4. Disjunctions

4.1. Or: taking common content

4.1.0. Overview

The third connective we will study, *disjunction*, might be thought of as a logical mirror image conjunction; more precisely, the relation between them is another example of duality.

4.1.1. Hedging

While the components of a conjunction contribute their content to the whole, a disjunction asserts only the content its components have in common.

4.1.2. Inclusive and exclusive disjunction

The distinction between implications and implicatures is especially important when assessing the meaning of *or* in English.

4.1.3. Disjunction in English

Many of the other issues that arise for disjunction are like those that arise for conjunction; and one of the ways of expressing disjunction in English suggests a use of connectives to express certain numerical claims.

4.1.4. Further examples

We now have the means to give natural analyses to a wide variety of patterns in English, including a more natural analysis of sentences involving neither-nor.

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4.1.1. Hedging

Although, as was noted in 3.1.4, conjunction and negation are, by themselves enough to give us the effect of any connective for which has a truth table, these two are not the only connectives that are marked by special vocabulary in English. We will introduce special notation for two further connectives. The first is expressed by the English word or. This word has a range of grammatical uses comparable to those of and. It can join words and phrases with various grammatical functions, and the force of most of these uses can be captured by a use of or to join sentences. For example,

The weight is at or near the limit

can be paraphrased as

The weight is at the limit or the weight is near the limit

and we will study all uses of or by way of its use to join sentences.

The connective corresponding to or is called *disjunction;* we will use the symbol ∨ (the *logical or*) for it and represent it also with the English notion either … or (in which either plays a role like that of both). As in the case of conjunction we will sometimes use a special term for the components of a disjunction: they are *disjuncts*.

The effect of disjoining a sentence with another is to back off from a definite claim by leaving open a second alternative. The sentence above, instead of asserting The weight is at the limit in an unqualified way, adds the alternative The weight is near the limit to leave open a further range of possibilities. In general, we can regard a sentence $\varphi \vee \psi$ as leaving open all possibilities left open by φ as well as all those left open by ψ. As a result, a disjunction $\varphi \vee \psi$ says no more—and usually less—than either of the components φ and ψ, and the difference can be extreme, as in the cowardly weather forecast It will rain tomorrow, or else it won't. Since $\varphi \vee \psi$ leaves open as many possibilities as either φ or ψ, it rules out no more and has no more content. In particular, it rules out only those possibilities that are ruled out by both φ and ψ; and we can say that the content of $\varphi \vee \psi$ is the common content of φ and ψ, the content shared by the two. For example, the following sentences are roughly equivalent

The temperature was very hot or very cold
The temperature was extreme

and the second expresses the common content of The temperature was very hot and The temperature was very cold, the two components of the first.

Disjunction, then, adds the possibilities left open by one component to those left open by the other and selects as the possibilities ruled out those that are ruled out by both components. This is shown in Figure 4.1.1-1 below. The pictures of dice have the same significance as in Figure 2.1.2-1: they indicate regions consisting of the possible worlds in which a certain die shows one or another number. The proposition shown in 4.1.1-1B is The number shown by the die is odd ∨ the number shown by the die is less than 4 and 4.1.1-1A illustrates its two components.

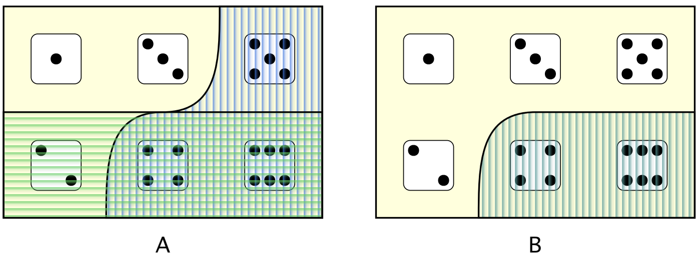

Fig. 4.1.1-1. Propositions expressed by two sentences (A) and their disjunction (B).

The possibilities ruled out by the components are shown in 4.1.1-1A shaded in different colors. 4.1.1-1B then shows the reduced set of possibilities ruled out by the disjunction and the enlarged set that are left open.

