6.2. Predicates and pronouns

6.2.0. Overview

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6.2.1. Abstracts

In our analyses so far, we have identified the places of a predicate with the blanks remaining when all largest individual terms have been removed. But, while this way of identifying the places of a predicate is best for a full analysis, it is not required by the concept of a predicate. For the greatest flexibility in identifying predicates, we need a notation that will allow us to specify an order for the places of a predicate that is different from the order of blanks in the English and that will allow us to associate a given place with more than one blank. What we will use is an extension of the ordinary algebraic use of variables. It is a simple idea that was used by Frege but it was first studied extensively by the American logician Alonzo Church (1903-1995) in the 1930s.

The usual form of definition for a function—of a polynomial, for example,

$$f(x, y) = x^2 + 3xy + 1$$

gives a name to the function and uses a variable or variables to indicate the input values, with the output specified by some sort of formula. An alternative notation represents the input and output more graphically

$$f: x, y \mapsto x^2 + 3xy + 1$$

The latter definition might be read

f is the function which, when given input x and y, yields the output $x^2 + 3xy + 1$

Church's notation, the notation of *lambda abstraction*, provides a symbolic version of the sort of definite description that appears in the English version of the definition. Using this notation, the symbolic definition could be written as

$$f = \lambda xy\ (x^2 + 3xy + 1)$$

That is, the expression "$\lambda xy\ (x^2 + 3xy + 1)$" can be read as the function which, when given input x and y, yields the output $x^2 + 3xy + 1$.

When we define a function by a formula, whether we use the traditional notation or Church's, we are interested in the way the meaning of the formula varies with changes in the reference of certain individual terms. This "way" is more abstract than any particular value the formula has when the reference of these terms is fixed, so the move from the formula to the function is reasonably described as "abstraction." The notation of lambda abstraction identifies a function without immediately introducing a name for it. This idea has been important in the development of computer programming languages and, in that context, the right-hand side of the second equation above would now often be described as an "anonymous function." So, when a defining equation is expressed in the notation of lambda abstraction, it abstracts a function anonymously by using the expression "$\lambda xy\ (x^2 + 3xy + 1)$" and then assigns it the name "f."

Since predicates express functions, the same idea can be applied to them, and it will provide the sort of flexibility we need in identifying predicates. However, our notation for abstraction will be a little different from Church's. We will write the variables that follow the lambda in Church's notation as subscripts on brackets. For example, for the function defined above, we can write

$$[x^2 + 3xy + 1]_{xy}$$

something that might be read as $x^2 + 3xy + 1$ as a function of x and y.

As an example of a predicate in this notation, consider the following:

$$[x \text{ introduced } x \text{ to } y]_{xy}$$

If we give this the input Bill and Ann, it will generate an output sentence by putting Bill in place of x and Ann in place of y. The output will then have Bill in the first and second blanks of the sentence-with-blanks _ introduced _ to _, and it will have Ann in the third blank. So we will get as output the sentence Bill introduced Bill to Ann—or, more idiomatically, Bill introduced himself to Ann.

That is, the expression,

$$[x \text{ introduced } x \text{ to } y]_{xy}\ \underline{\text{Bill}}\ \underline{\text{Ann}}$$

provides an alternative analysis of the first example of 6.1.5 in which use a two-place predicate instead the three-place predice [_ introduced _ to _]. The chief application of this sort of flexibility in analysis will be in later chapters; but this example shows that it captures some aspects of English predications better than the analysis we will most often use. In particular, like the English sentence, this analysis indicates a double reference to Bill without repeating his name. We will look at this aspect of abstraction further in 6.2.3 .

We will call an expression formed with these subscripted brackets an *abstract*. We will speak of a *predicate abstract* when the brackets enclose a sentence-with-variables and of a *functor abstract* when they enclose an individual-term-with-blanks. The general form of an abstract with *n* places is

$$\underbrace{[\ \ ---\ \ ]}_{}{}_{x_1 \ldots x_n}$$

body *abstractor*

It has two parts, a *body*, which specifies the output of the expression, and an *abstractor*, consisting of the brackets and subcripted list of variables. The variables listed in the abstractor may appear in the body in any order and may occur several times.

And they need not occur in the body at all. To get the effect of a definition like $f(x) = 2$, we use an abtract like $[2]_x$ to indicate a function whose output is 2 for any input. Abraction like this is said to be *vacuous*.

