject the predicate [_ ran into an iceberg and sank] to further analysis. And, because we cannot repeat a ship without changing the meaning, we must introduce a pronoun with a ship as its antecedent. So not only are we often prevented from eliminating pronouns with quantifier phrase antecedents, we are often forced to introduce such pronouns in order to analyze sentences. Because of this abstracts will play a central role in the analysis of quantifier phrases: often sentences containing quantifier phrases can be analyzed only by identifying a complex predicate and subjecting it to analysis.

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7.1.3. Finding quantifier phrases

The examples of quantifier phrases we have been considering take an especially simple form: a quantifier word modifying a common noun. And these two elements form the core of most quantifier phrases. (Not all, because a sentence like Lions are carnivores achieves the same effect as the generalization Every lion is a carnivore by using the plural rather than a quantifier word.) Words and phrases other than quantifier words can appear as further modifiers of the common noun in a quantifier phrase; in particular, adjectives, relative clauses, and adjectival prepositional phrases often serve this function. For example, in

Every large dog in the neighborhood that was outside last night
  was barking

the underlined quantifier phrase is the bulk of the sentence. Besides the common noun dog and quantifier word every, it employs the adjective large, the prepositional phrase in the neighborhood and the relative clause that was outside last night to further specify the claim that is being made.

We encountered a same array of possible modifiers in the case of definite descriptions and the problem of finding the whole of a definite description has an analogue in the case of quantifier phrases. That is, when locating quantifier phrases, we must be on the lookout for modifiers that are part of the phrase. As was the case with definite descriptions, prepositional phrases and relative clauses are especially problematic here since they appear after the common noun. One test that was suggested for definite descriptions is particularly important in the case of quantifier phrases: try replacing the phrase you have isolated by the pronoun it. This pronoun will not accept modifiers, if you leave any behind, the result will be ungrammatical (or else change the meaning other parts of the sentence because the abandoned modifiers are understood to apply to them). Thus the first sentence below is grammatical, but the second and third are not.

  It was barking
* It that was outside last night was barking
* It in the neighborhood that was outside last night was barking

Since we will make replacements by pronouns as part of the analysis of a quantifier phrase, this test will be performed as a matter of course.

Of course, some contexts will require he, she, or the like, and the test will then be less clear-cut since these pronouns can accept modifiers. What you need to check in this sort of case is whether the pronoun when modified is able

to have an earlier antecedent. For example, in Everyone who hesitates is lost, replacing everyone by he gives us the sentence He who hesitates is lost, which is not only grammatical but has the same meaning as the original (when he is understood generically). But we cannot regard he in this context as having an earlier antecedent. In Sam is indecisive and he who hesitates is lost, the pronoun he must still be used to make a general claim and cannot refer only to Sam.

It is also important to avoid slipping between restrictive and non-restrictive relative clauses. For example, Sam was indecisive and he, who hesitated, was lost comes close to being grammatical and he has Sam as its antecedent. But this only works because the relative clause who hesitated is non-restrictive here (note the commas), and it is restrictive in Everyone who hesitated was lost.

Prepositional phrases present a special problem when quantifier phrases are in predicates since prepositional phrases can have adverbial as well as adjectival roles and a prepositional phrase left behind in the predicate when a quantifier phrase is analyzed need not make the sentence ungrammatical. Sentences can even be ambiguous in this respect. For example, Larry heard of a new band in Indianapolis might speak of Indianapolis either as the home of the band or as the place where Larry learned of them. The difference is captured by the following expansions:

A new band in Indianapolis is such that (Larry heard of it)
A new band is such that (Larry heard of it in Indianapolis)

The difference can be brought out also by another test suggested in the case of definite descriptions: converting the prepositional phrase into a relative clause. (In the example above, the clause would be that is in Indianapolis or that was in Indianapolis.) Doing so will force the prepositional phrase to be understood as adjectival and thus show the effect of treating it as part of the quantifier phrase. On the other hand, moving a prepositional phrase to the beginning of the sentence will to force to be understood adverbially and thus as not part of the quantifier phrase.

The results of these tests are shown for two sentences below.

