5. Conditionals

5.1. If: trimming content

5.1.0. Overview

The last connective we will consider is an asymmetric one whose asymmetry gives it an important role in deductive reasoning.

5.1.1. Conditions
In its simplest form, the conditional trims the content of one component by limiting the worlds it rules out to ones that the other component leaves open.

5.1.2. The conditional as a truth-functional connective
The trimming of content is naturally described by an asymmetric truth table.

5.1.3. Doubts about truth-functionality
The truth table just associated with the condition has been controversial since antiquity because the conditional is closely associated with certain implicatures that can seem to add further content.

5.1.4. Examples
The chief task in analyzing the English conditionals marked by if alone is to assign the correct order to the components.

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5.1.1. Conditions

The use of or is not the only way of hedging what we say. Instead of hedging a claim by offering an alternative, we can limit what we rule out to a certain range of possibilities. For example, instead of saying It will rain tomorrow, a forecaster might say It will rain tomorrow if the front moves through. The subordinate clause if the front moves through limits the forecaster's commitment to rain tomorrow to cases where the front does move through. If it does not move through, the forecaster's prediction cannot be faulted even if it does not rain.

We will refer to the connective marked by if as the *(if-)conditional* and to sentences of the form ψ if φ as *(if-)conditionals*. The qualification if- is used here to distinguish this connective from connectives associated with only if and unless that we will consider in 5.2. The three connectives are closely related, we will refer to all three as *conditionals*. However, the if-conditional is the most important of the three we will consider, and a reference to "the conditional" without qualification will be to it. Outside of contexts where we are discussing several sorts of conditional sentence, a reference to "conditionals" will be to the various compounds formed using it rather than to the three sorts of connective. In fact, we will analyze the other two connectives in a way that makes the if-conditional the main component of the result, so compounds formed using the other two connectives will count as special sorts of if-conditionals.

Although we take the word if, like the words and and or, to mark a two-place connective, it raises somewhat different grammatical issues. Since it is used mainly to join full clauses, there is less often a need to fill out the expressions it joins to get full sentences (though, of course, pronominal reference from one component to another must still be removed). And there are special problems associated with it. The conditional is an asymmetric connective: it makes a difference which component is having its content trimmed and which expresses the condition used to trim the content. For example, there is a considerable difference between the following sentences:

Mike entered the contest if he won the prize
Mike won the prize if he entered the contest.

The first is a truism about contests and merely rules out cases of Mike winning the prize without entering the content. On the other hand, the second suggests confidence in Mike's success and rules out cases where he entered the contest without winning.

Still, no fixed order between the two clauses of a conditional is imposed by English syntax. Like other subordinate clauses, if-clauses can be moved to the beginning of the sentence. Thus the two sentences above could be rephrased, respectively, as the following:

If Mike won the prize, he entered the contest
If Mike entered the contest, he won the prize

Sometimes the word then will precede the main clause when conditionals are stated in this order; but, as the examples above show, this is not necessary.

We will use the asymmetric notation $\rightarrow$ (the *rightwards arrow*) or $\leftarrow$ (the *leftwards arrow*) for the conditional. The subordinate if-clause will contribute the component at the tail of the arrow, and the main clause of a conditional sentence will contribute the component at the head. We will refer to these two components, respectively, as the *antecedent* (i.e., what comes before, in the direction of the arrow) and the *consequent* (what comes after, again in the direction of the arrow).

Since the difference between the conditioned claim and what it is conditional on is marked by the difference between the two ends of the arrow, the order in which we write these components makes no difference provided that the arrow points from the antecedent to the consequent. For example, Adam opened the package if it had his name on it could be written as either of the following:

Adam opened the package $\leftarrow$ the package had Adam's name on it
The package had Adam's name on it $\rightarrow$ Adam opened the package

This means that the reordering of clauses in English can be matched by our symbolic notation, with $\varphi \rightarrow \psi$ corresponding to If φ then ψ and $\psi \leftarrow \varphi$ corresponding to ψ if φ. When we are not attempting to match the word order of English sentence, the rightwards arrow will be the preferred notation, and generalizations about conditionals will usually be stated only for the form $\varphi \rightarrow \psi$.

