Appendices

Appendix A. Reference

A.0. Overview

A.1. Basic concepts
Definitions of entailment and related ideas

A.2. Logical forms
Forms expressed using one or more logical constants together with symbolic and English notation or readings

A.3. Truth tables
Tables that stipulate the meaning of the constants of truth-functional logic

A.4. Derivation rules
A guide to the use of derivation rules with links to the rules themselves

A.1. Basic concepts

Concept	*Negative definition*	*Positive definition*
φ is *entailed* by Γ $\Gamma \vDash \varphi$	There is no logically possible world in which φ is false while all members of Γ are true.	φ is true in every logically possible world in which all members of Γ are true.
φ and ψ are *(logically) equivalent* $\varphi \simeq \psi$	There is no logically possible world in which φ and ψ have different truth values.	φ and ψ have the same truth value as each other in every logically possible world.
φ is a *tautology* $\vDash \varphi$ (*or* $\top \vDash \varphi$)	There is no logically possible world in which φ is false.	φ is true in every logically possible world.
φ is *inconsistent with* Γ $\Gamma, \varphi \vDash$ (*or* $\Gamma, \varphi \vDash \bot$)	There is no logically possible world in which φ is true while all members of Γ are true.	φ is false in every logically possible world in which all members of Γ are true.
Γ is *inconsistent* $\Gamma \vDash$ (*or* $\Gamma \vDash \bot$)	There is no logically possible world in which all members of Γ are true.	In every logically possible world, at least one member of Γ is false.
φ is *absurd* $\varphi \vDash$ (*or* $\varphi \vDash \bot$)	There is no logically possible world in which φ is true.	φ is false in every logically possible world.
Σ is *rendered exhaustive* by Γ $\Gamma \vDash \Sigma$	There is no logically possible world in which all members of Σ are false while all members of Γ are true.	At least one member of Σ is true in each logically possible world in which all members of Γ are true

A.2. Logical forms

Forms for which there is symbolic notation

	Symbolic notation	English notation *or* English reading	
Negation	¬ φ	not φ	
Conjunction	φ ∧ ψ	both φ and ψ	(φ and ψ)
Disjunction	φ ∨ ψ	either φ or ψ	(φ or ψ)
The conditional	φ → ψ ψ ← φ	if φ then ψ yes ψ if φ	(φ implies ψ) (ψ if φ)
Identity	τ = υ	τ is υ	
Predication	$\theta\tau_1\ldots\tau_n$	θ fits $\tau_1, \ldots, \tau_n$	A series of terms $\tau_1, \ldots, \tau_n$ can be read (series) $\tau_1, \ldots,$ ən τ_n (using the expression ən to distinguish this use of and from its use in conjunction and adding series when necessary to avoid ambiguity)
Compound term	$\gamma\tau_1\ldots\tau_n$	γ of $\tau_1, \ldots, \tau_n$ γ applied to $\tau_1, \ldots, \tau_n$	
Predicate abstract	$[\varphi]_{x_1\ldots x_n}$	what φ says of $x_1\ldots x_n$	
Functor abstract	$[\tau]_{x_1\ldots x_n}$	τ for $x_1\ldots x_n$	
Universal quantification	∀x θx	forall x θx everything, x, is such that θx	
Restricted universal	(∀x: ρx) θx	forall x st ρx θx everything, x, such that ρx is such that θx	
Existential quantification	∃x θx	forsome x θx something, x, is such that θx	
Restricted existential	(∃x: ρx) θx	forsome x st ρx θx something, x, such that ρx is such that θx	
Definite description	Ix ρx	the x st ρx the thing, x, such that ρx	

Some paraphrases of other forms

	Truth-functional compounds
neither φ nor ψ	¬ (φ ∨ ψ) ¬ φ ∧ ¬ ψ
ψ only if φ	¬ ψ ← ¬ φ
ψ unless φ	ψ ← ¬ φ
	Generalizations
All Cs are such that (… they …)	(∀x: x is a C) … x …
No Cs are such that (… they …)	(∀x: x is a C) ¬ … x …
Only Cs are such that (… they …)	(∀x: ¬ x is a C) ¬ … x …
with: among Bs	add to the restriction: x is a B
except Es	¬ x is an E
other than τ	¬ x = τ
	Numerical quantifier phrases
At least 1 C is such that (… it …)	(∃x: x is a C) … x …
At least 2 Cs are such that (… they …)	(∃x: x is a C) (∃y: y is a C ∧ ¬ y = x) (… x … ∧ … y …)
Exactly 1 C is such that (… it …)	(∃x: x is a C) (… x … ∧ (∀y: y is a C ∧ ¬ y = x) ¬ … y …) *or* (∃x: x is a C) (… x … ∧ (∀y: y is a C ∧ … y …) x = y)
	Definite descriptions (on Russell's analysis)
The C is such that (… it …)	(∃x: x is a C ∧ (∀y: ¬ y = x) ¬ y is a C) … x … *or* (∃x: x is a C ∧ (∀y: y is a C) x = y) … x …