Diane studied a stellar object at the edge of the known universe

expansions A stellar object at the edge of the known universe is such that (Diane studied it)
A stellar object is such that (Diane studied it at the edge of the known universe)

conversion to relative clause Diane studied a stellar object <u>that was at the edge of the known universe</u>

movement to the front <u>At the edge of the known universe</u>, Diane studied a stellar object

Diane studied a dinosaur <u>in her paleontology class</u>

expansions A dinosaur <u>in her paleontology class</u> is such that (Diane studied it)

A dinosaur is such that (Diane studied it <u>in her paleontology class</u>)

conversion to relative clause Diane studied a dinosaur <u>that was in her paleontology class</u>

movement to the front <u>In her paleontology class</u>, Diane studied a dinosaur

These should convince you that (outside of science fiction) the prepositional phrase in the first sentence is probably intended to be adjectival and part of the quantifier phrase while the prepositional phrase of the second is most likely to be adverbial and part of the predicate.

Glen Helman 01 Aug 2011

7.1.4. Kinds of generalizations

There are two broadly applicable ways of saying what a quantifier phrase says about the extensions of predicates. On the one hand, all quantifier phrases can be seen to say something about the number of objects in or outside of these extensions, and this is what justifies the term quantifier phrase. However, this characterization applies more naturally to some quantifier phrases than to others. For example, to restate Every dog likes bones as a numerical claim, we must resort to Zero dogs fail to like bones. On the other hand, we can describe the role of every dog in Every dog likes bones more naturally by saying that it is used to state a *generalization* about the property of liking bones. And all the quantifier phrases we will consider in this course could be described, again some more naturally than others, in terms of the making and denying of generalizations. For example, Some dog has been digging in the garden could, in a pinch, be regarded as the denial of the negative generalization No dog has been digging in the garden.

We will begin our study of quantifier phrases by looking at them in the latter way, in terms of the idea of generalization. This will make the properties of phrases like every dog and no dog stand out more clearly than those of phrases like some dog. So we will reserve full attention to the latter sort of phrase for the next chapter, where we look at quantifier phrases as ways of making more obviously numerical claims. For the time being, we will think of sentences like Some dog has been digging in the garden as statements that claim the existence of falsifying examples, or *counterexamples*, claims which deny generalizations.

We will begin our study of generalizations by developing some terminology for describing them informally before offering symbolic representations. A generalization claims that a certain property holds of all objects in a certain collection. We will refer to the collection of objects over which the generalization is made as the *domain* of the generalization and refer to the property that is said to hold generally as the *attribute* of the generalization. The chief problem in analyzing generalizations will be to identify the domain and the attribute. In a simple case, like Every dog likes bones, the domain will be the class of objects picked out by the common noun of the quantifier phrase together with its modifiers (here it is the class of dogs), and the attribute will be the property expressed by the predicate to which the quantifier phrase is applied (here it is the property of liking bones). We will refer to the common noun together with its modifiers as the *class indicator* of the generalization and

the predicate to which the quantifier phrase is applied as the *quantified predicate*.

In the simple case of Every dog barks we would have

class indicator
↓
quantifier phrase → Every dog barks ← quantified predicate
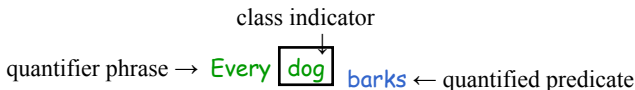
but with a more complex quantifier phrase we might have something like

AI cursed every large dog in the neighborhood that was outside barking last night

with modifiers of the common noun included in the class indicator.

There are two common ways that the quantified predicate of a generalization is related to its attribute. When the attribute is the property expressed by the quantified predicate, we will say that the generalization is *affirmative*. So Every dog barks is an affirmative generalization since barking is both the attribute that is said to hold generally of dogs and the property expressed by the predicate [_ barks].

On the other hand, No dog climbs trees is not affirmative. The domain of this generalization is also the class of dogs but what is said to hold generally of them is that they do not climb trees. That is, the attribute of this generalization is the denial of the property expressed by the quantified predicate. When this is the relation between the quantified predicate and the attribute—i.e., when the attribute is expressed by the negation of the quantified predicate—we will say that the generalization is *negative*.