We will use if φ then ψ as English notation for $\varphi \rightarrow \psi$. Here the word if plays the role of a left parenthesis (as both and either do). We will not often use English notation for the leftwards arrow, but it can help in understanding the relation of the two to have some available. If we are to have anything corresponding to the form $\psi \leftarrow \varphi$, we will put if between the two components, so we need another word to the role of left parenthesis. English usage provides no natural choices, so we will have to be a bit arbitrary. The interjection yes does not disturb the grammar of the surrounding sentence, so it can be easily placed where we want it. So we will write yes ψ if φ as our English notation for the form $\psi \leftarrow \varphi$. This way of tying the words yes and if is not backed up by an intuitive understanding of English, so the yes in the form yes ψ if φ does not help in understanding the symbolic form. On the other hand, it does not interfere with the help that if provides; and, as an interjection, yes can help to mark breaks in a sentence in much the way punctuation does.

On the other hand, the leftwards arrow $\leftarrow$ is the easier of the two to accommodate if we look for a simple English substitute to use along with parentheses, for $\leftarrow$ corresponds directly to if. We will not often need to use English notation with parentheses in the case of conditionals, so finding something for the rightwards arrow $\rightarrow$ is not a pressing practical problem. However, the way this problem is typically solved emphasizes an important point about the conditional

Of course, we cannot use if also for the rightwards arrow. And, even if we were not using if for the leftwards arrow, it would not work for $\rightarrow$ since if in English must precede rather than follow the subordinate clause. And then will not do either since it is if that bears the meaning of the connective in English. The usual approach is to look further afield and employ the word implies. Lacking a better alternative, we will follow this practice and use the word implies (in this typeface) as an English version of $\rightarrow$ to use with parentheses.

There is some danger of confusion in doing this, for we have used implies as a synonym for entails in the case of a single premises, and the signs $\rightarrow$ and $\vDash$ have quite different meanings. In particular, the notation $\varphi \rightarrow \psi$ refers to a sentence that speaks only of the actual world while, in saying that $\varphi \vDash \psi$, we make a claim about all possible worlds. One way to avoid the confusion is to say that $\varphi \rightarrow \psi$ expresses *material implication* while, when saying that $\varphi \vDash \psi$, we express *logical implication*. We will discuss this distinction further in 5.3.1; but, for now, we can note that this terminology is intended to capture a distinction between a claim about what is a matter of fact on the one hand and a claim about logical necessity on the other. And, however we describe the difference, this is a case where the typeface definitely matters, for

φ implies ψ

is the use of an English word to provide an alternative notation for $\varphi \rightarrow \psi$ while

φ implies ψ

is our way of saying in ordinary English what is expressed in notation as $\varphi \vDash \psi$.

To give an example of some of this notation in action, let us return to the idea that a conditional serves to trim the content of its consequent. This can be expressed in symbolic notation as the entailment

$$\psi \vDash \varphi \to \psi$$

which says that the argument ψ / $\varphi \to \psi$ is a valid one. If we use English notation for the conditional, we might express the same entailment as either

$$\psi \vDash \text{if } \varphi \text{ then } \psi$$

or

$$\psi \vDash \varphi \text{ implies } \psi$$

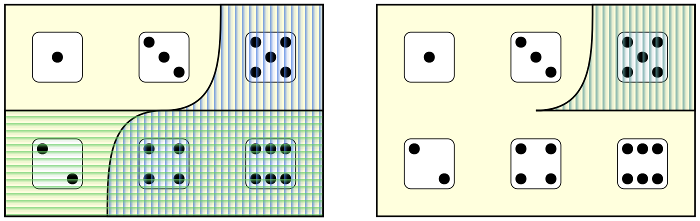
and we express the relation in English, using implies to express entailment, by saying that ψ implies $\varphi \to \psi$, that ψ implies if φ then ψ, or that ψ implies φ implies ψ. Of course, because we have all these options, we have many ways of avoiding potentially confusing expressions; but trying to discern the meaning of a potentially confusing but really unambiguous expression is a good exercise in sorting out the range of concepts we are working with.

5.1.2. The conditional as a truth-functional connective

We have looked at ψ if φ as a way of hedging the claim ψ by limiting our liability, leaving ourselves in danger of error only in cases where φ is true. If this perspective on the conditional is correct, we cannot go wrong in asserting ψ if φ except in cases where ψ is false while φ is true. Thus, the forecaster who predicts that it will rain tomorrow if the front goes through is wrong only if it does not rain even though the front goes through. That suggests that the truth conditions of the conditional are captured by the table below. The only cases where $\varphi \to \psi$ has a chance of being false are those where φ is true; and, in these cases, $\varphi \to \psi$ has the same truth value as ψ.

