8.2. Uniform generality

8.2.0. Overview

The ambiguity of sentences containing both existentials and universals was an important motivation in the development of logic, and we are now in a position to consider it directly.

Glen Helman 25 Nov 2009

8.2.1. General and uniformly general exemplification

When first discussing quantifier phrases in 7.1.1, we considered the ambiguity of sentences like

A reporter interviewed each juror.

Quantifiers were designed to represent the alternative interpretations of sentences like this, and we are now in a position to see how they provide an account of the ambiguity in this example.

Since this sentence contains two quantifier phrases, we have two places to begin its analysis; and two different logical forms can result.

A reporter interviewed each juror

A reporter is such that (he or she interviewed each juror)

($\exists$x: x is a reporter) x interviewed each juror

($\exists$x: x is a reporter) ($\forall$y: y is a juror) x interviewed y

($\exists$x: Rx) ($\forall$y: Jy) Ixy

$\exists x (Rx \land \forall y (Jy \rightarrow Ixy))$

A reporter interviewed each juror

Each juror is such that (a reporter interviewed him or her)

($\forall$y: y is a juror) a reporter interviewed y

($\forall$y: y is a juror) ($\exists$x: x is a reporter) x interviewed y

($\forall$y: Jy) ($\exists$x: Rx) Ixy

$\forall y (Jy \rightarrow \exists x (Rx \land Ixy))$

I: [_ interviewed _]; J: [_ is a juror]; R: [_ is a reporter]

Symbolically, the only difference in the analyses that use restricted quantifiers lies in the order of those quantifiers. The difference this makes can be seen best by looking at the second step in each analysis:

A reporter is such that (he or she interviewed each juror)

Each juror is such that (a reporter interviewed him or her).

If we use terms that reflect the medieval theories of reference discussed in 7.1.1, we can say that the difference is due to the fixed indefinite reference of a reporter in the first and the variably indefinite reference of the same quantifier phrase in the second sentence. Since the latter sentence says only that each juror was interviewed without claiming that any one reporter conducted all the interviews, it is entailed by the first but does not entail it. Thus the first of the claims is the stronger of the two.

An analysis of logical form using quantifiers is capable of much more than the simple dichotomy between fixed and variably indefinite reference, but a distinction between the sorts of claims represented by the sentences above will be useful in organizing the richer range of possibilities we now have available. In the terms we have been using recently, each of the two interpretations of the sentence is both a generalization and a claim of exemplification. In each case, one of these two aspects is recognized as the overall form of the sentence while the other remains part of the quantified predicate. The first of these two sorts of claims, represented by the first interpretation of the original sentence, says that the property of interviewing each juror is exemplified. This is a general property, one whose predication is expressed by a generalization, so the first sort of claim says that a general property is exemplified. The second sort of claim makes a generalization, but each instance of this generalization is a claim of exemplification that asserts that a particular juror was interviewed. We will describe this second sort of statement as a claim of *general exemplification*: it says that a relative property is exemplified generally with respect to some domain. In this case, the property of being an interviewing reporter is exemplified generally with respect to jurors; that is, an example of such a reporter can be found for each juror.

This way of looking at the two claims puts them in a parallel position, but we know that they do not stand on the same level as far as their content goes. The first implies the second but is not implied by it. In other words, the first adds information to the second: it says that the second is true in a special way. Let us capture this idea by saying that, while the second is a claim of general exemplification, the first is a claim of *uniformly general exemplification*. In the example above, the second claim says that an example of an interviewing reporter is available generally for jurors, and the first claims that this sort of example is not only available generally but can be chosen in a uniform way, the same reporter can serve as an example no matter what juror we consider. In symbolic terms, we have a claim of general exemplification whenever a universal and existential have overlapping scope. If their scopes overlap, the scope of one includes the other, and we have a claim of uniformly general exemplification when it is the existential whose scope includes the universal.