Notice that this does not characterize the claim made by the generalization so much as the way this claim is expressed. The generalization Every box was unopened is affirmative because its attribute is the property of not being opened and this is the property expressed by the quantified predicate [_ was unopened]. On the other hand the generalization No box was opened is negative even though it makes the same claim.

There are also two common ways the domain of a generalization is related to the class indicator. A *direct generalization* is one whose domain is identical with the class of objects picked out by its class indicator; that is, it is identical with the *indicated class*. Thus, both Every dog barks and No dog climbs trees are direct because in both cases the domain, dogs, is picked out directly by their class indicators.

However, the generalization

Only trucks were advertised

is not direct. Its domain is not the class of trucks, but the class of non-trucks, whose members are said not to have been advertised. To see this, think what the counterexamples to the generalization would be like: non-trucks that were advertised. We will refer to such generalizations as *complementary*. A complementary generalization will often have the corresponding direct generalization as an implicature; for example, Only new listings were distributed suggests that *all* new listings were distributed. But Only trucks were advertised carries no such implicature, and the implicature is easily canceled in other cases; the sentence Only new listings were distributed and not even all of them were is not at all incoherent, much less self-contradictory.

We use the term *complementary* for a generalization like Only trucks were advertised because its domain is the complement of the indicated class. The *complement of* a set X *relative to* a set Y is the class of all members of Y that are not in X (see Figure 7.1.4-1).
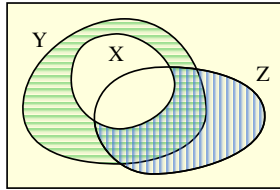

Fig. 7.1.4-1. The complements of a set X relative to sets Y and Z (each shaded).

It will sometimes be useful to turn this idea around and think of the complement of X relative to Y as Y *with* X *subtracted*. When only the class X is specified, the complement must be taken relative to the set of all reference values; this yields the *full complement of* a set, the class of every value not in the set. Thus a complementary generalization makes a claim about the full complement of the indicated class.

The basic classification of generalizations we have considered is summarized in Table 7.1.4-1 below. Each entry shows an English form that can be used as a standard paraphrase for that sort of generalization.

	Affirmative (the attribute is the property expressed by the quantified predicate)	*Negative* (the attribute is the denial of the property expressed by the quantified predicate)
Direct (the domain is the indicated class)	Every C is such that …it…	No C is such that …it…
Complementary (the domain is the complement of the indicated class)		Only Cs are such that …they…

Table 7.1.4-1. The classification of generalizations.

There seems to be no quantifier word that indicates an affirmative complementary generalization, but some of the modifying phrases we will consider next can be used to state an affirmative generalization about the complement of a given class.

Glen Helman 28 Oct 2011

7.1.5. Bounds and exceptions

In general, the class indicator and quantified predicate of the generalization will be our starting points for identifying the domain and attribute, and the quantifier word appearing in the quantifier phrase will be our chief guide in doing this. For example, the quantifier words every and all will lead us to presume that the domain and attribute can be determined in the simple way associated with an affirmative direct generalization.

However, this is only a presumption—as is shown by the following example:

> Among members of the House, all Republicans except Midwesterners supported the bill.

Here the class indicator is Republicans but the domain is the class of non-Midwestern Republicans in the House. One way to see this is to ask yourself what sort of thing would be a counterexample; here it would be a non-Midwestern Republican House member who did not support the bill. (This is not the first time we have seen that our sense of the content of a generalization can be sharpened by asking what a counterexample would have to be like. However, we can now say more about why this is so: a counterexample will tell you something about the domain and attribute of the generalization because it must be something in the domain that fails to have the attribute.)

The same generalization could be stated by the sentence Every non-Midwestern Republican who was a member of the House supported the bill, which builds the full specification of the domain into its class indicator. In the example displayed above, our path from the class indicator to the domain is less direct. It has been inflected by the phrases among members of the House and except Midwesterners, which function as modifiers of the quantifier phrase. We will call phrases like these *bounds indicators* and *exception indicators*, respectively. Each will consist of a common noun phrase, usually in the plural, together with a word showing that the class picked out by this common noun phrase is a class bounding the generalization, a *bounding class*, or that it is a *class of exceptions*. The words among and of are often used to marks bounds, and except and but can serve to mark exceptions. Although other than functions much like except and but, phrases formed with it have the grammatical status of adjectives and can therefore be regarded as modifiers of the common noun that are part of the class indicator.