φ	ψ	$\varphi \to \psi$
T	T	T
T	F	F
F	T	T
F	F	T

This can be seen in another way by diagramming the propositions expressed by conditionals, as in Figure 5.1.2-1. Adapting the example used with this sort of illustration before, 5.1.2-1B represents the proposition expressed by The number shown by the die is less than 4 if it is odd.



A B

Fig. 5.1.2-1. Propositions expressed by two sentences (A) and a conditional (B) whose consequent rules out the possibilities at the right of A.

The possibilities ruled out by the main clause or consequent of the conditional form the hatched region at the right of 5.1.2-1A and those ruled out by the antecedent or condition form the lower half. In 5.1.2-1B, the region at the right is whittled down to the portion containing possibilities left open by the antecedent, showing how the conditional weakens the claim made by the consequent alone (in the example, The number shown by the die is less than 4). Since the consequent is the second component of the conditional $\varphi \to \psi$, the rows of the truth table correspond to the top left and right and bottom left and right regions of 5.1.2-1A, respectively.

Apart from compositionality, the principles of implication and equivalence for the conditional are quite different from those we saw for conjunctions and disjunctions.

COVARIANCE WITH THE CONSEQUENT. *A conditional implies the result of replacing its consequent with anything that component implies.* That is, if $\psi \vDash \chi$, then $\varphi \to \psi \vDash \varphi \to \chi$.

CONTRAVARIANCE WITH THE ANTECEDENT. *A conjunction implies the result of replacing its antecedent with anything that implies that component.* That is, if $\chi \vDash \psi$, then $\psi \to \varphi \vDash \chi \to \varphi$.

CURRY'S LAW. *A conjunct of a conditional's antecedent may be made instead a condition on its consequent.* That is, $(\varphi \wedge \psi) \to \chi \simeq \varphi \to (\psi \to \chi)$.

COMPOSITIONALITY. *Conditionals are equivalent if their corresponding components are equivalent.* That is, if $\varphi \simeq \varphi'$ and $\psi \simeq \psi'$, then $\varphi \to \psi \simeq \varphi' \to \psi'$.

The asymmetry of the conditional (e.g., the fact that it is false in the second row of its table but true when the values of its components are reversed in the third) means that we would not expect it to obey a principle of commutativity. That asymmetry is also responsible for the fact that it obeys a principle of covariance for one component but contravariance for the other. It makes sense that a conditional varies in the same direction as its consequent since it's hedged assertion of that consequent. And it varies in the opposite direction from its antecedent because a condition that rules out more and will be harder to fulfill, so a commitment to the truth of the consequent will happen in fewer possibilties.

The asymmetry of the conditional also makes it no surprise that a principle of associativity does not hold because such a principle would involve several shifts between the roles of antecedent and consequent. A principle for regrouping that can be stated is here named after the logician Haskell Curry who made extensive use of an analgous operation on functions (and also directed people's attention to the analogy between certain operations on functions and principles governing conditionals). The operation on functions is sometimes called "currying," and you might think of the transition from the left to right of Curry's law as a matter of taking a pair of conditions clumped together as a conjunction in the antecedent of a conditional and combing them out into separate antecedents. The principle is also sometimes referred as an "import-export" principle because it tells us how to export a component of the antecedent to the consequent or import a component of the consequent into the antecedent. Curry's law holds because each side can be false only when both φ and ψ are true and χ is false. And this shows that both sides have the effect of hedging χ by the two conditions φ and ψ. Such a statement might then be called a *double conditional*.

5.1.3. Doubts about truth-functionality

The account of the truth conditions of $\varphi \rightarrow \psi$ considered in the last subsection was proposed by the Greek logician Philo (who was active around 300 BCE). It was immediately subjected to criticisms by other logicians—Diodorus Cronus in particular—on the grounds that not having φ true along with ψ false is not sufficient for the truth $\varphi \rightarrow \psi$; some further connection between φ and ψ was felt to be necessary. The later report of this dispute by Sextus Empiricus contains the example

If it is day, I am conversing.