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8.2.2. Quantifier scope ambiguities

One of the points of section 7.1.1 was that a simple dichotomy is not sufficient to account for the possible ambiguities when more than two quantifier phrases are present. So we need to extend the ideas developed in 8.2.1 to distinguish more than a single pair of claims. Consider the example cited in 7.1.1:

<p align="center" style="color:green">Every reporter asked a question of each juror.</p>

This could be said to generalize along two dimensions (reporters and jurors) and the exemplification (of a question that was asked) might be claimed to be uniform in either or both of these dimensions. That is, the exemplification may be set forth as doubly uniform (the same question could be used as an example in all cases) or as uniform in one dimension only (e.g., we might have to vary the question cited as an example from reporter to reporter but would not need to vary it from juror to juror provided we keep the reporter fixed). This means that there are four interpretations here: the basic claim of doubly general exemplification and three stronger claims, citing uniformity of the example in one or the other dimension or in both of them.

With three quantifier phrases, there will be six different symbolic representations of this sentence since there are three choices for the first quantifier phrase to be analyzed and, for each of these, two orders in which the remaining two can be analyzed. The results of these choices are shown below in a way that reflects their logical relations, with stronger claims lower on the page and equivalent claims grouped together.

$$(\forall x: Rx)\ (\forall y: Jy)\ (\exists z: Qz)\ Axzy$$
$$(\forall y: Jy)\ (\forall x: Rx)\ (\exists z: Qz)\ Axzy)$$
[no uniformity claimed]

$(\forall x: Rx)\ (\exists z: Qz)\ (\forall y: Jy)\ Axzy$ $(\forall y: Jy)\ (\exists z: Qz)\ (\forall x: Rx)\ Axzy$
[claims uniformity with respect to jurors]   [claims uniformity with respect to reporters]

$$(\exists z: Qz)\ (\forall x: Rx)\ (\forall y: Jy)\ Axzy$$
$$(\exists z: Qz)\ (\forall y: Jy)\ (\forall x: Rx)\ Axzy)$$
[claims uniformity with respect to both jurors and reporters]

A: [_ asked _ of _]; J: [_ is a juror]; Q: [_ is a question]; R: [_ is a reporter]

Two pairs of these six forms are equivalent; for, when the two universals are side by side (and neither binds variables in the restriction of the other), we can interchange them without altering the proposition expressed. The distinguishing feature of the forms that are not equivalent is the location of the existential quantifier used to represent a question—whether it is outside the scope of one or the other of the universal quantifier phrases, outside the scope of both, or

outside the scope of neither. And these four non-equivalent symbolic possibilities correspond to the four possibilities of uniformity we have noticed in the sentence.

The use of relative scope that we have seen in this example applies generally: when a claim of general exemplification is uniform with respect to a given dimension of generality, the existential quantifier representing the claim of exemplification should have wider scope than the universal quantifier corresponding to the relevant dimension of generality. So, when you are faced with choosing the order in which to represent several quantifier phrases and you wonder what effect the order you choose will have on the meaning, you can proceed as follows. First, identify the quantifier phrases making existential claims and the quantifier phrases that generalize on one or another dimension. Then ask, for each existential quantifier phrase and each generalizing one, whether the existential claims that exemplification is uniform on the dimension referred to by the generalizing phrase. If the existential makes this claim, it should be given wider scope than the universal; if it does not, the universal should be given wider scope. The answers to these questions will settle the relative order of treatment for each pair consisting of an existential and a universal.

This approach may not settle all questions about the order in which quantifier phrases are to be treated in claims of general exemplification, but the remaining questions can be settled arbitrarily without any effect on the meaning ascribed to the sentence. For example, if **a question** is held to claim an exemplification that is uniform with respect to both reporters and jurors, we know that the existential quantifier phrase must be treated before the two universals. Nothing is implied about the order in which we go on to handle **every reporter** and **each juror**, but that order also has no effect on the content of the result.