Our description of the domains of direct and complementary generalizations applies only in cases where the are no bounds or exceptions to the generaliza-

tion. When there is a bounding class, the true domain is the region where the domain we described earlier overlaps, or *intersects*, the bounding class; it is the *intersection* of the two classes. For example, the direct generalization

> Among members of the House, all Republicans supported the bill

has as its domain the class of members of the House who are Republicans, and this is the intersection of the class of Republicans with the class of House members. The complementary generalization

> Of GM products, only trucks were advertised

has as its domain the class of GM products that are non-trucks, the intersection of the class of non-trucks with the class of GM products.

The bounding class need not appear as explicitly as it does in these examples. For example, it may be supplied by the context of use rather than by an explicit bounds indication. For example, someone who asserts Everyone was affected by the drought probably has in mind only the population of a particular region. Also, it is not unusual for the class indicator of a generalization to indicate both the bounding class and the complemented class through the use of emphasis. As examples, read the following with emphasis on the underlined expressions:

> Only cans of mushrooms <u>from that plant</u> were recalled
> Only cans <u>of mushrooms</u> from that plant were recalled

In each case, the attribute is the property of *not* having been recalled. In the first, the domain consists of cans of mushrooms that are not from the plant in question. In the second, it consists of cans from the plant that are not cans of mushrooms. (To confirm this for yourself, think what counterexamples to the claims would be like.)

We can get from the class indicators to the domains of these examples by taking the bounding class as the class picked out by the unemphasized part of the class indicator (e.g., cans of mushrooms or cans from that plant, respectively). The emphasis serves to mark the further specifications that move us from the bounding class to the narrower class that is complemented relative to it. Thus the domain in the second case can be described as the class of all cans from the plant that are not cans of mushrooms from the plant. Notice that this can be described either as the intersection of the full complement of the indicated class (i.e., cans of mushrooms from the plant) and the bounding class (i.e., all cans from the plant) or as the complement of the indicated class relative to the bounding class (i.e., all cans from the plant). In general, the intersection of the complement of a set X with a set Y is the complement of X rela-

tive to Y. (You can use Figure 7.1.4-1 to help you think about this.)

The same device can appear with direct generalizations but its effect is primarily on implicatures. The sentence

<p style="text-align:center;color:green;">All <u>Republicans</u> in the House supported the bill</p>

ends up having the same domain as

<p style="text-align:center;color:green;">All Republicans <u>in the House</u> supported the bill</p>

but, while the first suggests (and it is only a suggestion) that some Democrats in the House did not support the bill, the second suggests that support faltered among Republicans in the Senate. In fact, what we are seeing here is again the effect of emphasis on complementary generalizations but its effect is on the complementary generalizations that are often implicated by direct generalizations.

The domain of a generalization is also modified when there is a class of exceptions—though this happens only with direct generalizations. If we set aside the effect of a bounding class, the domain of a generalization with exceptions is the indicated class with the class of exceptions subtracted; that is, the domain consists of the members of the indicated class that are not in the class of exceptions. For example, the domain of the generalization

<p style="text-align:center;color:green;">All Republicans except Midwesterners supported the bill</p>

is the class of Republicans with the class of Midwesterners subtracted; it is the class of non-Midwestern Republicans. In the presence of the bounds indicator among members of the House, as in the example beginning this subsection, the domain is restricted further, to the members of the House who are non-Midwestern Republicans. Both bounds and exceptions modify a direct generalization by narrowing the domain, and they do so from independent directions so their effects accumulate without interaction. In particular, there is no need for concern about the order in which these modifications are applied; bounding and then excepting comes to the same thing as excepting and then bounding.