We can use these ideas to describe the truth conditions of disjunctions. If $\varphi \vee \psi$ is to leave open all possibilities left open by φ as well as all those left open by ψ, it must be true in all cases where φ is true and also in all cases where ψ is true. And if $\varphi \vee \psi$ captures the content common to φ and ψ—if it rules out the possibilities ruled out by both—it must be false whenever both φ and ψ are false. This is enough to tell us that disjunction is a connective with the table below. That is, $\varphi \vee \psi$ is true whenever at least one of φ and ψ is true and is false only when both are false.

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

This table should be compared to the diagram above; the worlds covered by the four rows of the table appear in 4.1-1A as the four regions at the top left and right and bottom left and right, respectively when φ rules out world at the bottom of the rectangle and ψ rules out worlds at the right.

Disjunction shares many of its logical properties with conjunction. In particular, analogues of the laws stated for conjunction at the end of 2.1.2 hold for it, too:

COMMUTATIVITY. *The order of disjuncts in a disjunction does not affect the content.* That is, $\varphi \vee \psi \simeq \psi \vee \varphi$.

ASSOCIATIVITY. *When a disjunction is a disjunct of a larger disjunction, the way components are grouped does not affect the content.* That is, $\varphi \vee (\psi \vee \chi) \simeq (\varphi \vee \psi) \vee \chi$.

IDEMPOTENCE. *Disjoining a sentence to itself does not change the content.* That is, $\varphi \vee \varphi \simeq \varphi$.

COVARIANCE. *A disjunction implies the result of replacing a component with anything that component implies.* That is, if $\psi \vDash \chi$, then $\varphi \vee \psi \vDash \varphi \vee \chi$ and $\psi \vee \varphi \vDash \chi \vee \varphi$.

COMPOSITIONALITY. *Disjunctions are equivalent if their corresponding components are equivalent.* That is, if $\varphi \simeq \varphi'$ and $\psi \simeq \psi'$, then $\varphi \vee \psi \simeq \varphi' \vee \psi'$.

There is nothing surprising in this. Conjunction shared analogues of these properties with both the minimum and the maximum operations on numbers, and conjunction and disjunction differ in the way the minimum and maximum operations do. In particular, a conjunction implies each of its components while a disjunction is implied by them (just as the minimum of two numbers is less than or equal to both while their maximum has both less than or equal to it).

4.1.2. Inclusive and exclusive disjunction

The fact that the table for $\varphi \vee \psi$ gives the value **T** when both φ and ψ are **T** may raise doubts about its correctness as an account of or. For we sometimes say things like

Al will go to France or Germany, or both;

and there are contexts where the expression and/or seems to capture our meaning better than or. But, if φ or ψ is already true when both φ and ψ are true, what does the alternative or both add? And, if φ or ψ is already true when φ and ψ is, why does and/or seem to differ from or?

Considerations like these have led logicians, from the Stoics on, to be interested in a connective with the table below.

φ	ψ	
T	T	F
T	F	T
F	T	T
F	F	F

This is the table of *exclusive disjunction*—so-called because it excludes the possibility that both components are true. The connective $\vee$ is known as *inclusive disjunction* because it leaves this possibility open. It has often been suggested that the English word or, in at least some of its uses, is a sign for exclusive rather than inclusive disjunction. If this were true, it would explain why we add the phrase or both or resort to and/or when we wish to express inclusive disjunction; for a sentence of the form Both φ and ψ is true in exactly the case in which inclusive and exclusive disjunction differ.

But in spite of this apparent evidence for regarding or as a sign of exclusive disjunction, there are strong reasons for thinking that it is always a sign for inclusive disjunction. That is, there are reasons for thinking that φ or ψ in English does not imply Not both φ and ψ (as it would if it were an exclusive disjunction of φ and ψ) but instead has the not-both claim an implicature in some contexts. The arguments we will look at touch on three features of a sentence that help to distinguish its implications among its implicatures: the effect of denying the sentence, yes-no questions concerning its truth, and the possibility of canceling implicatures.

Let us first look at the denial of the sentence Al will go to France or Germany. The most straightforward denial of this is Al will not go to France or Germany, but we could just as well say this:

Al will go to neither France nor Germany.

And we can paraphrase the latter as

Al will not go to France, and he will not go to Germany.

Now, we have seen that this sort of sentence can be analyzed as a not-and-not form, specifically, as $\neg F \wedge \neg G$ (F: Al will go to France; G: Al will go to Germany). And, it seems reasonable to suppose that the denial of φ or ψ can always be expressed as Neither φ nor ψ or, equivalently, as $\neg \varphi \wedge \neg \psi$.