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A.3. Truth tables

Tautology

$\top$
T

Absurdity

$\bot$
F

Negation

φ	$\neg\varphi$
T	F
F	T

Conjunction

φ	ψ	$\varphi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional

φ	ψ	$\varphi \to \psi$
T	T	T
T	F	F
F	T	T
F	F	T

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A.4. Derivation rules

Basic system

Rules for developing gaps	*for resources*	*for goals*
atomic sentence		IP
negation $\neg\varphi$	CR (if φ not atomic & goal is $\bot$)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \to \psi$	RC (if goal is $\bot$)	CP
universal $\forall x\, \theta x$	UI	UG
existential $\exists x\, \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and $\neg\varphi$	$\bot$	Nc
		$\top$	ENV
	$\bot$		EFQ
τ—υ		$\tau = \upsilon$	EC
τ—υ	$\neg\, \tau = \upsilon$	$\bot$	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$	$P\upsilon_1 \ldots \upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$, $\neg P\upsilon_1 \ldots \upsilon_n$	$\bot$	Nc=

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
$\varphi \to \psi$	φ	MPP
$\varphi \to \psi$	$\neg^{\pm}\psi$	MTT
$\varphi \vee \psi$	$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$	MTP
$\neg(\varphi \wedge \psi)$	φ or ψ	MPT

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

Additional rules (not guaranteed to be progressive)

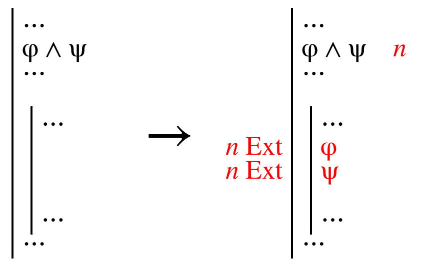
Attachment rules	
added resource	*rule*
$\varphi \wedge \psi$	Adj
$\varphi \to \psi$	Wk
$\varphi \vee \psi$	Wk
$\neg(\varphi \wedge \psi)$	Wk
$\tau = \upsilon$	CE
$\theta\upsilon_1 \ldots \upsilon_n$	Cng
$\exists x\, \theta x$	EG

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	LFR

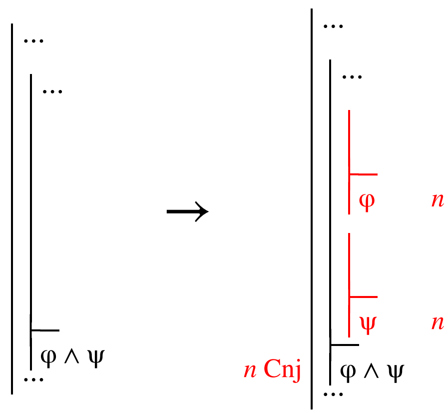
Diagrams

Rules from chapter 2

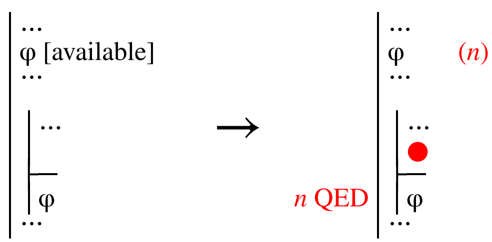
Extraction (Ext)



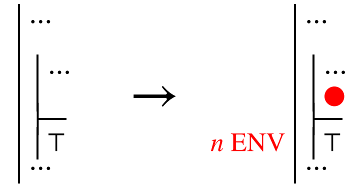
Conjunction (Cnj)



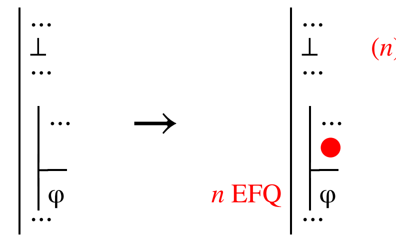
Quod Erat Demonstrandum (QED)



