Overview

Basic system

Exploitation and planning rules		
sentence	*as a resource*	*as a goal*
atomic sentence	*none*	IP
negation $\neg\varphi$	CR (if φ not atomic & goal is $\bot$)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \rightarrow \psi$	RC (if goal is $\bot$)	CP
universal $\forall x\, \theta x$	UI	UG
existential $\exists x\, \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and $\neg\varphi$	$\bot$	Nc
	any	$\top$	ENV
	$\bot$	*any*	EFQ
τ—υ	*any*	$\tau = \upsilon$	EC
τ—υ	$\neg\,\tau = \upsilon$	$\bot$	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$	$P\upsilon_1 \ldots \upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$ $\neg\, P\upsilon_1 \ldots \upsilon_n$	$\bot$	Nc=

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
$\neg(\varphi \wedge \psi)$	φ or ψ	MPT
$\varphi \vee \psi$	$\neg^{\pm}\varphi$ or $\neg^{\pm}\psi$	MTP
$\varphi \rightarrow \psi$	φ	MPP
	$\neg^{\pm}\psi$	MTT

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

Additional rules

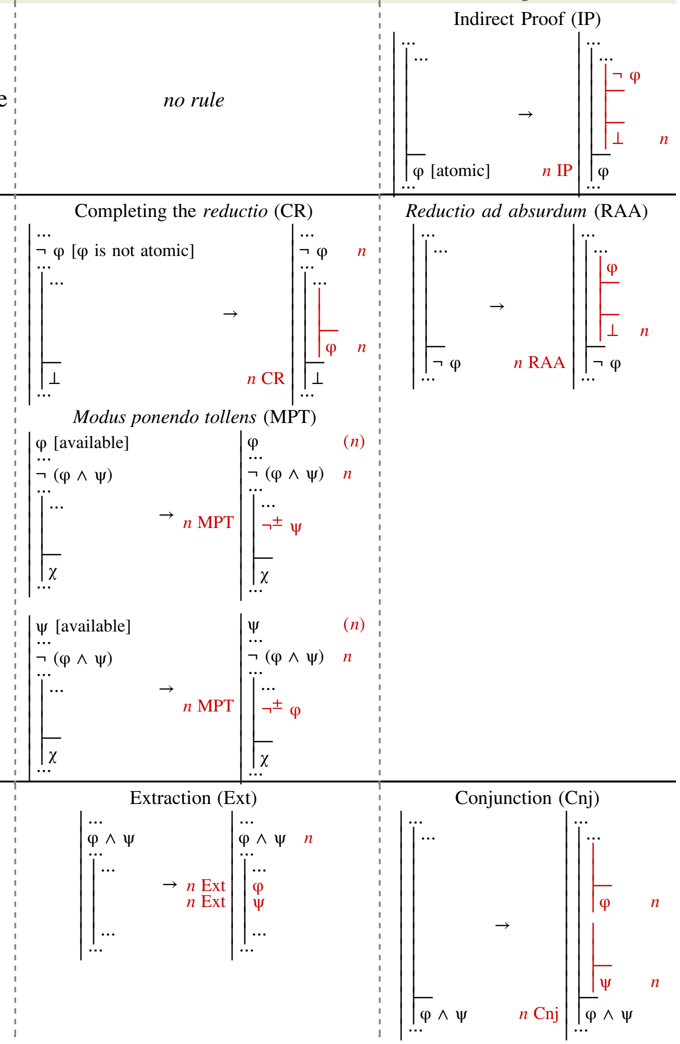
Attachment rules	
added resource	*rule*
$\varphi \wedge \psi$	Adj
$\neg(\varphi \wedge \psi)$	Wk
$\varphi \vee \psi$	Wk
$\varphi \rightarrow \psi$	Wk
$\tau = \upsilon$	CE
$\theta\upsilon_1 \ldots \upsilon_n$	Cng
$\exists x\, \theta x$	EG