According to the table above, this is true whenever its speaker is engaged in conversation during the daytime as well as being true throughout the night under all conditions. On the other hand, according to the view of conditionals offered by Diodorus Cronus, this sentence is true at a given time only if its speaker is and always will be conversing from sunrise to sunset. If Diodorus' claim is correct, the truth of the sentence depends on more than the current truth values of its components and, since the current truth values are the only input in a truth table, no truth table is possible for a conditional as he understood it.

The controversy apparently became widespread enough in antiquity to be noticed by people other than logicians, and it has reappeared whenever the logic of conditionals has been given serious attention. In recent years, quite a bit of thought has been devoted to the issue, and a consensus may be emerging. It is widely granted that certain conditional sentences are in fact false in cases beyond those indicated in the table for $\rightarrow$. Other conditionals are held to obey the table but to carry implicatures that obscure this fact.

The clearest failures of the table occur with what are known variously as *subjunctive* or *counterfactual* conditionals. The difference in both form and content between these conditionals and ordinary *indicative* conditionals can be seen clearly in the following pair of examples (due to Ernest Adams):

If Oswald didn't shoot Kennedy, someone else did.
If Oswald hadn't shot Kennedy, someone else would have.

The first conditional, which grammarians would say is in the indicative mood, will be affirmed by anyone who knows Kennedy was shot by someone; but the second, which is in the subjunctive mood, would be asserted only by someone who believes there was a conspiracy to assassinate him (or who believes that his assassination was likely for other reasons). Notice also that the first suggests that the speaker is leaving open to question the identity of Kennedy's assassin while the second suggests the conviction that Oswald did shoot Kennedy. The antecedent of the second does not function simply as a hedge on what is claimed by the consequent; instead, it directs attention to possibilities inconsistent with what its speaker holds to be fact—in this case, possible worlds in which Oswald did not shoot Kennedy. That is the reason why conditionals like the second one are referred to as "contrary-to-fact" or counterfactual.

Now, if subjunctive conditionals are asserted primarily in cases where their antecedents are held to be false, it is clear that the table we have given is not appropriate for them. According to the table, a sentence of the form $\varphi \rightarrow \psi$ is bound to be true when its antecedent is false and therefore cannot provide any information about such cases; but subjunctive conditionals seem designed to provide information about just this sort of case.

We have to be a little careful here and remember that we can derive information from an assertion not only by considering what it implies (which is what a truth table is intended to capture) but also what it implicates. So we might consider the possibility that counterfactual conditionals really do not imply anything at all about the cases where their antecedents are false, and the information we get about such cases comes from their implicatures. But it is not hard to see that this is not so. Consider, for example, the following survey question (with X replaced by the name of a politician):

If the election were held today, would you vote for X?

This asks the respondent to evaluate the truth of the conditional If the election were held today, I would vote for X, and it makes sense to ask such a question only if a conditional like this can be false in cases where it has a false antecedent.

If the truth table above does not tell us the truth conditions of subjunctive conditionals, what are their truth conditions? A full discussion of this question would lead us outside the scope of this course, but I can outline what seems to be the most common current view. Like most good ideas, this account is hard to attribute; but two recent philosophers, Robert Stalnaker and David Lewis, did much to develop and popularize it (in slightly different versions). When evaluating the truth of a subjunctive conditional of the form If it were the case that φ, it would be the case that ψ in a given possible world, we do not limit our consideration to the truth values of φ and ψ in that world. We consider other possible worlds, too, and see whether we find φ true and ψ false in any of them. However, we do not consider all possible worlds (as we do when deciding whether φ entails ψ). Some possibilities are closer to the world in which we are evaluating the conditional than others are; and, as we broaden our horizons past a given possible world, we can move to more and more distant alternatives. When evaluating a subjunctive conditional, we extend our view just far enough to find possible worlds in which its antecedent is true and check to see whether its consequent is false in any of these. In short, a subjunctive conditional is true if its consequent is true in the nearest possible worlds in which its antecedent is true.

As an example, consider the following:

If we were in the Antarctic, we would have very cold summers.
If we were in the Antarctic, the Antarctic would have warm summers.