The predicate abstract $[x \text{ introduced } x \text{ to } y]_{xy}$ might be read as

what "x introduced x to y" says about x and y

and we will take as our English notation for predicate abstracts an abbreviated form of this reading:

what --- says of $x_1 \ldots x_n$

so the English notation for this predicate would be

what x introduced x to y says of x ən y

(again using the contraction ən to distinguish this use of and from its use in conjunction). The predication that applies this predicate to Ann and Bill then takes the form

what x introduced x to y says of x ən y fits Bill ən Ann

Our English notation for functor abstracts is simply

--- for $x_1 \ldots x_n$

which is a compact version of a reading suggested earlier. The application of $[x^2 + 3xy + 1]_{xy}\ \underline{2}\ \underline{3}$ could be written in this notation as

$x^2 + 3xy + 1$ for x ən y applied to 2 ən 3

This is a case where the alternative English notation applied to for compound functors reads better than the simpler of.

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6.2.2. Bound variables

If a variable in an abstractor appears in the body of an abstract, its occurrences in the body are said to be *bound* to the abstractor. So any occurrence of x_1, …, x_n in the body --- of the following abstract is bound to the abstractor $x_1…x_n$:

$$[\text{ --- }]_{x_1…x_n}$$

If a variable is in the scope of more than one abstractor containing it, it is bound to the one with narrowest scope. So the first occurrence of x and the occurrence of y are bound to the abstractor xy in the following while both the occurrences of x outside the inner abstract and the occurrence of z within its body are bound to the abstractor xz:

$$[[\text{y introduced x to z}]_{xy}\ \text{xx}]_{xz}$$

A variable that is not bound to any abstractor is said to be *free*. So z is free in $[\text{y introduced x to z}]_{xy}$, and when an expression like "$x^2 + 3xy + 1$" or "x introduced x to y" is considered by itself outside the context of an abstract, all variables in it are free.

Variables have the grammatical status of individual terms but have no definite reference values. In the context of a formula like "$x^2 + 3xy + 1$," free variables are naturally thought of as variable quantities (hence their name) since, when they vary in their reference, the value of the formula varies as a result. When variables are bound in an abstract like $[x^2 + 3xy + 1]_{xy}$, there is no longer this sort of variation. The abstract makes a reference to a mathematical object, a polynomial function, that incorporates the variation but does not itself vary. Because of this, an older terminology referred to bound variables as "apparent" variables.

The notation for predicates and functors used in 6.1 can be thought of as a variation on the notation for abstracts that deals with the "apparent" character of bound variables by removing them entirely. We will understand a bracketed sentence- or individual-term-with-blanks to represent an abstract in which each of the blanks is filled with a different variable and the variables appear in the same order in the body and the abstactor. So [_ introduced _ to _] would come to the same thing as the abstract

$$[\text{x introduced y to z}]_{xyz}$$

Because the blanks in the English expression correspond one for one and in the same order to the places of the predicate or functor, there is no need for bound variables to indicate the relation between the two.

Bracketing alone is not sufficient in cases where the places of a predicate do not correspond one for one to the blanks. However, we might supplement it by lines showing how places correspond to blanks.

This is clearer than the corresponding use of bound variables

$$[\text{x introduced x to y}]_{xy}$$

but it is significantly less convenient. Still, it is worth bearing in mind, even when bound variables are used, since the lines in the graphical notation depict the pattern of binding of variables by the abstractor.

Because bound variables only mark a correspondence between locations in the body of the abstract and the abstactor, the bound variables of different abstracts have no connection with one another. This means that, for example, the following abstracts express the same predicate:

$$[\text{x introduced x to y}]_{xy}$$
$$[\text{y introduced y to z}]_{yz}$$

Each says that for any input terms τ and υ (in that order), the output sentence should be τ introduced τ to υ, and pattern of binding in each would be depicted in the same way in the graphical notation.

Expressions, like these, that use different variables to indicate the same correspondence between blanks in the body and places for input will be referred to as *alphabetic variants*. Notice that alphabetic variants can use a given variable in different ways. For example, although the variable y appears in both of the abstracts above, it would be replaced by a different one of the input terms in each case.

The body of a predicate abstract is grammatically like a sentence even though it may contain free variables. It is standard to speak of an expression as *closed* if any variables it contains are bound within it and call an expression *open* if one or more of its variables is free. Logicians typically use the term *formula* for any expression that is grammatically like a sentence whether it is open or closed, and reserve the term *sentence* for closed formulas. Since all formulas are grammatically like sentences, the grammatical vocabulary applied to sentences in previous chapters applies to all formulas. In particular, formulas can be built from formulas by use of connectives, so formulas can be compound and have components.

The distinction between open and closed expressions applies to term-like expressions also, but the terminology is handled differently. Both open and closed expressions are classified as *(individual) terms* with closed expressions distinguished simply as *closed terms*.