But, if this is so, the word or must express inclusive disjunction. For the truth value of φ or ψ must be the opposite of the truth value of its denial, and we have seen reasons to believe that the truth value of its denial is given by the table below.

φ	ψ	$\neg \varphi \wedge \neg \psi$
T	T	F
T	F	F
F	T	F
F	F	T

If, on the other hand, the word or indicated exclusive disjunction, there would be two ways for a sentence φ or ψ to be false—i.e., when φ and ψ were both false and also when they were both true—and, therefore, two ways for its denial to be true. But the form Neither φ nor ψ, does not seem to leave open the possibility that both φ and ψ are true. In short, if the possibility that Al will go to both France and Germany must not be ruled out by the disjunction, because it is not left open by the corresponding neither-nor sentence.

A second argument concerns questions. Imagine that you intend to visit both France and Germany this summer and are filling out a questionnaire that includes the following:

Will you visit France or Germany this year? __ Yes __ No

The correct answer in this case seems to be yes. But this means that the sentence I will visit France or Germany this year is true if you will visit both.

A final argument concerns the following way of making it clear that Al might visit both France and Germany.

Al will visit France or Germany, and he may visit both.

Notice that instead of hedging the claim (as is done or both is added), this sentence uses and and thereby adds a second claim Al may visit both France and Germany. Now, if Al will visit France or Germany implied Al won't visit both France and Germany, the sentence displayed above would imply the following:

> Al won't visit both France and Germany, but he may visit both.

This sentence may not have fallen into self-contradiction, but it is teetering on the edge. On the other hand, Al will visit France or Germany, and he may visit both is neither a self-contradiction nor anything close to one.

If these arguments are correct, when a disjunction φ or ψ does convey the idea that φ and ψ are not both true, it does so by means of an implicature rather than an implication. Moreover, it seems possible to cancel any such implicature by adding a phrase like and maybe both. This possibility of cancellation is a sign that the implicature is of a special kind that Grice distinguished as a *conversational implicature*. A conversational implicature does not attach to a particular word as do the special implicatures that come with the use of even and but. Instead, it is produced by an interaction between the content of the claim being made and the conversational setting in which it is made. Conversational implicatures may be canceled while implicatures attaching to particular words typically cannot be canceled without lapsing into the sort incoherence exhibited by Even John was laughing, but John always laughs. Although it is not easy to say exactly how conversational implicatures arise in the case of disjunction, it does seem clear that any suggestion that the alternatives are not both true depends on the setting in which the disjunction is asserted. For example, if it was clear to everyone that the speaker's knowledge of Al's plans was derived from his responses on the kind of questionnaire described above, Al will visit France or Germany would carry no suggestion that Al would not visit both.

Of course, to assume that or in English always expresses inclusive disjunction is to not claim that exclusive disjunction cannot be expressed in English. We can, of course, always rule out the possibility that two alternatives are both true if we choose to do so. But, if this is to be done through the truth conditions of what we say (rather than through an implicature), we must rule out the possibility explicitly by, for example, saying something of the form φ or ψ but not both. And, in our notation, we have the following two forms:

Inclusive disjunction	*Exclusive disjunction*
$\varphi \lor \psi$	$(\varphi \lor \psi) \land \neg (\varphi \land \psi)$
either φ or ψ	both either φ or ψ and not both φ and ψ

But, for the remainder of this text, the term disjunction without qualification will always refer to inclusive disjunction—i.e., to the form $\varphi \lor \psi$.

4.1.3. Disjunction in English

Once we set aside controversies about the meaning of or, there are few special problems that arise in analyzing sentences as disjunctions. Of course, we must continue to be careful that the components we identify are independent sentences and that they really may be combined by disjunction to capture the content of the original sentence. This can keep us from analyzing a sentence as a disjunction even though it contains the word or. For example, Everyone stood at either the port or the starboard railing may not be analyzed as Everyone stood at the port railing ∨ everyone stood at the starboard railing.