I take the first of these sentences to be true and the second false, because I take the nearest possibilities in which we are in the Antarctic to be ones in which it has retained its location and climate but we have traveled to it. There are, no doubt, possible worlds in which the Antarctic is a continent in the northern temperate zone (and perhaps even some in which we have stayed here and it has traveled to meet us) but they are much more distant possibilities.

This account of truth conditions of counterfactual conditionals cannot be stated in a truth table because, when judging the truth value of a subjunctive conditional in a given possible world, it forces us to consider the truth values of its components in other possible worlds. And the failure to have a truth table puts the logical properties of subjunctive or counterfactual conditionals outside the scope of this course.

But what about indicative conditionals? The argument just given to show that subjunctive conditionals do not have a truth table does not apply to indicate conditionals. However, we are still not prepared to assert indicative conditionals in all cases when Philo's table would count them as true. This can be seen by considering examples such as If Kennedy was west of the Mississippi when shot, he was shot in Texas. This sentence is true according to the table but suggests a belief on the part of the speaker that somehow ties being west of the Mississippi and Texas together in the matter of Kennedy's assassination, and it would be inappropriate for a speaker who did not have such a belief to utter the conditional. (Notice that the tie here can lie with the speaker as much as with the events. The sentence If Kennedy was west of the Mississippi when shot, he was shot in Texas would be appropriately asserted by someone who believed that Kennedy was shot while travelling in Florida and Texas but did not know the precise location.)

Still, inappropriateness as a result of false suggestions need not mean falsity through false implications, and there is reason for holding that a connection between Indiana and Texas is not implied by this example, only implicated. I hope you will grant that the following sentences are equivalent:

If Kennedy was west of the Mississippi when shot, he was shot in Texas.
Either Kennedy wasn't west of the Mississippi when shot or he was shot in Texas.
Kennedy wasn't west of the Mississippi when shot without being shot in Texas.

And this suggests that the content of an indicative conditional can be captured by compounds that do have truth tables.

Indeed, the restrictions that we feel on the use of indicative conditionals are ones that can arise even if the truth table for → gives an accurate account of its truth conditions. They are found in the second and third sentences above, and the tables for ¬, ∧, and ∨ gives those sentences the truth conditions that are given to the first sentence by the table for →. Moreover, it is possible to see the restrictions on the appropriateness of indicative conditional as arising naturally from these truth conditions. A speaker who knows whether the components φ and ψ are true or false, generally ought to say so rather than assert the conditional (or disjunction or not-without form). For information about the truth values of at least one clause will usually be relevant to the conversation if the conditional is. As a result, someone who asserts only a conditional is assumed not to know the truth values of its components. But a speaker must have some basis for an assertion if it is to be appropriate. So we assume that anyone asserting a conditional is basing this assertion on some knowledge of φ and ψ that is sufficient to rule out the case where φ is true and ψ is false without settling the truth value of either φ or ψ. And this sort of knowledge concerning φ and ψ could only be knowledge of some connection between them. So assertion of a conditional will often be appropriate only when the speaker knows some connection between its two components, and the conditional will thus often carry the existence of such a connection as an implicature. An argument similar to this was one of Grice's chief applications of his idea of implicature.

We will pursue this a little further in 5.2.2 but, for now, we can say that one possible account of the indicative conditional is to say that its truth conditions and what it says or implies is captured by the truth table for → but that an the conditional suggests or implicates something more, and the content of this implicature cannot be captured by a truth table. Indeed, the corresponding subjunctive conditional often seems to roughly capture this implicature of an indicative conditional. However, it is hard to tell whether the correspondence is more than rough. Subjunctive conditionals have their own implicatures —e.g., that the antecedent is false—and these can make the comparison difficult. And the content of a subjunctive conditional depends on what possibilities are counted as nearer than others, something that can vary with the context in which a subjunctive conditional is asserted. So, while If Kennedy were west of the Mississippi when shot, he would have been shot in Texas may not seem to be an implicature of If Kennedy was west of the Mississippi when shot, he was shot in Texas, that may be because the relations among possibilities corresponding to the normal context of the first assertion are not the ones required to capture what the second implicates.

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5.1.s. Summary

1 One way to hedge a claim is to make it conditional on another one, limiting responsibility for the truth of the first claim to cases where the second is true. The English word if is used for this purpose. We will refer to a compound of this sort (and the connective used to form it) as a conditional. Its two components are distinguished as the antecedent (which expresses the condition placed on the claim and appears as a subordinate clause in English) and the consequent (which is the claim that is made conditional and appears as a main clause). Although, the two components have a different significance in the compound, they can be stated in either order in English, with the antecedent preceded by if.