It is time to update our notion of atomic sentences or, more generally, *atomic formulas*. Now that we analyze sentences and other formulas into components like predicates and individual terms, the atomic formulas will no longer be simply the unanalyzed sentences (though any sentences that go unanalyzed will still count as atomic). We will now also count as atomic any predication. Predications are compound and can even have formulas as components (albeit not immediate components), but the role of predications in derivations is sufficiently analogous to that of unanalyzed sentences for it to make sense to put them both in the same category. This analogy lies behind our use of capital letters for predicates, and it can be built into our syntactic categories: an unanalyzed sentence can be thought of as a *zero-place predicate*, one that requires no input to yield a sentence as output.

6.2.3. Variables and pronouns

English has devices which function like bound variables. The force of the abstract

[_ introduced _ to _]

or the equivalent

$[x_1 \text{ introduced } x_2 \text{ to } x_3]_{x_1 x_2 x_3}$

can be captured in English by the expression

what is said about three people by saying that (the first introduced the second to the third)

which uses expressions like the first, the second, and so on, instead of subscripted variables. (Parentheses were used in the English displayed above simply to mark the portion corresponding to the body of the abstract.) No particular group of people is in question here, and the expressions the first, etc., do not refer to anything outside the sentence. Instead, these expressions function here much like pronouns that have three people as their antecedant. The word order differs from that used in the English notation for abstracts, but that was done merely to put the phrase "three people" before the "pronouns" that refer to it.

In the case of a one-place predicate abstract, the corresponding English can be stated with a genuine pronoun:

$[\text{Tom bought } x]_x$

what is said about a thing when it is said that (Tom bought it)

The blank that is marked by x in the body of the symbolic abstract is filled in the English with the pronoun it, which has a thing as its antecedent. Since a thing makes no definite reference, neither does the pronoun; the pronoun "refers back" to its antecedent only in the sense that their references are linked in their indefiniteness and cannot be indefinite in independent ways. The general moral is that the variables used in the bodies of abstracts are like pronouns, and the ones in abstractors are like their antecedents. One consequence of this reiterates a point made in the last subsection: you should not expect variables bound to different abstractors to be linked in their reference any more than you would expect this of pronouns that have different antecedents.

We can also move in the other direction and use abstracts to represent the contribution of pronouns to the logical form of a sentence. We can get a hint of how they might do this by looking at a particular English rendering of the sample predicate abstract discussed in 6.2.1

$$[x \text{ introduced } x \text{ to } y]_{xy}$$

what is said about two people when it is said that (the former introduced the former to the latter)

where we use another common pronoun-like device. Now consider the following restatement:

what is said about two people when it is said that (the former introduced him- or herself to the latter)

The reflexive pronoun in this expression corresponds to the repeated variable in the symbolic abstract.

Of course this English expression was a rather artificial one constructed to correspond to an abstract, but there are ways to apply abstracts more broadly. To see how, let us look at three further English expressions corresponding to the abstracts we have been considering. This time the English expressions are predicates (rather than noun phrases that refer to the contents of predicates):

___, ___, and ___ are such that (the first introduced the second to the third)
___ is such that (Tom bought it)
___ and ___ are such that (the former introduced the former to the latter)

Of course, these predicates themselves are also artificial, but they employ a device, the various forms of the phrase is such that, that is sometimes unavoidable. And, while there are usually better ways of saying what may be said using it, it can be easily understood and may be applied to virtually any English sentence to restate it (in English) in a way that corresponds to the use of an abstract.

Because the first element of a sentence often indicates the topic under discussion, languages have many devices for restating sentences with various elements at the front. One common device in English is the use of passive voice. If we wish to say who wrote a book but focus attention on the book rather than its author, we might say something like *Moby Dick* was written by Melville. Here we take the direct object of Melville wrote *Moby Dick* and move it to the front of the sentence by changing the verb from active to passive voice. Passive voice can be used similarly to move more than direct objects to the front, but it has limitations, as do many of the other devices English has for making noun phrases into subjects. The use of is such that—which we will call *expansion*—enables us to make a great variety and arbitrary number of noun phrases into the subject of a sentence. This phrase is written after the subject and is itself followed by the result of replacing the noun phrases in the original sentence by pronouns or pronoun-like devices. For example, Melville wrote *Moby Dick* can be converted into any of

Moby Dick is such that (Melville wrote it)
Melville is such that (he wrote *Moby Dick*)
Melville and *Moby Dick* are such that (the former wrote the latter)

The result of expansion is an *expanded form*, and we will often write it, as has been done here, with the residue of the original sentence in parentheses. When we need to distinguish among alternative ways of expanding a sentence, we will speak of expanding on a particular noun phrase. The opposite of expansion is *reduction*, and we will describe the original sentence as being in *reduced form* relative to that expansion. The idea of reduced form is relative because, in principle, expansion can be applied more than once, and a reduced form may be reduced still further. For example, the first expanded form above is also the result of reducing *Moby Dick* is such that (Melville is such that (he wrote it)).