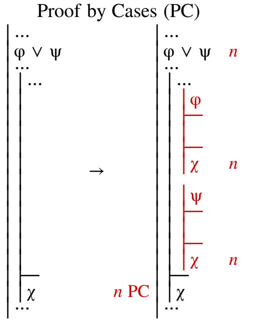
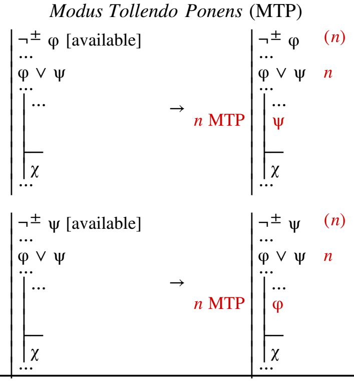
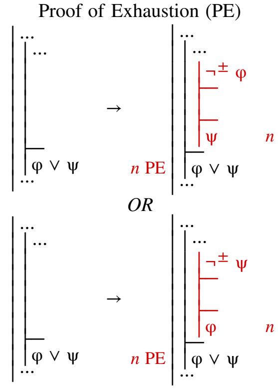
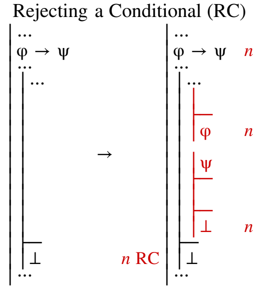
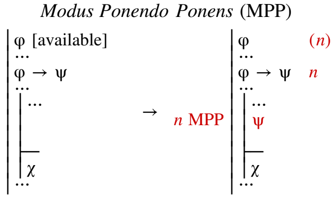
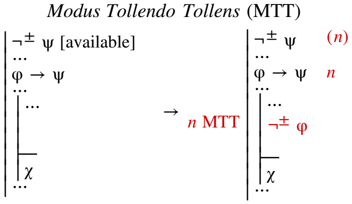
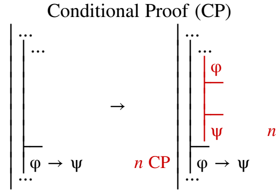
Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	LFR

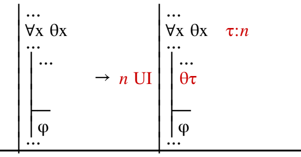
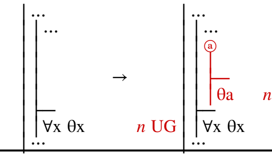
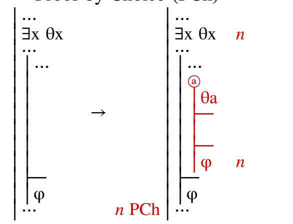
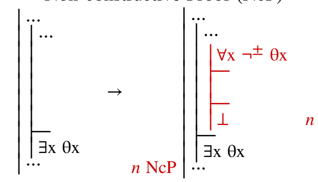
Derivation rules

Basic system

Rules for developing gaps		
logical form	*as resource*	*as goal*
atomic sentence	*no rule*	Indirect Proof (IP)
negation $\neg\varphi$	Completing the *reductio* (CR); *Modus ponendo tollens* (MPT)	*Reductio ad absurdum* (RAA)
conjunction $\varphi \wedge \psi$	Extraction (Ext)	Conjunction (Cnj)



Rules for developing gaps		
logical form	*as resource*	*as goal*
disjunction $\varphi \vee \psi$	Proof by Cases (PC)  *Modus Tollendo Ponens* (MTP) 	Proof of Exhaustion (PE) 
conditional $\varphi \to \psi$	Rejecting a Conditional (RC)  *Modus Ponendo Ponens* (MPP)  *Modus Tollendo Tollens* (MTT) 	Conditional Proof (CP) 

Rules for developing gaps		
logical form	*as resource*	*as goal*
universal $\forall x\ \theta x$	Universal Instantiation (UI) 	Universal Generalization (UG) 
existential $\exists x\ \theta x$	Proof by Choice (PCh) 	Non-constructive Proof (NcP) 

The parameter a used in UG and PCh should be new to the derivation; that is, it should appear only to the right of the scope line it labels

Rules for closing gaps (truth-functional logic)

when to close		*rule*
resources	*goal*	
φ	φ	*Quod Erat Demonstrandum* (QED) … φ [available] … | … | —— | φ … → … φ (n) … | … | ● | —— | φ n QED …
φ and $\neg\,\varphi$	$\bot$	Non-contradiction (Nc) … $\neg\,\varphi$ [available] … φ [available] … | … | —— | $\bot$ … → … $\neg\,\varphi$ (n) … φ (n) … | … | ● | —— | $\bot$ n Nc …
any	$\top$	*Ex Nihilo Verum* (ENV) … | … | —— | $\top$ … → … | … | ● | —— | $\top$ n ENV …
$\bot$	*any*	*Ex Falso Quodlibet* (EFQ) … $\bot$ … | … | —— | φ … → … $\bot$ (n) … | … | ● | —— | φ n EFQ …

Rules for closing gaps (equations)

In addition to the following rules for closing gaps, if the conditions for applying any rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "=" to its label; QED= and Nc= below are examples of this in the case of rules for closing gaps.