The word or may be used in English to join a series of items and our approach to such serial disjunctions will similar to that used for serial conjunctions. We need to use two disjunctions and impose some grouping, but it will not matter which disjunction we take to have the wider scope. The parentheses indicating the grouping we impose may be suppressed when an analysis is written—so Al will visit England, France, or Germany could be analyzed using a run-on disjunction as

Al will visit England ∨ Al will visit France ∨ Al will visit Germany

However, we must recognize the grouping again in order to apply laws of entailment stated for two-component disjunctions.

There are few stylistic variants of or in English, but there is one especially clear way of stating an inclusive disjunction that deserves some comment. We might avoid any suggestion that Al will not visit both France and Germany by restating our earlier example as follows.

Al will visit at least one of France and Germany.

That we can have any chance at all of avoiding the implicature requires some explanation because, even though conversational implicatures are not part of the content of what we say, they derive from it. So it is hard to avoid them (in a given conversational context) by saying the same thing in different words. Perhaps we succeed in the case at hand because the phrase at least one is slightly stilted and would be appropriate only if the simpler form or could not be used. The stilted language could provide a clue to the audience that the speaker wants to avoid the implicatures ordinarily carried by a disjunction, and the implicature that is carried by the content of the assertion would then end up being canceled by the way that content was expressed.

The phrase at least one seems stilted in part because it presents a simple disjunction as if it was chosen from a whole family of similar claims, each saying something about how many alternatives from a list are true. For example, we might say that Al will not visit both countries by means of the following:

Al will visit at most one of France and Germany.

And we could state an exclusive disjunction as follows:

Al will visit exactly one of France and Germany.

Notice that this last sentence can be analyzed as the conjunction of the two preceding it.

With a list of more than two alternatives, there is a greater variety of claims of this sort; but, like the examples above, all of them can be expressed quite directly using conjunction, negation, and disjunction. For example, let us try to express the following sentence as a compound of the three abbreviated below it:

Exactly two of Dan, Ed, and Fred will make the finals

D: Dan will make the finals;
E: Ed will make the finals;
F: Fred will make the finals

As a first step in analyzing this sentence, we may note that it can be regarded as a conjunction of two claims, one saying that at least two of the three will make it and the other saying that at most two will.

A claim that at most two will make it denies that all three will make it and can be expressed as $\neg (D \wedge E \wedge F)$. The claim that at least two will make it tells us that there is at least one true sentence of the form a and b will make the finals where a and b are different names chosen from among Dan, Ed, and Fred. Now there are three non-equivalent sentences of this form—namely, $D \wedge E$, $D \wedge F$, and $E \wedge F$—so what we wish to say is that at least one of these three sentences is true. This can be expressed by the run-on disjunction $(D \wedge E) \vee (D \wedge F) \vee (E \wedge F)$. Putting the two analyses together, we get

$$((D \wedge E) \vee (D \wedge F) \vee (E \wedge F)) \wedge \neg (D \wedge E \wedge F)$$

as an analysis of the claim that exactly two will make it.

This analysis is admittedly complex, and no one would choose to carry out an analogous analysis for even a moderately long list of alternatives; but the fact that it would be theoretically possible to carry it out is interesting, for it shows that we can understand some implications that seem to depend on numerical reasoning—for example, the validity of

$$\frac{\text{Exactly two of Dan, Ed, and Fred will make the finals}}{\text{At least one of Dan, Ed, and Fred will make the finals}}$$

solely in terms of the logical properties of and, or, and not. In 8.3.2, we will see that this idea can be carried further by using other logical constants. The possibility of understanding numerical reasoning as an aspect of purely logical reasoning was one of the key reasons for Frege's interest in logic and one of the chief motivations for its development at the end of the 19th and beginning of the 20th centuries.

4.1.4. Further examples

The first example below illustrates the difference between not both and neither-nor, but it does so with an analysis of the latter that is closer to English than the one that was used in the examples of 3.1.5 .

Ann and Bill didn't both enjoy the meal but neither complained about it

Ann and Bill didn't both enjoy the meal ∧ neither Ann nor Bill complained about the meal

¬ Ann and Bill both enjoyed the meal ∧ ¬ either Ann or Bill complained about the meal

¬ (Ann enjoyed the meal ∧ Bill enjoyed the meal) ∧ ¬ (Ann complained about the meal ∨ Bill complained about the meal)

$$\neg (A \land B) \land \neg (C \lor D)$$

not both A and B and not either C or D

A: Ann enjoyed the meal; B: Bill enjoyed the meal; R: Ann complained about the meal; S: Bill complained about the meal

The second example is a sample of the complexity of structure we are now in a position to find in even fairly ordinary sentences.