Expansion will serve us in a number of different ways in the rest of the course. For now, the fact that it uses pronouns and is analogous to the use of abstracts will help in using abstracts to analyze the role of pronouns in a sentence. To see how, let us analyze the sentence Bill told Ann his name in a way that employs a predicate abstract to reflect the use of a pronoun.

Bill told Ann his name
<u>Bill</u> is such that (he told Ann his name)
$[\, x \text{ told } \underline{\text{Ann}}\ \underline{x\text{'s name}}\,]_x\ \underline{\text{Bill}}$
$[\, [\, \_ \text{ told } \_\ \_\,]\ x\ \underline{\text{Ann}}\ \underline{x\text{'s name}}\,]_x\ \underline{\text{Bill}}$
$[Txa([\, \_\text{'s name}]x)]_x\ \underline{\text{Bill}}$

$$[Txa(nx)]_x b$$

T: [_ told _ _]; n: [_'s name]; a: Ann; b: Bill

Once the sentence as a whole has been analyzed as the predication of an abstract, the formula x told Ann x's name that is the body of the abstract is analyzed in the same way as Bill told Ann Bill's name would be. The final analysis departs from the original sentence in having the equivalents of two pronouns instead of one (as does Bill is such that (he hold Ann his name), but it is like the original in having only a single occurrence of Bill. So, in this respect, it is closer to the English than the alternative analysis as Tna(nb), which is what we would get if we analyzed the sentence, Bill told Ann Bill's name, that is the result of replacing the pronoun his by its antecedent. It is in this way that expansion and analysis by abstracts reflects the use of pronouns.

We might also have expanded on both Bill and Ann to get Bill and Ann are such that he told her his name, with the analysis

$$[Txy(nx)]_{xy}ba$$

That would have added no enlightenment in the case of this sentence, but consider the following ambiguous sentence, given with abbreviated analyses of two interpretations of it. (Imagine that the second concerns a case of amnesia.)

Bill told Al his name	Bill told Al his name
Bill and Al are such that (the former told the latter the former's name)	Bill and Al are such that (the former told the latter the latter's name)
[x told y x's name $]_{xy}$ Bill Al	[x told y y's name $]_{xy}$ Bill Al
$[Txy(nx)]_{xy}bl$	$[Txy(ny)]_{xy}bl$

T: [_ told _ _]; n: [_'s name]; b: Bill; l: Al

In each of these analyses, the names Bill and Al are separated completely from the abstracts, which use variables to show any patterns of coreference. The advantage of this sort of analysis is that it gives us an account of the ambiguity of this sentence that enables us to point to the same ambiguity in other sentences, such as Barb told Ann her name.

6.2.4. Expanded and reduced forms

We will use the ideas of expansion and reduction and of expanded and reduced form in connection with symbolic analyses as well as English sentences. (The context will usually indicate which use of the terms is intended but, if necessary, we can speak of symbolic expansion on the one hand or expansion using such that on the other.) As it applies to symbolic analyses, expansion is the process of restating an analysis using an abstract as we did when we moved from the analysis Tbab of Bill told Ann his name to $[Txa(nx)]_{x}b$.

From one point of view, there is no need to use expansion to study the ambiguity of Bill told Al his name. A pair of simple reduced forms like Tba(nb) and Tba(na) is quite sufficient. And even the point that Barb told Ann her name shares the same ambiguity can be captured by referring to a pair of logical forms $T\tau\upsilon(n\tau)$ and $T\tau\upsilon(n\upsilon)$ that are exhibited by each of the two pairs.

Of course, this simpler approach would ignore the fact that the ambiguity lies in the pattern of coreference marked by anaphoric pronouns. Abstracts capture this, but in a rather crude way since they introduce extra pronouns to do so. While the English sentence Bill told Al his name has a single pronoun, our analyses each had three bound variables. The notation could be modified to be more subtle if our main interest was in anaphoric pronouns with individual terms as antecedents. However, the prime application of abstracts will be in later chapters where we will use abstracts in connection with our analysis of quantifier phrases.

In order to analyze a sentence as a truth-functional compound, we must be able to identify components that function independently. In particular, a pronoun in one component cannot have its antecedent in another. The approach we took before employing abstracts was to simply replace a pronoun by its antecedent when this was possible and avoid analysis when it was not. The prime example of a pronoun we could not replace is one whose antecedent is a quantifier phrase. The sort of analysis we will eventually use in this case employs abstracts behind the scenes, and the use of abstracts for cases where pronouns have individual terms as antecedents brings those cases closer to our handling of cases where the antecedents are quantifier phrases.

