Overview

Basic system

Exploitation and planning rules		
sentence	*as a resource*	*as a goal*
atomic sentence	*none*	IP
negation $\neg\varphi$	CR (if φ not atomic & goal is $\bot$)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \rightarrow \psi$	RC (if goal is $\bot$)	CP
universal $\forall x\ \theta x$	UI	UG
existential $\exists x\ \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and $\neg\varphi$	$\bot$	Nc
	any	$\top$	ENV
	$\bot$	*any*	EFQ
τ—υ	*any*	$\tau = \upsilon$	EC
τ—υ	$\neg\,\tau = \upsilon$	$\bot$	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1\ldots\tau_n$	$P\upsilon_1\ldots\upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1\ldots\tau_n$ $\neg\,P\upsilon_1\ldots\upsilon_n$	$\bot$	Nc=

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
$\neg(\varphi \wedge \psi)$	φ or ψ	MPT
$\varphi \vee \psi$	$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$	MTP
$\varphi \rightarrow \psi$	φ $\neg^{\pm}\psi$	MPP MTT

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

Additional rules

Attachment rules	
resource to be added	*rule*
$\varphi \wedge \psi$	Adj
$\neg(\varphi \wedge \psi)$	Wk
$\varphi \vee \psi$	Wk
$\varphi \rightarrow \psi$	Wk
$\tau = \upsilon$	CE
$\theta\upsilon_1\ldots\upsilon_n$	Cng
$\exists x\ \theta x$	EG

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	LFR

Derivation rules

Basic system

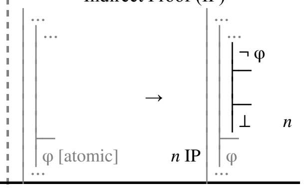
Rules for developing gaps

logical form | *as resource* | *as goal*

atomic sentence

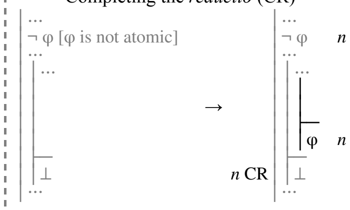
As resource: *no rule*

As goal: Indirect Proof (IP)

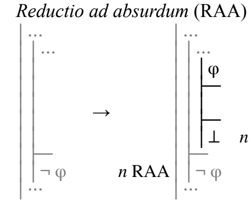


negation $\neg\varphi$

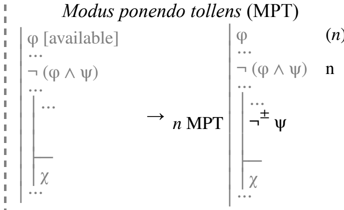
As resource: Completing the *reductio* (CR)

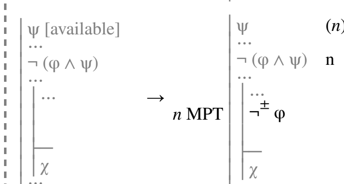


As goal: *Reductio ad absurdum* (RAA)



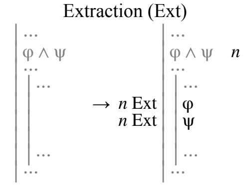
As resource: *Modus ponendo tollens* (MPT)