Either Smith went ahead without Jones or Hardy backing him, or else Brown knew of his wishes and carried them out without consulting him

Smith went ahead without Jones or Hardy backing him ∨ Brown knew of Smith's wishes and carried them out without consulting him

(Smith went ahead ∧ ¬ Jones or Hardy backed Smith) ∨ (Brown knew of Smith's wishes ∧ Brown carried out Smith's wishes without consulting him)

(Smith went ahead ∧ ¬ (Jones backed Smith ∨ Hardy backed Smith)) ∨ (Brown knew of Smith's wishes ∧ (Brown carried out Smith's wishes ∧ ¬ Brown consulted Smith))

$$(A \land \neg (J \lor H)) \lor (K \land (C \land \neg N))$$

either both A and not either J or H or both K and both C and not N

A: Smith went ahead; C: Brown carried out Smith's wishes; H: Hardy backed Smith; J: Jones backed Smith; K: Brown knew of Smith's wishes; N: Brown consulted Smith

Notice how often it was necessary to replace a pronoun by its antecedent in order to uncover components that were independent sentences. If this replacement changed the meaning, analysis would be impossible.

Consider a sentence like the one above but having a certain partner where that one has the name Smith.

Either a certain partner went ahead without Jones or Hardy backing him, or else Brown knew of his wishes and carried them out without consulting him

We can analyze this as a disjunction A certain partner went ahead without Jones or Hardy backing him $\vee$ Brown knew of a certain partner's wishes and carried them out without consulting him; but we can go no further with the analysis until we have other sorts of logical form at our disposal.

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4.1.s. Summary

1 While the logical word or is grammatically similar to and, its logical role is to weaken claims by hedging them with a second alternative rather than to strengthen them by adding with a second assertion. This difference from conjunction is expressed by the truth table of the connective disjunction, according to which a disjunction is true when at least one true sentence among its components, which are called disjuncts. The symbol ∨ (logical or) is our notation for the operation of disjunction, and its scope is marked by parentheses. Alternatively, we can write a disjunction $\varphi \lor \psi$ as either φ or ψ, where either serves (like both with conjunction) to indicate scope.

2 The truth of a disjunction when both its components are true distinguishes inclusive disjunction from another logical form, exclusive disjunction, whose compounds are true only when exactly one component is true. While English sentences stated with or often convey the idea that two alternatives are not both true, it can be argued that this information is conveyed as an implicature rather than an implication and that, as far as its truth conditions are concerned, the English word or may be taken as a sign of inclusive disjunction.

3 As is true of conjunction, there are cases where a word like or marking disjunction appears in a sentence but the sentence cannot be analyzed as a disjunction due to our inability to replace pronouns by their antecedents. Also, English has serial disjunctions just as it has serial conjunctions; and serial disjunction in English can be mimicked to some degree by run-on disjunctions, which suppress parentheses. Disjunction can be expressed in English by the phrase at least one, one of the group of related phrases indicating numerical compounding operations. In some cases, sentences containing these phrases can be analyzed by employing disjunction along with conjunction and negation.

4 Finally, disjunction provides an alternative, and more natural, way of analyzing *neither-nor* claims.

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4.1.x. Exercise questions

1. Analyze each of the following sentences in as much detail as possible.

- **a.** Either Tommy ate his vegetables or he didn't get any dessert.
- **b.** Mike heard neither the phone nor the doorbell.
- **c.** Either Mike wasn't home or he wasn't answering the phone.
- **d.** The package was sent, but either it's still on its way or it's been lost in the mail.
- **e.** Neither the House nor the Senate had acted on the bill, but the White House expressed confidence that it would pass.
- **f.** Sam won't pass through without either stopping by or calling.
- **g.** Either Davis or Edwards will take you or give you directions.
- **h.** We'll have either a can without an opener or an opener without a can.
- **i.** Neither Jan nor Ken had matches or a lighter.
- **j.** Both Ann and Bill were in town but neither knew the other was.
- **k.** Either Tom, Dick, or Harry will handle both the scheduling and the publicity.
- **l.** The scheduling will be handled by either Tom, Dick, or Harry—as will the publicity.

2. Restate each of the following forms, putting English notation into symbols and vice versa. Indicate the scope of connectives in the result by underlining.