Still, one of the key points to be made about abstracts with regard to individual terms is the very fact that they are dispensible, so let us look more closely at how to dispense with them once we have used them in an analysis. For example, consider the sentence Ann visited the class and she spoke to Davie. If we use an abstract to capture the coreference of she and Ann, we can analyze this as follows:

Ann visited the class and she spoke to Davie

Ann is such that (she visited the class and she spoke to Davie)

[_ is such that (she visited the class and she spoke to Davie)] Ann

[x visited the class and x spoke to Davie $]_x$ Ann

[x visited the class ∧ x spoke to Davie $]_x$ Ann

[[_ visited _] x the class ∧ [_ spoke to _] x Davie $]_x$ Ann

$$[Vxc \land Sxd]_x a$$

what both Vxc and Sxd says of x fits a

S: [_ spoke to _]; V: [_ visited _]; a: Ann; c: the class; d: Davie

The formula x visited the class and x spoke to Davie can be analyzed as a truth-functional compound because the two occurences of the variable x are independent of each other (though each is bound to the abstractor).

The approach we used earlier would have led us to analyze the sentence as the compound Ann visited the class ∧ Ann spoke to Davie in which she is replaced by Ann, and this sentence would receive a symbolic analysis of the form Vac ∧ Sad. Now, if we compare the symbolic analyses

$$[Vxc \land Sxd]_x a \qquad Vac \land Sad$$

we can see that the second is the result of putting the term a in place of the variable x in the body of the abstract in the first. That is, the second is the reduced form of the first.

When we reduce the predication of an abstract, we take the body of the abstract and put the term of which it is predicated in the blanks marked by the variable. An analogous description applies to the reduction of compound terms formed by applying functor abstracts, and the description can be extended to apply to abstracts on any number of variables. Schematically, the general pattern is as follows:

$$[\text{---}x_1\text{---}\ldots\text{---}x_n\text{---}]_{x_1\ldots x_n}\tau_1\ldots\tau_n \qquad \text{---}\tau_1\text{---}\ldots\text{---}\tau_n\text{---}$$

When interpreting the schema, remember that the variables of the abstractor can appear in the body in any order and may each appear any number of times (including not at all). The expression on the right is the result of using each term τ_i to replace all occurrences of the corresponding variable.

Special care is needed when performing a reduction if the body contains abstracts and a term to be substituted contains free variables. The short account of this sort of case is that no free variable should become bound as a result of reduction and that abstracts should be replaced by alphabetic variants as necessary to avoid this happening. The easiest way to insure this is to choose bound variables so that they are all different from each other and from any free variables. However, our use of abstracts will be limited to much simpler situations, so a detailed rule is not important. Moreover, we will regard reduced and expanded expressions as two ways of writing the same formula or term, so no rule at all is needed as part of our rules for derivations, where sentences will be written only in fully reduced form.

Let us now return to the issue of pronouns and truth-functional connectives. From our present point of view, the fact that pronouns can always be replaced by individual term antecedents can be seen as the result of the fact the reduction is always possible. The analyses of sentences involving quantifier phrases that we will go on to develop in the next couple of chapters will employ predicate abstracts but not by way of predication, so nothing analogous to reduction will be in question. That can be cited as the reason a pronoun often cannot be replaced by a quantifier phrase antecedent—as in A mother visited the class and she spoke to Davie, which is not equivalent to A mother visited the class and a mother spoke to Davie. In cases where replacement by a quantifier phrase antencedent is possible without changing the meaning—as in A mother visited the class or she spoke to Davie on the phone—this will be due to special interactions between the quantifier phrase and other logical constants in the sentence.

Finally, although our focus has been on pronouns, much of what we have seen applies also to sentences containing compound predicates and other compound phrases. The sentence Ann visited the class and spoke to Davie can also be analyzed as $[Vxc \land Sxd]_x a$. While this analysis introduces the symbolic analogues pronouns that do not appear in the English, it does capture the form of the English in one respect: it treats it as a predication whose predicate contains the connective. And the possibility of restating the sentence as Ann visited the class and Ann spoke to Davie can be seen as due to the reduction of this form to Vac ∧ Sad.

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6.2.s. Summary

1 We adapt the notation of lambda abstraction to provide a flexible way of linking the places of a predicate to blanks in an English sentence. An expression formed using our notation

$$[\ldots x_1 \ldots x_n \ldots]_{x_1 \ldots x_n}$$

is an abstract (in this use, a predicate abstract); it consists of a abstractor applied to a bracketed body. In English notation, a predicate abstract takes the form what … x_1… x_n … says of x_1 … x_n, and a functor abstract takes the form … x_1… x_n … for x_1 … x_n. (Variables in an abstractor that do not appear in the body are cases of vacuous abstraction.)