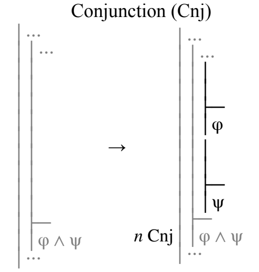


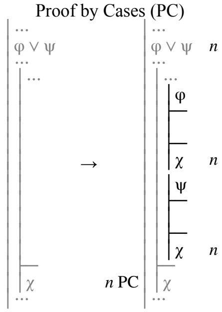
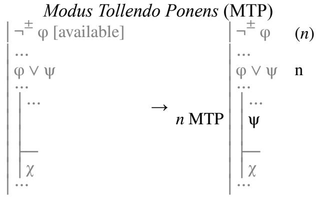
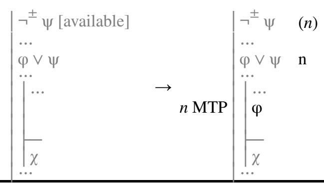
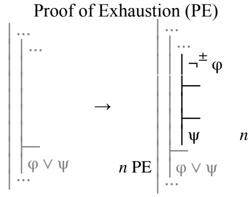
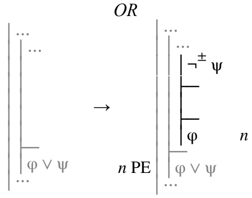
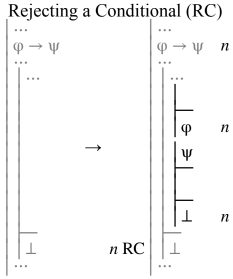
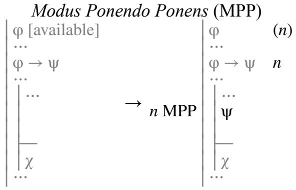
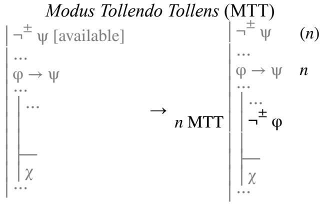
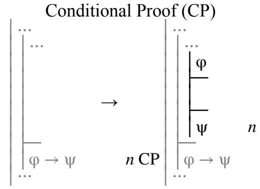
conjunction $\varphi \wedge \psi$

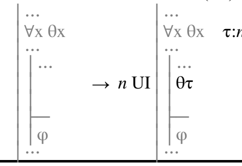
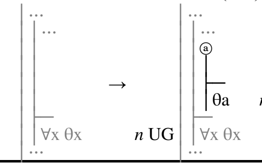
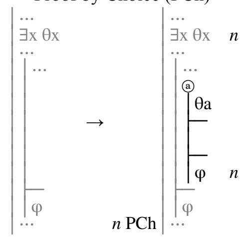
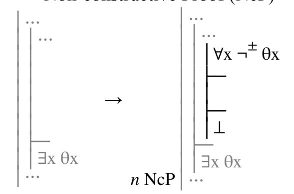
As resource: Extraction (Ext)



As goal: Conjunction (Cnj)



Rules for developing gaps		
logical form	*as resource*	*as goal*
disjunction $\varphi \vee \psi$	Proof by Cases (PC)  *Modus Tollendo Ponens* (MTP)  	Proof of Exhaustion (PE)  *OR* 
conditional $\varphi \to \psi$	Rejecting a Conditional (RC)  *Modus Ponendo Ponens* (MPP)  *Modus Tollendo Tollens* (MTT) 	Conditional Proof (CP) 

Rules for developing gaps		
logical form	*as resource*	*as goal*
universal $\forall x\ \theta x$	Universal Instantiation (UI) 	Universal Generalization (UG) 
existential $\exists x\ \theta x$	Proof by Choice (PCh) 	Non-constructive Proof (NcP) 

The parameter a used in UG and PCh should be new to the derivation; that is, it should appear only to the right of the scope line it labels

Rules for closing gaps (truth-functional logic)

when to close		*rule*
resources	*goal*	
φ	φ	*Quod Erat Demonstrandum* (QED) … φ [available] … │ … │ φ … → … φ (n) … │● │ φ … n QED
φ and ¬ φ	⊥	Non-contradiction (Nc) … ¬ φ [available] … φ [available] … │ … │ ⊥ … → … ¬ φ (n) … φ (n) … │● │ ⊥ … n Nc
any	⊤	*Ex Nihilo Verum* (ENV) … │ … │ ⊤ … → … │● │ ⊤ … n ENV
⊥	*any*	*Ex Falso Quodlibet* (EFQ) … ⊥ … │ … │ φ … → … ⊥ (n) … │● │ φ … n EFQ

Rules for closing gaps (equations)

In addition to the following rules for closing gaps, if the conditions for applying any rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "=" to its label; QED= and Nc= below are examples of this in the case of rules for closing gaps.