The language we have been using to speak about the process of settling the relative scope of quantifier phrases is open to one sort of misinterpretation. Although there is no way to arrange overlapping scopes to claim uniformity in each of two dimensions without claiming uniformity in both, this does not mean that separate claims of uniformity in each of two dimensions together entail a claim of uniformity in both. Since there might be different examples exhibiting uniformity in each of two dimensions, there can be situations where a claim of uniformity holds for each dimension even though there is no example that works uniformly for both. For example, it may be that reporters had favorite questions so that it is true that

$$(\forall x\text{: } Rx)\,(\exists z\text{: } Qz)\,(\forall y\text{: } Jy)\, Axzy$$

—i.e., each reporter asked a question uniformly for all jurors—and it may also be that there was an obvious question for each juror so that it is true that

$$(\forall y\text{: } Jy)\,(\exists z\text{: } Qz)\,(\forall x\text{: } Rx)\, Axzy$$

—i.e., each juror was asked a question uniformly by all reproters—while still there was no one question that appeared across interviews in which both the reporter and juror differed. In short, while the conjunction

$$(\forall x\text{: } Rx)\,(\exists z\text{: } Qz)\,(\forall y\text{: } Jy)\, Axzy \wedge (\forall y\text{: } Jy)\,(\exists z\text{: } Qz)\,(\forall x\text{: } Rx)\, Axzy$$

says more than either of its conjuncts, it still says less than the claim

$$(\exists z\text{: } Qz)\,(\forall x\text{: } Rx)\,(\forall y\text{: } Jy)\, Axzy$$

of doubly uniform exemplification.

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8.2.3. Controlling ambiguity

Although ambiguity is common in English sentences when claims of exemplification are combined with generalization, there are some indicators that tend to make a given interpretation more likely even though this is rarely more than a tendency. There are, of course, many contextual indicators of the correct interpretation, and these can be quite strong, but explicit verbal indication is rarely conclusive.

Probably the most important verbal indication of relative scope is the simplest, word order. All things being equal, the first quantifier phrase is understood to be the main one. This means that the passive voice plays an important role in indicating the sort of claim we wish to make since it enables us to alter word order and promote a given quantifier phrase to subject position. Subject-predicate expansion can help in the same way when it is stylistically acceptable.

The effect in subject-predicate expansion is due not only to word order but also to another syntactic indicator: quantifier phrases within relative clauses are usually understood to have narrower scope than those outside them. In the case of existentials, use of the there-is form, which typically also involves a relative clause, will accomplish the same thing as an expanded form, and usually with better style. For example, There is [or was] a reporter who interviewed each juror makes a pretty definite claim of uniformity.

In addition to such syntactic indicators, word choice can play a role. The words each, every, and all (and any when it is grammatically possible) in the case of generalizations and the words some and a in the case of existential claims lend varying degrees of force to a quantifier phrase's claim to a wide scope. Perhaps these words never overcome the effects of word order, but they can moderate it, as may been seen with the following four restatements of the example discussed in 8.2.1:

Some reporter interviewed every juror	A reporter interviewed each juror
Every juror was interviewed by some reporter	Each juror was interviewed by a reporter

The guiding idea here is that the word some marks a stronger claim to wide scope than the word a does and that the word each marks a stronger claim than the word every. The sentence at the upper left is the most likely to be understood as a claim of uniformly general exemplification, and the one at the lower right is the least likely to be understood to claim uniformity; the other two cases are intermediate, with word order probably beating out word choice so that the sentence at the upper right is the second most likely to be understood to involve a claim of uniformity.

But, while the choices of wording mentioned so far are perhaps never enough to overcome the effects of word order, there are other words choices that are. There is a use of the word certain that seems to function only to mark an existential quantifier phrase as having wide scope. If we add this word to the existential quantifier phrase in the sentence at the lower right, we get Each juror was interviewed by a certain reporter and this sentence stands a very good chance of being interpreted as a claim of uniform exemplification in spite of word order and other choices of wording. On the other hand, if we add or other to the existential in the top left sentence, we get Some reporter or other interviewed every juror, which is less likely to carry an implication of uniformity.