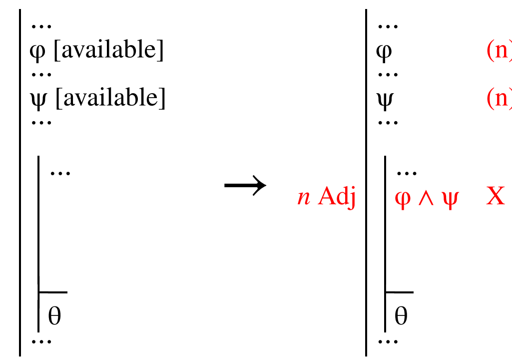
Ex Nihilo Verum (ENV)



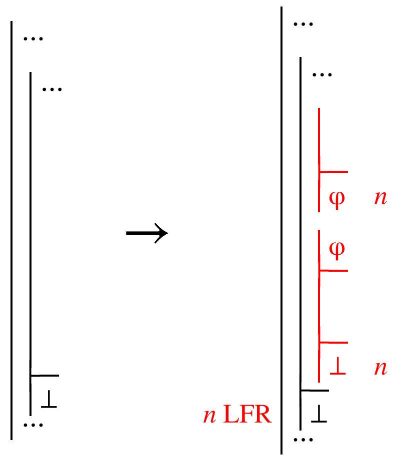
Ex Falso Quodlibet (EFQ)



Adjunction (Adj)

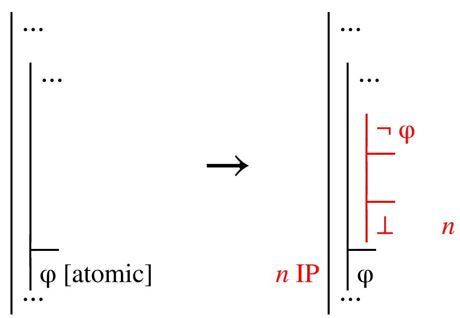


Lemma for *Reductio* (LFR)

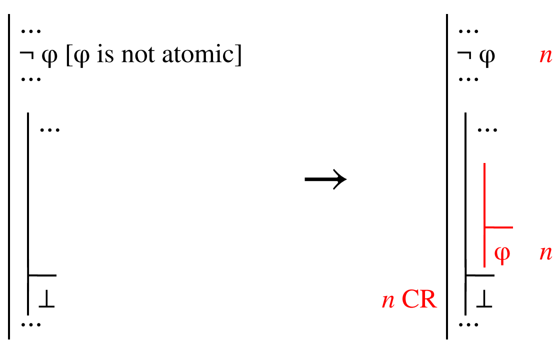


Rules from chapter 3

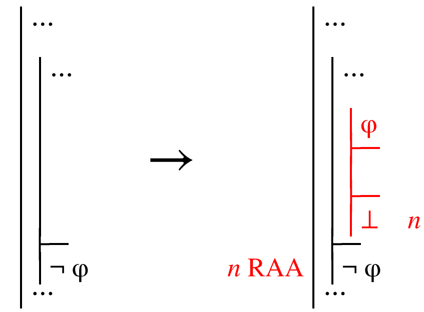
Indirect Proof (IP)



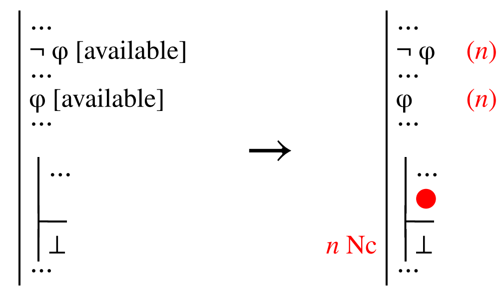
Completing the *Reductio* (CR)



Reductio ad Absurdum (RAA)

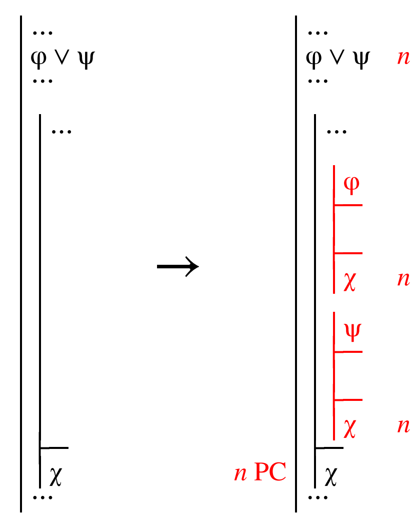


Non-contradiction (Nc)