Our only examples of complementary generalizations were negative because there seemed to be no quantifier word in English that indicates a complementary affirmative generalization. This gap is filled by a peculiar device: the class indicator can be dropped entirely when an exception indicator is present. For example, if there were no bounds provided by the context, the affirmative generalization All but the hardiest plants suffered would have as its domain the full referential range with the class of hardiest plants subtracted. This is the same thing as the complement of the class of hardiest

plants, so we have an affirmative generalization whose domain is a complement (though not the complement of the indicated class since there is no class indicated). We are rarely prepared to generalize affirmatively about everything outside a given class, so we can expect to find a bounding class in most cases. Here, it probably would be the class of all plants—though that would depend on the context and emphasis can play a role, too. The sentence All but Midwestern Republicans supported the bill leaves its bounding class to the context while All but Midwestern Republicans supported the bill marks its bounding class as the class of Republicans (though the context might bound it further).

When the class indicator is negative, the need for bounds applies somewhat differently. The sentence All but non-smokers are at risk amounts to the same thing as All smokers are at risk, so it needs no bounds. However, someone who asserts All non-smokers were relieved probably intends to limit its domain to people.

In general, a bounding class can be shown in a paraphrase by prefixing a phrase of the form Among Bs. The effect of this phrase is to limit the domain to the bounding class by intersecting this class with the class obtained from the other specifications of the domain. A class of exceptions can be shown by putting the phrase of the form except Es after the quantifier phrase. Its effect is to limit the domain by subtracting the class of exceptions from the class obtained from other specifications. In order to use these modifications grammatically when paraphrasing direct generalizations, we must first put the quantifier phrase in the plural (e.g., All Cs or No Cs). Paraphrases like these will often be stilted but they will make it easy to represent the form of the generalization symbolically.

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7.1.s. Summary

1 Not all singular noun phrases are individual terms. Many are quantifier phrases formed using quantifier words. These phrases are used to say how many objects of a certain kind have a certain property. The study of them dates to Aristotle's theory of syllogisms and was active again in medieval theories of "supposition" but some central problems were solved only with Frege's work a century ago. The problems he solved concern the role of multiple quantifier phrases in the same sentence. To describe this role, we must describe the contribution of individual phrases in a way that takes account of the interaction between phrases, something that can lead to multiple ambiguity. Frege's approach was to regard quantifier phrases as signs for operations that apply to predicates, with the interaction of phrases accounted for by the order in which these operations are applied. A particular order of application can usually be fixed in English (and different interpretations of a sentence can be distinguished) by expansions using the phrase is such that.

2 Complex predicates can be avoided when they apply to individual terms and all such sentences can be stated in reduced form, but this is not always possible in the case of quantifier phrases because a quantifier phrase may be used to state a relation between predicates that cannot be analyzed as a compound of separate claims about these predicates. For the same reason, pronouns with quantifier phrase antecedents often cannot be replaced by their antecedents, and we may even have to introduce pronouns with quantifier phrase antecedents when we paraphrase compound phrases by compound clauses.

3 The core of a quantifier phrase consists of a quantifier word and a common noun, but the noun may be modified by adjectives, relative clauses, or prepositional phrases. You can test to see if you have isolated the whole of a quantifier phrase by seeing if it would be grammatical to replace it by the pronoun it. Prepositional phrases in the predicate can often be understood to modify either a noun or a verb (and can be missed by the pronoun test); they are part of the quantifier phrase if they can be restated as relative clauses.

4 A generalization claims that every object in a certain class, the generalization's domain, has a certain property, the generalization's attribute. In the simplest cases the domain is the class (the indicated class) that is picked out by the class indicator, the common noun plus modifiers of the quantifier phrase; and the attribute is expressed by the quantified predicate to which the quantifier phrase is applied. The content of a generalization can often be

clarified by considering the sort of counterexample that would show it to be false. When the attribute of the generalization is expressed by the quantified predicate, the generalization is affirmative. The words all and every are used to express affirmative generalizations while no is used to express a negative generalization, one whose attribute is the denial of the quantified predicate. All these words express direct generalizations, whose domain is the indicated class; but, in other generalizations, complementary generalizations, the domain is outside the indicated class. In the simplest case, the domain is the full complement of this class, which is its complement relative to the class of all reference values—i.e., the result of subtracting it from this class.