- **a.** $A \land (B \lor C)$
- **b.** $(A \land B) \lor C$
- **c.** not either A or not B
- **d.** both either A or B and either A or C

3. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below by the intensional interpretations that follow them.

- **a.** $B \lor N$

  B: it was the butler; N: it was the nephew
- **b.** $\neg (A \lor S)$

  A: the alarm worked; S: the sprinkler worked
- **c.** $\neg A \lor \neg P$

  A: the part arrived; P: the part was the problem
- **d.** $A \lor \neg (B \land C)$

  A: Ann has a large car; B: Bill will ride with us; C: Carol will ride with us

e. $(R \vee D) \wedge W$

D: there was a heavy dew; R: it rained over night; W: it is wet

f. $(A \wedge Z) \vee (F \wedge \neg (A \vee Z))$

A: AAA ∧ Co. will profit from the deal; F: the deal will fall through; Z: ZZZ Inc. will profit from the deal

4.1.xa. Exercise answers

1. **a.** Tommy ate his vegetables ∨ Tommy didn't get any dessert

Tommy ate his vegetables ∨ ¬ Tommy got dessert

$$V \lor \neg D$$

either V or not D

D: Tommy got dessert; V: Tommy ate his vegetables

b. ¬ (Mike heard either the phone or the doorbell)

¬ (Mike heard the phone ∨ Mike heard the doorbell)

$$\neg (P \lor D)$$

not either P or D

D: Mike heard the doorbell; P: Mike heard the phone

c. Mike wasn't home ∨ Mike wasn't answering the phone

¬ Mike was home ∨ ¬ Mike was answering the phone

$$\neg H \lor \neg P$$

either not H or not P

H: Mike was home; P: Mike was answering the phone

d. The package was sent ∧ either the package is still on its way or it's been lost in the mail

The package was sent ∧ (the package is still on its way ∨ the package has been lost in the mail)

$$S \land (W \lor L)$$

both S and either W or L

L: the package has been lost in the mail; S: the package was sent; W: the package is still on its way

e. Neither the House nor the Senate had acted on the bill ∧ the White House expressed confidence that the bill would pass

¬ either the House or the Senate had acted on the bill ∧ the White House expressed confidence that the bill would pass

¬ (the House had acted on the bill ∨ the Senate had acted on the bill) ∧ the White House expressed confidence that the bill would pass

$$\neg (H \lor S) \land W$$

both not either H or S and W

H: the House had acted on the bill; S: the Senate had acted on the bill; W: the White House expressed confidence that the bill would pass

f. ¬ Sam will pass through without either stopping by or calling

¬ (Sam will pass through ∧ ¬ Sam will either stop by or call)

¬ (Sam will pass through ∧ ¬ (Sam will stop by ∨ Sam will call))

$$\neg (P \land \neg (S \lor C))$$

not both P and not either S or C

C: Sam will call; P: Sam will pass through; S: Sam will stop by

g. Davis will take you or give you directions ∨ Edwards will take you or give you directions

(Davis will take you ∨ Davis will give you directions) ∨ (Edwards will take you ∨ Edwards will give you directions)

$$(D \lor G) \lor (E \lor V)$$

either either D or G or either E or V

D: Davis will take you; E: Edwards will take you; G: Davis will give you directions; V: Edwards will give you directions

h. We'll have a can without an opener ∨ we'll have an opener without a can

(we'll have a can ∧ we won't have an opener) ∨ (we'll have an opener ∧ we won't have a can)

(we'll have a can ∧ ¬ we'll have an opener) ∨ (we'll have an opener ∧ ¬ we'll have a can)

$$(C \land \neg O) \lor (O \land \neg C)$$

either both C and not O or both O and not C

C: we'll have a can; O: we'll have an opener

i. ¬ either Jan or Ken had matches or a lighter

¬ (Jan had matches or a lighter ∨ Ken had matches or a lighter)

¬ ((Jan had matches ∨ Jan had a lighter) ∨ (Ken had matches ∨ Ken had a lighter))

$$\neg ((M \lor L) \lor (K \lor G))$$

not either either M or L or either K or G

G: Ken had a lighter; K: Ken had matches; L: Jan had a lighter; M: Jan had matches

j. Both Ann and Bill were in town ∧ neither Ann nor Bill knew the other was in town