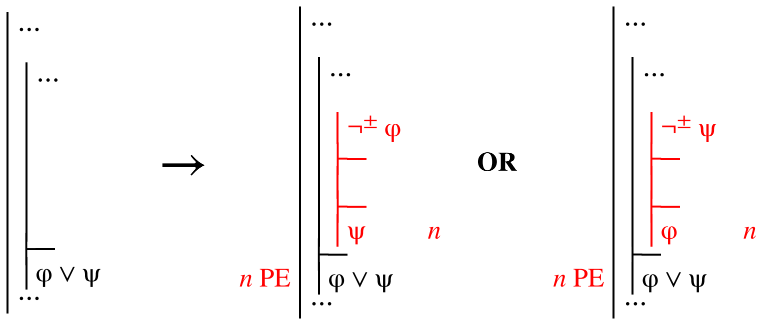


Rules from chapter 4

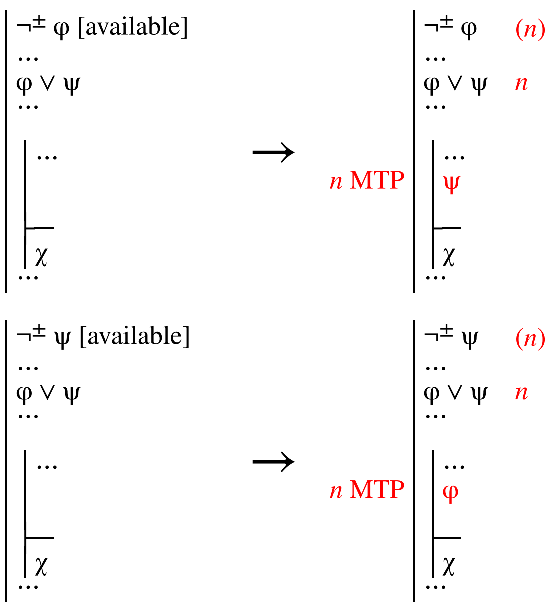
Proof by Cases (PC)



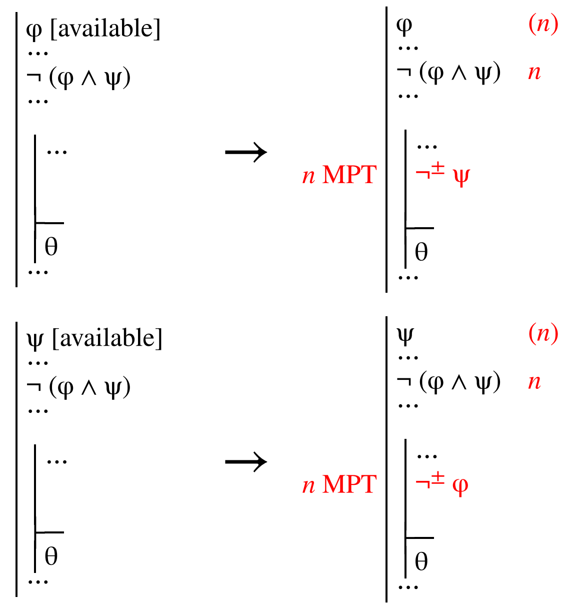
Proof of Exhaustion (PE)



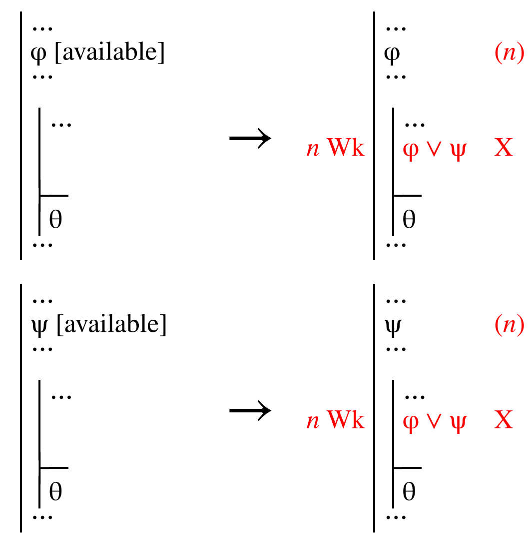
Modus Tollendo Ponens (MTP)