2 A variable in the body of an abstract that appears in an abstractor is bound to it, provided it is not already bound to one with narrower scope. Bound variables may be thought of as pronouns whose antecedents are in the abstractor. Expressions that establish the same patterns of binding using different variables are alphabetic variants. A expression that has variables not bound to any abstractor (such as the body of an abstract considered by itself) is open; otherwise, it is closed. A sentence-like expression that is open is not a sentence in the strict sense, but it does count as a formula. Formulas have many of the syntactic properties of sentences; in particular, they can be built from other formulas using connectives. And we can distinguish as atomic formulas not only unanalyzed sentences but all formulas that are predictions. (Indeed, unanalyzed sentences can be thought of as predications of zero-place predicates.)

3 Many pronouns in English function like the bound variables of the symbolic notation for abstracts, and the phrase is such that can be used to expand an English sentence by introducing them. The resulting expanded form is analogous to the predication of an abstract and can be reduced to a sentence in which the pronouns introduced by expansion are replaced by their antecedents. Because of the analogy between variables and anaphoric pronouns, abstracts can be used to represent the contribution of such pronouns to logical form.

4 Processes analogous to the expansion and reduction of English sentences apply to symbolic forms. In the simplest case, the application of an abstract can be reduced by replacing variables bound to it by the terms filling the corresponding places of the predicates. And a symbolic form may be expanded to introduce the predication of an abstract. Both operations help in comparing sentences in reduced form to logical forms studied in later chapters in which abstracts appear in contexts other than predication.

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6.2.x. Exercise questions

1. Expand each of the following in two different ways, (i) on a single occurrence of a single individual term, and (ii) on all terms together. In each case express the expanded form in English using is such that and in a partially symbolic way, as in

[x wrote *Moby Dick*]$_x$ Melville

- **a.** Romney is north of Linden.
- **b.** Mike gave the package to Nancy.
- **c.** Tom spoke of Ed to Sue.
- **d.** Sam traveled to Atlanta by way of Chicago.

2. Analyze each of the following in a way that uses abstracts and variables to represent pronouns instead of replacing them by their antecedents. Since you will not replace pronouns by their antecedents, you should end up with as many occurrences of each individual term in your result as in the original sentence. Also, restate your symbolic analysis in reduced form.

- **a.** Ann nominated herself
- **b.** Ralph tried the motor, and it started
- **c.** If the alarm is touched, it will go off
- **d.** Ralph fixed Sam's car, and he drove it back to him
- **e.** Ann and Bill each left a message for the other

3. Each of the following sentences exhibits an ambiguity (in pronoun reference) between meanings that can be indicated by alternative analyses using abstracts. Use abstracts to give two complete analyses of each sentence that express different interpretations of it. You will find it easier to distinguish interpretations if you expand for all terms involved in the ambiguity whether or not all have pronouns referring to them on each interpretation (see the last example of 6.2.3).

In **c**, the word so serves to apply the same predicate to Bill as was applied to Al, so each of your analyses of it should have a repeated abstract.

- **a.** Al called Bill, and he called Carol.
- **b.** Sam gave the book to Tom, but he didn't read it.
- **c.** Al washed his car, and so did Bill.

4. For each of the following abstracts (i) diagram the pattern of binding using lines rather than variables (in the manner shown in 6.2.2) and (ii) give an alphabetic variant (i.e., abstract which indicates the same pattern of binding using different variables).

In the case of **e**, remember that, as noted in 6.2.2, a bracketed sentence-with-blanks amounts to an abstract whose body has a different variable in each blank and whose abstractor lists the variables in the same order. Also, the lower-case f in **c** means that it is a functor rather than a predicate; but that won't make for any differences in the way you handle it.