The rightwards and leftwards arrows, → and ←, provide our signs for the if-conditional; the two components may be written in either order but the arrow should be chosen to point from the subordinate to the main clause. As English notation, we write if φ then ψ for φ → ψ and yes ψ if φ for ψ ← φ. When parentheses are to be used for grouping along with English for the connective itself, we can use if for ← but we must resort to implies for → (understanding this to indicate material implication rather than the logical implication that is a special case of entailment).

2 In its truth table, a conditional as false only when its antecedent is true and its consequent is false. This asymmetry means that it says more as its consequent is strengthen but also as its antecedent is weakened.

3 The truth table of the conditional was first suggested in antiquity and has been controversial ever since. Current thinking distinguishes between indicative and subjunctive conditionals. The latter are held not to have truth tables (but to instead be true when their consequents are true in all the nearest worlds in which the antecedent is true). Indicative conditionals are held to have truth tables even though implicatures obscure this fact.

4 The rule of the thumb that if precedes the antecedent is the key to analyzing English conditionals, but it may not be obvious how much of the sentence is being made conditional on this antecedent. English conditionals about the future usually have antecedents in the present tense, so the tense must be changed to get an independent component with the correct meaning. When a branching conditional is stated in English, the term otherwise (which amounts to if that is not the case) is often used to state one of the antecedents.

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5.1.x. Exercise questions

1. Analyze each of the following sentences in as much detail as possible.

a. If it was raining, the roads were slippery.

b. He was home if the light was on.

c. Ann and Bill helped if Carol was away

d. Sam will help—and Tom will, too, if we ask him.

e. If it was warm, they ate outside provided it didn't rain.

f. If the new project was approved, Carol started work on it and so did Dave if he was finished with the last one.

g. If he found the instructions, Tom set up the new machine; otherwise, he packed up the old one.

2. Restate each of the following forms, putting English notation into symbols and vice versa and indicating the scope of connectives in the result by underlining:

a. $A \wedge (B \rightarrow C)$ **c.** if A then both B and if C then D

b. $(A \wedge B) \rightarrow C$ **d.** both if A then B and if not A then not B

3. Synthesize idiomatic English sentences that express the propositions that are associated with the logical forms below by the intensional interpretations that follow them.

a. $\neg S \rightarrow \neg B$

S: I'll see it; B: I'll believe it

b. $S \rightarrow \neg (R \vee N)$

S: it was sunny; R: it rained; N: it snowed

c. $\neg W \leftarrow \neg (P \wedge \neg B)$

W: the set works; P: the set is plugged in; B: the set is broken

d. $\neg (A \vee B) \rightarrow (G \leftarrow \neg (C \vee D))$

A: Adams will back out; B: Brown will back out; G: the deal will go through; C: Collins will have trouble with financing; D: Davis will have trouble with financing

4. Calculate truth values for all components of the forms below on each possible extensional interpretation. Since the first two each have two unanalyzed components, there will be 4 interpretations and your table will have 4 rows of values; with three components, as in the third and fourth, there will be 8 interpretations giving 8 rows of values.

a. $(A \rightarrow B) \wedge (B \rightarrow A)$ **c.** $(A \rightarrow C) \wedge (B \rightarrow \neg C)$

b. $\neg (A \wedge B) \rightarrow (\neg B \vee A)$ **d.** $\neg (A \rightarrow C) \wedge (\neg B \rightarrow C)$

5.1.xa. Exercise answers

1. **a.** It was raining → the roads were slippery

$R \rightarrow S$

if R then S

R: it was raining; S: the roads were slippery

b. He was home ← the light was on

$H \leftarrow L$

$L \rightarrow H$

if L then H

H: he was home; L: the light was on

c. Ann and Bill helped ← Carol was away

(Ann helped ∧ Bill helped) ← Carol was away

$(A \wedge B) \leftarrow C$

$C \rightarrow (A \wedge B)$

if C then both A and B

A: Ann helped; B: Bill helped; C: Carol was away

d. Sam will help ∧ Tom will help if we ask him

Sam will help ∧ (Tom will help ← we will ask Tom to help)