Context can also play a role in the significance of word choice. For example, if we were discussing the events surrounding a sort of sensational trial that was typical of some historical era (rather than discussing a particular example of such a trial), there would be an implicit generalization concerning such trials in what we said. The use of or other might then simply cancel a claim of uniformity with respect to trials while allowing it to be maintained with respect to jurors.

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8.2.s. Summary

1 English sentences that involve both generalization and claims of exemplification are often ambiguous, and the differences between interpretations can be expressed in analyses by the relative scope of universal and existential quantifiers. We will refer to a sentence that mixes generalization and with a claim of exemplification as a claim of general exemplification. One in which the existential has wider scope than the universal can be thought of as a claim of uniformly general exemplification because it asserts that a single example can be given that suffices for all instances of the generalization.

2 When more than two quantifier phrases are present, an existential may be classified as making or not making a claim of uniformity with respect to each universal, giving rise to a variety of uniformity claims that a sentence may be understood to make. The issue of quantifier scope can thus be addressed by asking, for each of the dimensions of generality with which a claim of exemplification is asserted, whether the exemplification is claimed to be uniform in that dimension; this settles the relative scope of each existential with respect to each universal, and the relative scope of contiguous universals and contiguous existentials has no impact on what is said.

3 The ambiguity in sentences involving both existentials and universals is hard to eliminate, but syntax and word choice can help. The first quantifier phrase is usually understood to have widest scope, and a quantifier phrase in a relative clause usually has its scope limited to that clause (a fact that makes the there-is form useful). The choice of quantifier words can counteract the effect of word order to some extent, and the use of the special quantifier phrases a certain X and some X or other will tend to advance or to renounce a claim of uniformity, respectively, in a fairly strong way.

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8.2.x. Exercise questions

1. Analyze the following in as much detail as possible. Since it is difficult to completely avoid ambiguity in English sentences that both generalize and make existential claims, alternative non-equivalent analyses are possible in some cases. You should choose an analysis that captures the most likely interpretation (or one of the most likely ones). The answers will represent my own judgment about this.

   a. Everyone has seen a bear.
   b. Everyone was talking about a certain movie.
   c. A capital was chosen by each state.
   d. There is a capital that was chosen by each state.
   e. Someone who no reporter knew leaked the information.
   f. A head of a horse is the head of a mammal.
   g. Everyone who has seen a rainbow has seen a rainstorm.
   h. Every child was given a toy by each Santa.
   i. There is a toy that was given to every child by each Santa.

2. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below by the intensional interpretations that are provided for each sentence or group of sentences.

   a. $\forall x\, \exists y\, Dxy$ D: [_ depends on _]
   b. $\exists x\, \forall y\, Dxy$
   c. $\forall x\, \exists y\, Dyx$
   d. $\exists x\, \forall y\, Dyx$

   e. $(\forall x: Px \wedge Hx)\, (\exists y: Py)\, Axy$ A: [_ admires _]; H: [_ is humble]; P: [_ is a person]
   f. $(\exists y: Py)\, (\forall x: Px \wedge Hx)\, Axy$
   g. $\neg\, (\forall x: Px \wedge (\exists y: Py)\, Axy)\, Hx$

   h. $\neg\, (\exists x: Px)\, (\forall y: Py \wedge Syx)\, Sxy$ P: [_ is a person]; S: [_ has seen _]

   i. $\neg\, (\exists x: Px \wedge (\forall y: \neg\, (Py \wedge Syx))\, \neg\, Sxy)\, Ex$

   E: [_ is an extrovert]; P: [_ is a person]; S: [_ has spoken to _]

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8.2.xa. Exercise answers

1.