5 Other, less straightforward, relations between the wording of the generalization and the claim it makes can be indicated by modifiers of the quantifier phrase that we will label bounds indicators or exception indicators. The complement of an indicated class over which a generalization is made is often not its full complement but its complement relative to some bounding class, which may cited explicitly by a bounds indicator (such as among Bs) or may be implicit in the context. There seems to be no word designed to express generalizations that are both affirmative and complementary. Nevertheless, direct generalizations may be restricted to objects outside a class of exceptions by use of such phrases as except Es. Bounds and exceptions limit the domain by intersecting it with another class (the bounding class or the complement of the class of exceptions); the order in which these operations are carried out makes no difference.

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7.1.x. Exercise questions

1. In the sentences below underline all individual terms and quantifier phrases, and any pronouns that have such expressions as antecedents. Distinguish individual terms and quantifier phrases by marking them T or Q and indicate what antecedents any pronouns are bound to—as in the following example:

Ann sent a package to Bill but he hasn't received it
T · · · · · · · · · Q · · · · · · · T
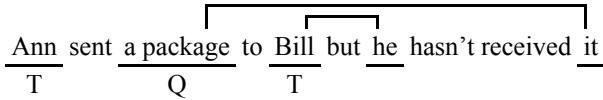
   a. Ann saw a movie and told Bill about it.
   b. Tim watched a dance troop from India.
   c. If anyone backs out, they will get a refund.
   d. Dave called everyone he knew.
   e. Every dog in the kennel was barking.
   f. Bill heard something and Carol heard it, too.
   g. Tim watched a dance troop from the balcony.

2. Check the sentences below for any ambiguity that may be traced to the order in which quantifier phrases have been applied; when more than one interpretation is possible, bring out the differences by means of expansion and indicate which of the interpretations imply which others.
   a. Everyone works toward some goal.
   b. Each member of the committee read each application.
   c. Someone eats at a restaurant every day.

3. For each of the following generalizations, take the following preliminary steps in its analysis: (i) separate the main quantifier phrase and quantified predicate through a restatement using is such that and mark the class indicator, (ii) describe the sort of thing that would count as a counterexample to the generalization, and (iii) determine whether the generalization is affirmative or negative and whether it is direct or complementary.
   a. Every book was checked out.
   b. Kathy spoke to each guest.
   c. No one in the lobby had seen the package before the explosion.
   d. Tod carefully noted everything he saw in the room.
   e. No one who was familiar with both France and Germany was surprised.

     f.   The committee accepted only entries submitted before the deadline.

4.   The generalizations below have quantifier phrases that make essential use of emphasis (marked here by underlining) to indicate the bounds on the generalization. Restate them so that these bounds are explicitly indicated. For example, Only patient children will complete the puzzle could be paraphrased by Among children, only patient ones will complete the puzzle.

     a.   Only new commercial vehicles are covered by the regulation.
     b.   Only French composers of the early Baroque used that device.
     c.   All but emergency vehicles were banned from the park.
     d.   Only new commercial vehicles are covered by the regulation.
     e.   Sam eats all but orange jelly beans.
     f.   Only new commercial vehicles are covered by the regulation.

5.   In each case below, use the information given about a generalization to determine its domain and attribute. Also, state such a generalization in English.

     a.   type: direct and affirmative
          class indicator: road
          quantified predicate: [_ is finished]
     b.   type: direct and negative
          class indicator: road
          quantified predicate: [_ is finished]
     c.   type: complementary and negative
          class indicator: road
          quantified predicate: [_ is finished]
     d.   type: direct and affirmative
          class indicator: road
          quantified predicate: [_ is finished]
          class of exceptions: urban freeways

e. type: direct and negative
class indicator: road
quantified predicate: [_ is well-maintained]
class of exceptions: urban freeways

f. type: direct and affirmative
class indicator: roads
quantified predicate: [_ is finished]
bounding class: federal projects
class of exceptions: urban freeways

g. type: direct and negative
class indicator: dentists
quantified predicate: [_ frowned]
bounding class: alumni
class of exceptions: orthodontists

7.1.xa. Exercise answers

1. **a.**

Ann saw a movie and told Bill about it
T   Q   T

(The bracket connects "a movie" and "it")

b.

Tim watched a dance troop from India
   T   Q   T

c.

If anyone backs out, they will get a refund
    Q   Q

(The bracket connects "anyone" and "they")

d.

Dave called everyone he knew
  T   Q

(The bracket connects "everyone" and "he")

e.