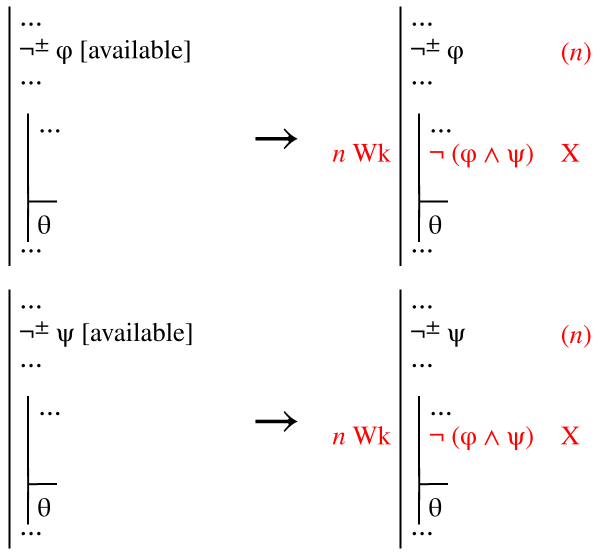
Modus Ponendo Tollens (MPT)



Weakening (Wk)

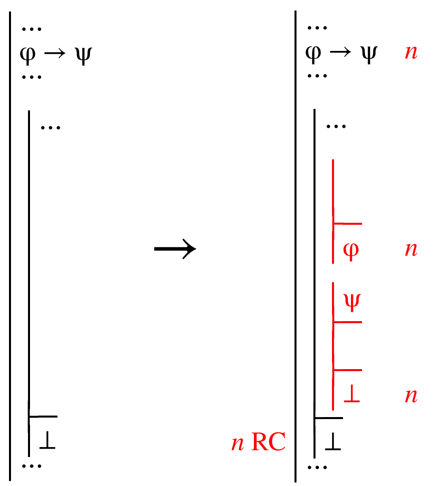


Weakening (Wk)

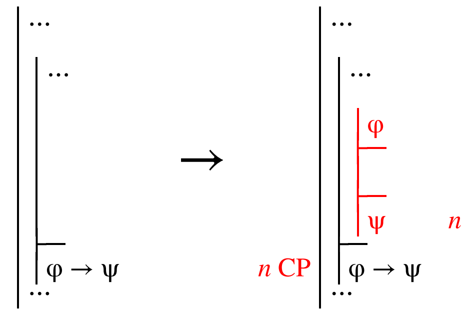


Rules from chapter 5

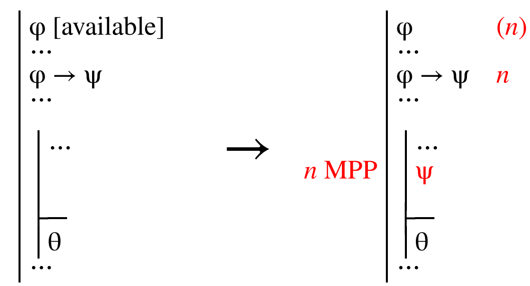
Rejecting a Conditional (RC)



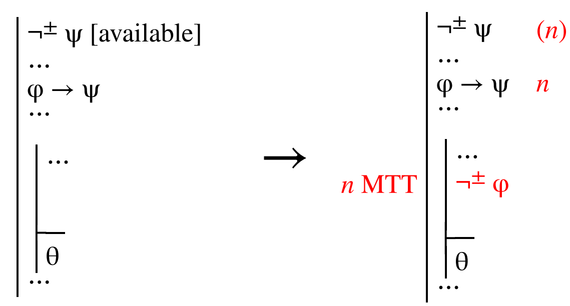
Conditional Proof (CP)



Modus Ponendo Ponens (MPP)



Modus Tollendo Tollens (MTT)



Weakening (Wk)

ψ [available]
…
…
θ
…

→

ψ (n)
…
…
n Wk φ → ψ X
θ
…

Weakening (Wk)

$\neg^{\pm}$ φ [available]
…
…
θ
…

→

$\neg^{\pm}$ φ (n)
…
…
n Wk φ → ψ X
θ
…

Rules from chapter 6

Equated Co-aliases (EC)