when to close			*rule*
co-aliases	*resources*	*goal*	
τ—υ	*any*	τ = υ	Equated Co-aliases (EC) … [τ and υ are co-aliases] … │ … │ τ = υ … → … [τ and υ are co-aliases] … │● │ τ = υ … n EC
τ—υ	¬ τ = υ	⊥	Distinguished Co-aliases (DC) … [τ and υ are co-aliases] … ¬ τ = υ … │ … │ ⊥ … → … [τ and υ are co-aliases] … ¬ τ = υ (n) … │● │ ⊥ … n DC
τ_1—υ_1, ⋯, τ_n—υ_n	P$\tau_1\cdots\tau_n$	P$\upsilon_1\cdots\upsilon_n$	QED given equations (QED=) … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1\cdots\tau_n$ … │ … │ P$\upsilon_1\cdots\upsilon_n$ … → … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1\cdots\tau_n$ (n) … │● │ P$\upsilon_1\cdots\upsilon_n$ … n QED=
τ_1—υ_1, ⋯, τ_n—υ_n	P$\tau_1\cdots\tau_n$ ¬ P$\upsilon_1\cdots\upsilon_n$	⊥	Non-contradiction given equations (Nc=) … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1\cdots\tau_n$ … ¬ P$\upsilon_1\cdots\upsilon_n$ … │ … │ ⊥ … → … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1\cdots\tau_n$ (n) … ¬ P$\upsilon_1\cdots\upsilon_n$ (n) … │● │ ⊥ … n Nc=

Additional rules (not guaranteed to be progressive)

Attachment rules		
what is required	*added resource*	*rule*
φ and ψ are both available	$\varphi \wedge \psi$	Adjunction (Adj) … / φ [available] / … / ψ [available] / … / │ … / │ χ / … $\rightarrow$ n Adj … / φ (n) / … / ψ (n) / … / │ … / │ $\varphi \wedge \psi$ X / │ χ / …
$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$ is available	$\neg(\varphi \wedge \psi)$	Weakening (Wk) … / $\neg^{\pm}\varphi$ [available] / … / │ … / │ χ / … $\rightarrow$ n Wk … / $\neg^{\pm}\varphi$ (n) / … / │ … / │ $\neg(\varphi \wedge \psi)$ X / │ χ / … … / $\neg^{\pm}\psi$ [available] / … / │ … / │ χ / … $\rightarrow$ n Wk … / $\neg^{\pm}\psi$ (n) / … / │ … / │ $\neg(\varphi \wedge \psi)$ X / │ χ / …
φ or ψ is available	$\varphi \vee \psi$	… / φ [available] / … / │ … / │ χ / … $\rightarrow$ n Wk … / φ (n) / … / │ … / │ $\varphi \vee \psi$ X / │ χ / … … / ψ [available] / … / │ … / │ χ / … $\rightarrow$ n Wk … / ψ (n) / … / │ … / │ $\varphi \vee \psi$ X / │ χ / …
$\neg^{\pm}\varphi$ or ψ is available	$\varphi \rightarrow \psi$	$\neg^{\pm}\varphi$ [available] / … / │ … / │ χ / … $\rightarrow$ n Wk $\neg^{\pm}\varphi$ (n) / … / │ … / │ $\varphi \rightarrow \psi$ X / │ χ / … ψ [available] / … / │ … / │ χ / … $\rightarrow$ n Wk ψ (n) / … / │ … / │ $\varphi \rightarrow \psi$ X / │ χ / …

Attachment rules		
what is required	*added resource*	*rule*
τ and υ are co-aliases	$\tau = \upsilon$	Co-alias Equation (CE) … / [τ and υ are co-aliases] / … / │ … / │ φ / … $\rightarrow$ n CE … / [τ and υ are co-aliases] / … / │ … / │ $\tau = \upsilon$ X / │ φ / …
have co-alias relations τ_1—υ_1, …, τ_n—υ_n and $\theta\tau_1\cdots\tau_n$ is available	$\theta\upsilon_1\cdots\upsilon_n$	Congruence (Cng) … / [have co-alias relations: τ_1—υ_1, …, τ_n—υ_n] / … / $\theta\tau_1\cdots\tau_n$ / … / │ … / │ φ / … $\rightarrow$ n Cng … / [have co-alias relations: τ_1—υ_1, …, τ_n—υ_n] / … / $\theta\tau_1\cdots\tau_n$ (n) / … / │ … / │ $\theta\upsilon_1\cdots\upsilon_n$ X / │ φ / …
$\theta\tau$ is available	$\exists x\, \theta x$	Existential Generalization (EG) … / $\theta\tau$ / … / │ … / │ φ / … $\rightarrow$ n EG … / $\theta\tau$ (n) / … / │ … / │ $\exists x\, \theta x$ X / │ φ / …

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	Lemma for *Reductio* (LFR) … / │ … / │ $\bot$ / … $\rightarrow$ … / │ … / │ │ φ n / │ │ │ φ / │ │ │ $\bot$ n / n LFR │ $\bot$ / …