(Ann was in town ∧ Bill was in town) ∧ ¬ either Ann or Bill knew the other was in town

(Ann was in town ∧ Bill was in town) ∧ ¬ (Ann knew Bill was in town ∨ Bill knew Ann was in town)

$$(A \land B) \land \neg (K \lor N)$$

both both A and B and not either K or N

A: Ann was in town; B: Bill was in town; K: Ann knew Bill was in town; N: Bill knew Ann was in town

k. Tom will handle both the scheduling and the publicity ∨ Dick will handle both the scheduling and the publicity ∨ Harry will handle both the scheduling and the publicity

(Tom will handle the scheduling ∧ Tom will handle the publicity) ∨ (Dick will handle the scheduling ∧ Dick will handle the publicity) ∨ (Harry will handle the scheduling ∧ Harry will handle the publicity)

$$(T \land P) \lor (D \land B) \lor (H \land L)$$

(both T and S) or (both D and C) or (both T and S)

[B: Dick will handle the publicity; D: Dick will handle the scheduling; H: Harry will handle the scheduling; L: Harry will handle the publicity; P: Tom will handle the publicity; T: Tom will handle the scheduling]

Note: this sentence is ambiguous and could also be interpreted as equivalent to the following one.

l. The scheduling will be handled by either Tom, Dick, or Harry ∧ the publicity will be handled by either Tom, Dick, or Harry

(the scheduling will be handled by Tom ∨ the scheduling will be handled by Dick ∨ the scheduling will be handled by Harry) ∧ (the publicity will be handled by Tom ∨ the publicity will be handled by Dick ∨ the publicity will be handled by Harry)

$$(T \lor D \lor H) \land (P \lor B \lor L)$$

both (T or D or H) and (P or B or L)

B: the publicity will be handled by Dick; D: the scheduling will be handled by Dick; H: the scheduling will be handled by Harry; L: the publicity will be handled by Harry; P: the publicity will be handled by Tom; T: the scheduling will be handled by Tom

2. **a.** both A and either B or C

b. either both A and B or C

c. $\neg (A \vee \neg B)$

d. $(A \vee B) \wedge (A \vee C)$

3. **a.** It was the butler ∨ it was the nephew

It was either the butler or the nephew

b. ¬ (the alarm worked ∨ the sprinkler worked)

¬ (either the alarm or the sprinkler worked)

Neither the alarm nor the sprinkler worked

c. ¬ the part arrived ∨ ¬ the part was the problem

The part didn't arrive ∨ the part wasn't the problem

Either the part didn't arrive or it wasn't the problem

d. Ann has a large car ∨ ¬ (Bill will ride with us ∧ Carol will ride with us)

Ann has a large car ∨ ¬ Bill and Carol will ride with us

Ann has a large car ∨ Bill and Carol won't both ride with us

Either Ann has a large car or Bill and Carol won't both ride with us

Note: both is introduced here to help distinguish this sentence from $A \vee (\neg B \wedge \neg C)$

e. (it rained over night ∨ there was a heavy dew) ∧ it is wet

It rained over night or there was a heavy dew ∧ it is wet

It rained over night or there was a heavy dew but, either way, it is wet

Note: either way here serves to indicate that the scope of the disjunction has ended and that the final clause is unhedged and but reinforces this by marking the contrast between the indefinite disjunction and the definite final clause.

f. (AAA ∧ Co. will profit from the deal ∧ ZZZ Inc. will profit from the deal) ∨ (the deal will fall through ∧ ¬ (AAA ∧ Co. will profit from the deal ∨ ZZZ Inc. will profit from the deal))

AAA ∧ Co. and ZZZ Inc. will both profit from the deal ∨ (the deal will fall through ∧ ¬ (either AAA ∧ Co. or ZZZ Inc. will profit from the deal))

AAA ∧ Co. and ZZZ Inc. will both profit from the deal ∨ (the deal will fall through ∧ neither AAA ∧ Co. nor ZZZ Inc. will profit from the deal)

AAA ∧ Co. and ZZZ Inc. will both profit from the deal ∨ the deal will fall through and neither AAA ∧ Co. nor ZZZ Inc. will profit from it)

Either AAA ∧ Co. and ZZZ Inc. will both profit from the deal, or the deal will fall through and neither will profit from it