…
[τ and υ are co-aliases]
…
…
τ = υ
…

→

…
[τ and υ are co-aliases]
…
…
●
n EC τ = υ
…

Distinguished Co-aliases (DC)

…
[τ and υ
are co-aliases]
…
¬ τ = υ
…
…
⊥
…

→

…
[τ and υ
are co-aliases]
…
¬ τ = υ (n)
…
…
●
n DC ⊥
…

QED given equations (QED=)

…
[$\tau_1 \ldots \tau_n$ and
$\upsilon_1 \ldots \upsilon_n$ are
co-alias series]
…
$P\tau_1 \ldots \tau_n$
…
…
$P\upsilon_1 \ldots \upsilon_n$
…

→

…
[$\tau_1 \ldots \tau_n$ and
$\upsilon_1 \ldots \upsilon_n$ are
co-alias series]
…
$P\tau_1 \ldots \tau_n$ (n)
…
…
●
n QED= $P\upsilon_1 \ldots \upsilon_n$
…

Note: Two series of terms are co-alias series when their corresponding members are co-aliases.

Non-contradiction given equations (Nc=)

…
[$\tau_1 \ldots \tau_n$ and $\upsilon_1 \ldots \upsilon_n$
are co-alias series]
…
¬ $P\tau_1 \ldots \tau_n$
…
$P\upsilon_1 \ldots \upsilon_n$
…
…
⊥
…

→

…
[$\tau_1 \ldots \tau_n$ and $\upsilon_1 \ldots \upsilon_n$
are co-alias series]
…
¬ $P\tau_1 \ldots \tau_n$ (n)
…
$P\upsilon_1 \ldots \upsilon_n$ (n)
…
…
●
n Nc= ⊥
…

Note: Two series of terms are co-alias series when their corresponding members are co-aliases.

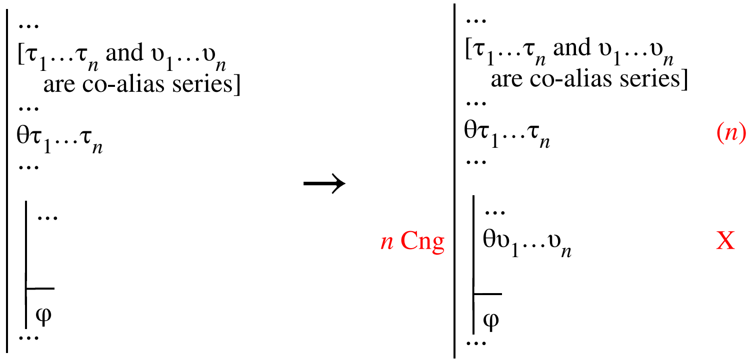
Co-alias Equation (CE)

…
[τ and υ
are co-aliases]
…
…
φ
…

→

…
[τ and υ
are co-aliases]
…
…
n CE τ = υ X
φ
…

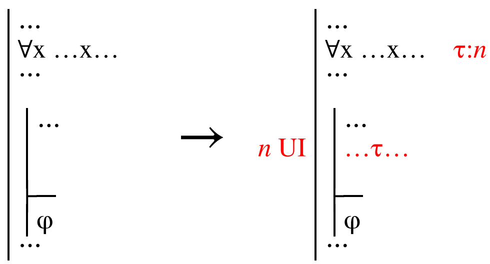
Congruence (Cng)



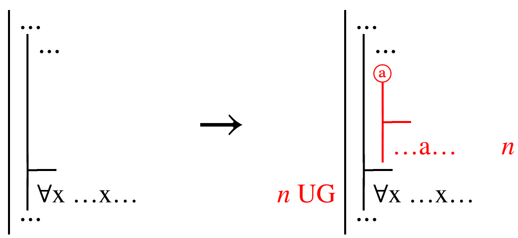
Note: θ can be an abstract, so $\theta\tau_1\ldots\tau_n$ and $\theta\upsilon_1\ldots\upsilon_n$ are any formulas that differ only in the occurrence of terms and in which the corresponding terms are co-aliases.