when to close			*rule*
co-aliases	*resources*	*goal*	
τ—υ	*any*	$\tau = \upsilon$	Equated Co-aliases (EC) … [τ and υ are co-aliases] … | … | —— | $\tau = \upsilon$ … → … [τ and υ are co-aliases] … | … | ● | —— | $\tau = \upsilon$ n EC …
τ—υ	$\neg\,\tau = \upsilon$	$\bot$	Distinguished Co-aliases (DC) … [τ and υ are co-aliases] … $\neg\,\tau = \upsilon$ … | … | —— | $\bot$ … → … [τ and υ are co-aliases] … $\neg\,\tau = \upsilon$ (n) … | … | ● | —— | $\bot$ n DC …
τ_1—υ_1, ⋯, τ_n—υ_n	$P\tau_1 \cdots \tau_n$	$P\upsilon_1 \cdots \upsilon_n$	QED given equations (QED=) … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ … | … | —— | $P\upsilon_1 \cdots \upsilon_n$ … → … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ (n) … | … | ● | —— | $P\upsilon_1 \cdots \upsilon_n$ n QED= …
τ_1—υ_1, ⋯, τ_n—υ_n	$P\tau_1 \cdots \tau_n$ $\neg\,P\upsilon_1 \cdots \upsilon_n$	$\bot$	Non-contradiction given equations (Nc=) … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ … $\neg\,P\upsilon_1 \cdots \upsilon_n$ … | … | —— | $\bot$ … → … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … $P\tau_1 \cdots \tau_n$ (n) … $\neg\,P\upsilon_1 \cdots \upsilon_n$ (n) … | … | ● | —— | $\bot$ n Nc= …

Additional rules (not guaranteed to be progressive)

Attachment rules		
what is required	*added resource*	*rule*
φ and ψ are both available	$\varphi \wedge \psi$	Adjunction (Adj) … φ [available] … ψ [available] … | … | χ … → *n* Adj … φ (n) … ψ (n) … | … | $\varphi \wedge \psi$ X | χ …
$\neg^{\pm}\,\varphi$ or $\neg^{\pm}\,\psi$ is available	$\neg\,(\varphi \wedge \psi)$	Weakening (Wk) … $\neg^{\pm}\,\varphi$ [available] … | … | χ … → *n* Wk … $\neg^{\pm}\,\varphi$ (*n*) … | … | $\neg\,(\varphi \wedge \psi)$ X | χ … … $\neg^{\pm}\,\psi$ [available] … | … | χ … → *n* Wk … $\neg^{\pm}\,\psi$ (*n*) … | … | $\neg\,(\varphi \wedge \psi)$ X | χ …
φ or ψ is available	$\varphi \vee \psi$	… φ [available] … | … | χ … → *n* Wk … φ (*n*) … | … | $\varphi \vee \psi$ X | χ … … ψ [available] … | … | χ … → *n* Wk … ψ (*n*) … | … | $\varphi \vee \psi$ X | χ …
$\neg^{\pm}\,\varphi$ or ψ is available	$\varphi \to \psi$	$\neg^{\pm}\,\varphi$ [available] … | … | χ … → *n* Wk $\neg^{\pm}\,\varphi$ (*n*) … | … | $\varphi \to \psi$ X | χ … ψ [available] … | … | χ … → *n* Wk ψ (*n*) … | … | $\varphi \to \psi$ X | χ …

Attachment rules		
what is required	*added resource*	*rule*
τ and υ are co-aliases	$\tau = \upsilon$	Co-alias Equation (CE) … [τ and υ are co-aliases] … | … | φ … → *n* CE … [τ and υ are co-aliases] … | … | $\tau = \upsilon$ X | φ …
have co-alias relations τ_1—υ_1, …, τ_n—υ_n and $\theta\tau_1\cdots\tau_n$ is available	$\theta\upsilon_1\cdots\upsilon_n$	Congruence (Cng) … [have co-alias relations: τ_1—υ_1, …, τ_n—υ_n] … $\theta\tau_1\cdots\tau_n$ … | … | φ … → *n* Cng … [have co-alias relations: τ_1—υ_1, …, τ_n—υ_n] … $\theta\tau_1\cdots\tau_n$ (*n*) … | … | $\theta\upsilon_1\cdots\upsilon_n$ X | φ …
$\theta\tau$ is available	$\exists x\ \theta x$	Existential Generalization (EG) … $\theta\tau$ … | … | φ … → *n* EG … $\theta\tau$ (*n*) … | … | $\exists x\ \theta x$ X | φ …

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	Lemma for *Reductio* (LFR) … | … | $\bot$ … → *n* LFR … | … | | φ *n* | | φ | | $\bot$ *n* | $\bot$ …