- **a.** $[Fx]_x$
- **b.** $[Fz \rightarrow Gz]_z$
- **c.** $[Tyxy]_{xy}$
- **d.** $[fyz]_{zy}$
- **e.** [S _ _ _]
- **f.** $[[Rxy]_x a \land Rby]_y$
- **g.** $[[Rcy]_y a \land Rby]_y$

6.2.xa. Exercise answers

1. In each case, the English restatement appears first, followed by the partial symbolization.

a. **i.** Romney is such that (it is north of Linden)
[x is north of Linden]$_x$ Romney
or Linden is such that (Romney is north of it)
[Romney is north of x]$_x$ Linden
ii. Romney and Linden are such that (the former is north of the latter)
[x is north of y]$_{xy}$ Romney Linden

b. **i.** Mike is such that (he gave the package to Nancy)
[x gave the package to Nancy]$_x$ Mike
or the package is such that (Mike gave it to Nancy)
[Mike gave x to Nancy]$_x$ the package
or Nancy is such that (Mike gave the package to her)
[Mike gave the package to x]$_x$ Nancy
ii. Mike, the package, and Nancy are such that (the first gave the second to the third)
[x gave y to z]$_{xyz}$ Mike the package Nancy

c. **i.** Tom is such that (he spoke of Ed to Sue)
[x spoke of Ed to Sue]$_x$ Tom
or Ed is such that (Tom spoke of him to Sue)
[Tom spoke of x to Sue]$_x$ Ed
or Sue is such that (Tom spoke of Ed to her)
[Tom spoke of Ed to x]$_x$ Sue
ii. Tom, Ed, and Sue are such that (the first spoke of the second to the third)
[x spoke of y to z]$_{xyz}$ Tom Ed Sue

d. **i.** Sam is such that (he traveled to Atlanta by way of Chicago)
[x traveled to Atlanta by way of Chicago]$_x$ Sam
or Atlanta is such that (Sam traveled to it by way of Chicago)
[Sam traveled to x by way of Chicago]$_x$ Atlanta
or Chicago is such that (Sam traveled to Atlanta by way of it)
[Sam traveled to Atlanta by way of x]$_x$ Chicago
ii. Sam, Atlanta, and Chicago are such that (the first traveled to the second by way of the third)
[x traveled to y by way of z]$_{xyz}$ Sam Atlanta Chicago

2. **a.** Ann nominated herself
Ann is such that (she nominated herself)
[x nominated x]$_x$ Ann

$$[Nxx]_x a$$
reduced form: Naa

N: [_ nominated _]; a: Ann

b. Ralph tried the motor, and it started
the motor is such that (Ralph tried it, and it started)
[Ralph tried x and x started]$_x$ the motor
[Ralph tried x ∧ x started]$_x$ m

$$[Trx \land Sx]_x m$$
reduced form: Trm ∧ Sm

S: [_ started]; T: [_ tried _]; m: the motor; r: Ralph

The analysis $[Txy \land Sy]_{xy}rm$ is also correct, but a 2-place abstract is not needed in order to analyze pronouns since only the motor has a pronoun referring to it.

c. If the alarm is touched, it will go off
the alarm is such that (if it is touched, it will go off)
[if x is touched, x will go off]$_x$ the alarm
[x will be touched → x will go off]$_x$ a

$$[Tx \to Gx]_x a$$
reduced form: Ta → Ga

T: [_ will be touched]; G: [_ will go off]; a: the alarm

d. Ralph fixed Sam's car, and he drove it back to him
Ralph and Sam are such that (the former fixed the latters's car, and he drove it back to him)
[x fixed y's car, and x drove y's car back to y]$_{xy}$ Ralph Sam
[x fixed y's car ∧ x drove y's car back to y]$_{xy}$rs
[Fx(y's car) ∧ Dx(y's car)y]$_{xy}$rs

$$[Fx(cy) \land Dx(cy)y]_{xy} rs$$
reduced form: Fr(cs) ∧ Dr(cs)s

D: [_ drove _ back to _]; F: [_ fixed _]; c: [_'s car]; r: Ralph; s: Sam

e. Ann and Bill each left a message for the other
Ann and Bill are such that (they each left a message for the other)
[x and y each left a message for the other]$_{xy}$ Ann Bill
[x left a message for y ∧ y left a message for x]$_{xy}$ab

$$[Mxy \land Myx]_{xy} ab$$
reduced form: Mab ∧ Mba

M: [_ left a message for _]; a: Ann; b: Bill

The noun phrase a message is a quantifier phrase rather than an individual term so it must be left unanalyzed.

3. **a.** **i.** Al called Bill, and he called Carol

Al and Bill are such that (the former called the latter, and the former called Carol)

$[x$ called y, and x called Carol$]_{xy}$ Al Bill

$[x$ called $y \wedge x$ called Carol$]_{xy}ab$

$$[Cxy \wedge Cxc]_{xy}ab$$

ii. Al called Bill, and he called Carol

Al and Bill are such that (the former called the latter, and the latter called Carol)

$[x$ called y, and y called Carol$]_{xy}$ Al Bill

$[x$ called $y \wedge y$ called Carol$]_{xy}ab$

$$[Cxy \wedge Cyc]_{xy}ab$$

C: [_ called _]; a: Al; b: Bill; c: Carol

The second interpretation can be indicated in spoken English by emphasizing the pronoun. The first interpretation could be indicated unambiguously by adding too to the end of the sentence.