Rules from chapter 7

Universal Instantiation (UI)

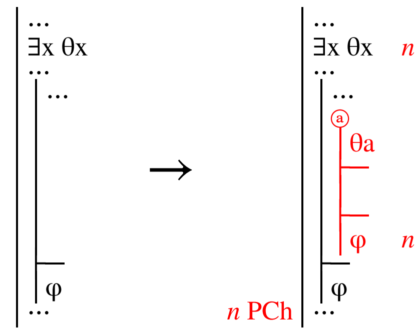


Universal Generalization (UG)

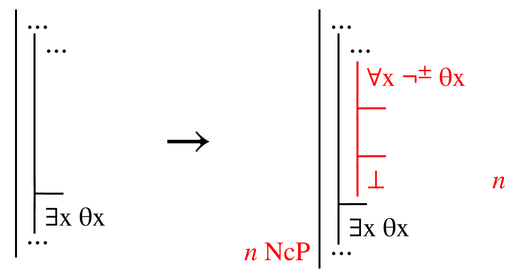


Rules from chapter 8

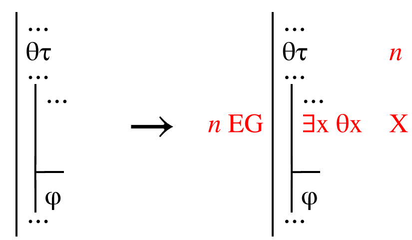
Proof by Choice (PCh)



Non-constructive Proof (NcP)



Existential Generalization (EG)



Glen Helman 28 Oct 2010

Appendix B. Laws for conditional exhaustiveness

Atomic sentences

The first of the following laws is stated only for unanalyzed sentences because laws of the same form for equations and other predications are special cases of the second and third laws:

$\Gamma, A \vDash A, \Sigma$
$\Gamma \vDash \tau = \upsilon, \Sigma$ (where τ and υ are co-aliases given the equations in Γ)
$\Gamma, P\tau_1 \ldots \tau_n \vDash P\upsilon_1 \ldots \upsilon_n, \Sigma$ (where τ_i and υ_i, for i from 1 to n, are co-aliases given the equations in Γ)

Non-atomic sentences

For each logical constant which forms non-atomic sentences, there are two laws, one for cases where it appears among the assumptions and one for cases where it appears among the alternatives.

Constant	*As an assumption*	*As an alternative*
$\top$	$\Gamma, \top \vDash \Sigma$ if and only if $\Gamma \vDash \Sigma$	$\Gamma \vDash \top, \Sigma$
$\bot$	$\Gamma, \bot \vDash \Sigma$	$\Gamma \vDash \bot, \Sigma$ if and only if $\Gamma \vDash \Sigma$
$\neg$	$\Gamma, \neg\varphi \vDash \Sigma$ if and only if $\Gamma \vDash \varphi, \Sigma$	$\Gamma \vDash \neg\varphi, \Sigma$ if and only if $\Gamma, \varphi \vDash \Sigma$
$\wedge$	$\Gamma, \varphi \wedge \psi \vDash \Sigma$ if and only if $\Gamma, \varphi, \psi \vDash \Sigma$	$\Gamma \vDash \varphi \wedge \psi, \Sigma$ if and only if both $\Gamma \vDash \varphi, \Sigma$ and $\Gamma \vDash \psi, \Sigma$
$\vee$	$\Gamma, \varphi \vee \psi \vDash \Sigma$ if and only if both $\Gamma, \varphi \vDash \Sigma$ and $\Gamma, \psi \vDash \Sigma$	$\Gamma \vDash \varphi \vee \psi, \Sigma$ if and only if $\Gamma \vDash \varphi, \psi, \Sigma$
$\rightarrow$	$\Gamma, \varphi \rightarrow \psi \vDash \Sigma$ if and only if both $\Gamma \vDash \varphi, \Sigma$ and $\Gamma, \psi \vDash \Sigma$	$\Gamma \vDash \varphi \rightarrow \psi, \Sigma$ if and only if $\Gamma, \varphi \vDash \psi, \Sigma$
$\forall$	$\Gamma, \forall x\, \theta x \vDash \Sigma$ if and only if $\Gamma, \forall x\, \theta x, \theta\tau \vDash \Sigma$	$\Gamma \vDash \forall x\, \theta x, \Sigma$ if and only if $\Gamma \vDash \theta\alpha, \Sigma$
$\exists$	$\Gamma, \exists x\, \theta x \vDash \Sigma$ if and only if $\Gamma, \theta\alpha \vDash \Sigma$	$\Gamma \vDash \exists x\, \theta x, \Sigma$ if and only if $\Gamma \vDash \theta\tau, \exists x\, \theta x, \Sigma$
where τ is any term and α is independent in the sense that it does not appear in θ, Γ, or Σ		