$S \wedge (T \leftarrow A)$

$S \wedge (A \rightarrow T)$

both S and if A then T

A: we will ask Tom to help; S: Sam will help; T: Tom will help

e. it was warm → they ate outside provided it didn't rain

it was warm → (they ate outside ← it didn't rain)

it was warm → (they ate outside ← ¬ it rained)

$W \rightarrow (O \leftarrow \neg R)$

$W \rightarrow (\neg R \rightarrow O)$

if W then if not R then O

O: they ate outside; R: it rained; W: it was warm

f. the new project was approved → Carol started work on the new probject and so did Dave if he was finished with the last one

the new project was approved → (Carol started work on the new probject ∧ Dave started work on the new probject if he was finished with the last one)

the new project was approved → (Carol started work on the new probject ∧ (Dave started work on the new probject ←

Dave was finished with the last project))

$$A \to (C \wedge (D \leftarrow F))$$

$$A \to (C \wedge (F \to D))$$

if A then both C and if F then D

A: the new project was approved; C: Carol started work on the new probject; D: Dave started work on the new probject; F: Dave was finished with the last project

g. If he found the instructions, Tom set up the new machine ∧ if Tom didn't find the instructions, he packed up the old machine

(Tom found the instructions → Tom set up the new machine) ∧ (Tom didn't find the instructions → Tom packed up the old machine)

(Tom found the instructions → Tom set up the new machine) ∧ (¬ Tom found the instructions → Tom packed up the old machine)

$$(F \to S) \wedge (\neg F \to P)$$

both if F then S and if not F then P

F: Tom found the instructions; P: Tom packed up the old machine; S: Tom set up the new machine

2. **a.** both A and if B then C

b. if both A and B then C

c. $A \to (B \wedge (C \to D))$

d. $(A \to B) \wedge (\neg A \to \neg B)$

3. **a.** ¬ I'll see it → ¬ I'll believe it

I won't see it → I won't believe it

If I don't see it, I won't believe it

b. It was sunny → ¬ (it rained ∨ it snowed)

It was sunny → ¬ it rained or snowed

It was sunny → it didn't rain or snow

If it was sunny, it didn't rain or snow

c. ¬ the set works ← ¬ (the set is plugged in ∧ ¬ the set is broken)

¬ the set works ← ¬ (the set is plugged in ∧ the set isn't broken)

¬ the set works ← ¬ (the set is plugged in and isn't broken)

The set doesn't work if it isn't both plugged in and unbroken

d. ¬ (Adams will back out ∨ Brown will back out) → (the deal will go through ← ¬ (Collins will have trouble with financing ∨ Davis will have trouble with financing))

¬ Adams or Brown will back out → (the deal will go through ← ¬ (Collins or Davis will have trouble with financing))

¬ Adams or Brown will back out → (the deal will go through ← neither Collins nor Davis will have trouble with financing)

¬ Adams or Brown will back out → the deal will go through provided neither Collins nor Davis has trouble with financing

If neither Adams nor Brown backs out, the deal will go through provided neither Collins nor Davis has trouble with financing

4. Numbers below the tables indicate the order in which values were computed.

a.

A	B	(A → B)	∧	(B → A)
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	T	T
		1	2	1

b.

A	B	¬	(A ∧ B)	→	(¬ B	∨ A)
T	T	F	T	T	F	T
T	F	T	F	T	T	T
F	T	T	F	F	F	F
F	F	T	F	T	T	T
		2	1	3	1	2

c.

A	B	C	(A	→	C)	∧	(B	→	¬	C)
T	T	T		T		F		F	F	
T	T	F		F		F		T	T	
T	F	T		T		T		T	F	
T	F	F		F		F		T	T	
F	T	T		T		F		F	F	
F	T	F		T		T		T	T	
F	F	T		T		T		T	F	
F	F	F		T		T		T	T	
				1		3		2	1	

d.

A	B	C	¬	(A	→	C)	∧	(¬	B	→	C)
T	T	T	F		T		F	F		T	
T	T	F	T		F		T	F		T	
T	F	T	F		T		F	T		T	
T	F	F	T		F		F	T		F	
F	T	T	F		T		F	F		T	
F	T	F	F		T		F	F		T	
F	F	T	F		T		F	T		T	
F	F	F	F		T		F	T		F	
			2		1		3	1		2	

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