b. **i.** Sam gave the book to Tom, but he didn't read it

Sam, the book and Tom are such that (the first gave the second to the third, but the first didn't read the second)

$[x$ gave y to z, but x didn't read $y]_{xyz}$ Sam the book Tom

$[x$ gave y to $z \wedge \neg\, x$ read $y]_{xyz}sbt$

$$[Gxyz \wedge \neg\, Rxy]_{xyz}sbt$$

ii. Sam gave the book to Tom, but he didn't read it

Sam, the book and Tom are such that (the first gave the second to the third, but the third didn't read the second)

$[x$ gave y to z, but z didn't read $y]_{xyz}$ Sam the book Tom

$[x$ gave y to $z \wedge \neg\, z$ read $y]_{xyz}sbt$

$$[Gxyz \wedge \neg\, Rzy]_{xyz}sbt$$

G: [_ gave _ to _]; R: [_ read _]; b: the book; s: Sam; t: Tom

It is hard to avoid this ambiguity in English without some rewording—e.g., by resorting to the former or the latter instead of he or by repeating one of the names.

c. **i.** Al washed his car, and so did Bill.

Al and Bill are such that (the former washed his car, and so did the latter).

$[x$ washed his car, and so did $y]_{xy}$ Al Bill

$[x$ is such that (he washed his car) $\wedge\ y$ is such that (he washed his car)$]_{xy}ab$

$[[z$ washed z's car$]_z x \wedge [z$ washed z's car$]_z y]_{xy}ab$

$[[Wz($z's car$)]_z x \wedge [Wz($z's car$)]_z y]_{xy}ab$

$$[[Wz(cz)]_z x \wedge [Wz(cz)]_z y]_{xy}ab$$

ii. Al washed his car, and so did Bill.

Al and Bill are such that (the former washed his car, and so did the latter).

$[x$ washed his car, and so did $y]_{xy}$ Al Bill

$[x$ is such that (he washed x's car) $\wedge\ y$ is such that (he washed x's car)$]_{xy}ab$

$[[z$ washed x's car$]_z x \wedge [z$ washed x's car$]_z y]_{xy}ab$

$[[Wz($x's car$)]_z x \wedge [Wz($x's car$)]_z y]_{xy}ab$

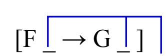
$$[[Wz(cx)]_z x \wedge [Wz(cx)]_z y]_{xy}ab$$

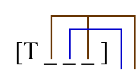
W: [_ washed _]; c: [_'s car]; a: Al; b: Bill

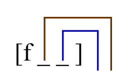
The abstracts here serve two different purposes. The one with largest scope is used to analyze the patterns of co-reference while the two inside its body are designed to capture the function of so did. The ambiguity in the sentence arises because the sameness claimed for Al's and Bill's actions might suggest washing a car related to the washer in the same way (the first interpretation) or, indeed, washing the very same car (the second interpretation). In particular, it's the difference between the idea of washing one's own car—i.e., $[Wz(cz)]_z$—and washing the car of someone, x—i.e., $[Wz(cx)]_z$—someone who, in this case, is the first person to whom the predicate is applied. It is the function of the abstract with wider scope to capture this idea of a reference to the first person to whom the predicate is applied.

4. Since each abstract has many (indeed, infinitely many) alphabetic variants, the answers **(ii)** below are only examples.

a. **i.**  **ii.** $[Fy]_y$

b. **i.**  **ii.** $[Fx \rightarrow Gx]_x$

c. **i.**  **ii.** $[Txyx]_{yx}$

d. **i.**  **ii.** $[fzx]_{xz}$

e. **i.** [S _ _ _] **ii.** $[Sxyz]_{xyz}$

f. **i.** [[R _ _] a ∧ Rb _] **ii.** $[[Ryx]_y a \wedge Rbx]_x$

g. **i.** [[R _] a ∧ Rb _] **ii.** $[[Rcx]_x a \wedge Rbz]_z$

In the original abstract for **(g)**, $[[Rcy]_y a \wedge Rby]_y$, the variable y in Rcy falls in the scope of two abstractors for y. It is bound to the one with narrower scope, so the one with wider scope binds only the y in Rby. The pattern in **(i)** shows that the variable in the first abstract is thoroughly “apparent” from the point of the abstractor with wider scope: since the latter binds no variables in the first abstract, it does not matter whether that abstract uses the same variable as it does or a different one. In **(f)**, on the other hand, the two abstractors must use different variables since one binds variables in the scope of the other.

Glen Helman 13 Oct 2011