Every dog in the kennel was barking
    Q   T

f.

Bill heard something and Carol heard it , too
T   Q   T

(The bracket connects "something" and "it")

g.

Tim watched a dance troop from the balcony
  T   Q   T

2. **a.** Two ways ambiguous:

     i) Everyone is such that (he or she works toward some goal).

     ii) Some goal is such that (everyone works toward it).
       (ii) implies (i).

b. Not ambiguous

c. Four ways ambiguous:

     i) Every day is such that (someone eats at a restaurant on that day).

     ii) Someone is such that (every day is such that (he or she eats at a restaurant on that day)).

       [Notice that, although that day is a demonstrative phrase (and might thus be counted an individual term), it does not have a reference independent of the quantifier phrase every day. It therefore functions here like a pronoun with every day as its antecedent. The phrase the truck has a similar function in A truck struck a car but only the truck was damaged.]

iii) A restaurant is such that (every day is such that (some-one eats at it on that day)).

iv) Some person and restaurant are such that (he or she eats at it every day).

(iv) implies all the others; (ii) and (iii) each imply (i).

3. Class indicators are boxed.

a. i. <u>Every book</u> was checked out.
Every ⟦book⟧ is such that (<u>it</u> was checked out).
ii. *counterexample:* a book that was *not* checked out
iii. *Direct and affirmative.*

b. i. Kathy <u>spoke</u> to <u>each guest</u>.
Each ⟦guest⟧ is such that (Kathy spoke to <u>him or her</u>).
ii. *counterexample:* a guest that Kathy *did not* speak to
iii. *Direct and affirmative.*

c. i. <u>No one in the lobby</u> had seen the package before the ex-plosion.
No ⟦one [i.e., person] in the lobby⟧ is such that (<u>he or she</u> had seen the package before the explosion).
ii. *counterexample:* a person in the lobby who *had* seen the pack-age before the explosion
iii. *Direct and negative.*

d. i. Tod carefully noted <u>everything he saw in the room</u>.
Every ⟦thing Tod saw in the room⟧ is such that (Tod carefully noted <u>it</u>).
ii. *counterexample:* something Tod saw in the room that he did *not* carefully note
iii. *Direct and affirmative.*

e. i. <u>No one who was familiar with both France and Germany was surprised.</u>
No ⟦one [i.e., person] who was familiar with both France and Germany⟧ is such that (<u>he or she</u> was surprised).
ii. *counterexample:* someone familiar with both France and Ger-many who *was* surprised
iii. *Direct and negative.*

f. i. The committee accepted <u>only entries submitted before the deadline</u>.
Only ⟦entries submitted before the deadline⟧ are such that (the committee accepted <u>them</u>).

      ii. *counterexample:* an entry *not* submitted before the deadline that the commitee *did* accept

      [*Note:* this reflects the simplest interpretation of the generalization, one that does not make distinctions among parts of the class indicator; that is, in analyzing it this way we do not look for the differences in emphasis that are the topic of the next exercise.]

      iii. *Complementary and negative.*

4. **a.** Among new vehicles, only commercial ones are covered by the regulation.

    b. Among French composers, only those of the early Baroque used that device.

    c. Among vehicles, all but emergency ones were banned from the park.

    d. Among commercial vehicles, only new ones are covered by the regulation.

    e. Among jelly beans, Sam eats all but orange ones.

    f. Among vehicles, only new commercial ones are covered by the regulation.

5. **a.** *domain:* roads
    attribute: the property of being finished
    Every road is finished

    b. *domain:* roads
    attribute: the property of being *un*finished
    No road is finished

    c. *domain:* things that are not roads
    attribute: the property of being *un*finished
    Only roads are finished

    d. *domain:* roads that are not urban freeways
    attribute: the property of being finished
    All roads, except urban freeways, are finished

    e. *domain:* roads that are not urban freeways
    attribute: the property of *not* being well maintained
    No roads, except urban freeways, are well-maintained

    f. *domain:* federal projects that are roads but not urban freeways
    attribute: the property of being finished
    Among federal projects, all roads, except urban freeways, are finished

g. *domain:* alumni who are dentists but not orthodontists
attribute: the property of *not* having frowned
Among alumni, no dentists, except orthodontists, frowned

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