a. Everyone has seen a bear
Everyone is such that (he or she has seen a bear)
($\forall$x: x is a person) x has seen a bear
($\forall$x: Px) a bear is such that (x has seen it)
($\forall$x: Px) ($\exists$y: y is a bear) x has seen y
($\forall$x: Px) ($\exists$y: By) Sxy
$\forall$x (Px → $\exists$y (By $\land$ Sxy))
B: [_ is a bear]; P: [_ is a person]; S: [_ has seen _]

b. Everyone was talking about a certain movie
A certain movie is such that (everyone was talking about it)
($\exists$x: x is a movie) everyone was talking about x
($\exists$x: Mx) everyone is such that (he or she was talking about x)
($\exists$x: Mx) ($\forall$y: y is a person) y was talking about x
($\exists$x: Mx) ($\forall$y: Py) Tyx
$\exists$x (Mx $\land$ $\forall$y (Py → Tyx))
M: [_ is a movie]; P: [_ is a person]; T: [_ was talking about _]

c. A capital was chosen by each state
Each state is such that (a capital was chosen by it)
($\forall$x: x is a state) a capital was chosen by x
($\forall$x: Sx) a capital is such that (it was chosen by x)
($\forall$x: Sx) ($\exists$y: y is a capital) y was chosen by x
($\forall$x: Sx) ($\exists$y: Cy) Hyx
$\forall$x (Sx → $\exists$y (Cy $\land$ Hyx))
C: [_ is a capital]; H: [_ was chosen by _]; S: [_ is a state]

d. There is a capital that was chosen by each state
Something is a capital that was chosen by each state
Something is such that (it is a capital that was chosen by each state)
$\exists$x x is a capital that was chosen by each state
$\exists$x (x is a capital $\land$ x was chosen by each state)
$\exists$x (Cx $\land$ each state is such that (x was chosen by it))
$\exists$x (Cx $\land$ ($\forall$y: y is a state) x was chosen by y)
$\exists$x (Cx $\land$ ($\forall$y: Sy) Hxy)
$\exists$x (Cx $\land$ $\forall$y (Sy → Hxy))
C: [_ is a capital]; H: [_ was chosen by _]; S: [_ is a state]

e. Someone who no reporter knew leaked the information
Someone who no reporter knew is such that (he or she leaked the information)
($\exists$x: x is a person who no reporter knew) x leaked the information
($\exists$x: x is a person $\land$ no reporter knew x) Lxi
($\exists$x: Px $\land$ no reporter is such that (he or she knew x)) Lxi
($\exists$x: Px $\land$ ($\forall$y: y is a reporter) ¬ y knew x) Lxi
($\exists$x: Px $\land$ ($\forall$y: Ry) ¬ Kyx) Lxi
$\exists$x ((Px $\land$ $\forall$y (Ry → ¬ Kyx)) $\land$ Lxi)
or: ($\exists$x: Px $\land$ ¬ ($\exists$y: Ry) Kyx) Lxi
K: [_ knew _]; L: [_ leaked y]; P: [_ is a person]; R: [_ is a reporter]; i: the information

f. A head of a horse is the head of a mammal
Every head of a horse is such that (it is the head of a mammal)
($\forall$x: x is the head of a horse) x is the head of a mammal
($\forall$x: a horse is such that (x is the head of it)) a mammal is such that (x is the head of it)
($\forall$x: ($\exists$y: y is a horse) x is the head of y) ($\exists$z: z is a mammal) x is the head of z
($\forall$x: ($\exists$y:Hy) x = the head of y) ($\exists$z: Mz) x = the head of z
($\forall$x: ($\exists$y: Hy) x = hy) ($\exists$z: Mz) x = hz
$\forall$x ($\exists$y (Hy $\land$ x = hy) → $\exists$z (Mz $\land$ x = hz))
or: ($\forall$x: ($\exists$y: Hy) Dxy) ($\exists$z: Mz) Dxz
D: [_ is a head of _]; H: [_ is a horse]; M: [_ is a mammal]; h: [the head of _]
In this interpretation, which seems most natural given the content of the sentence, the indefinite article a is understood to indicate a generalization rather than a claim of exemplification. That is, it amounts to a use of any but a use of that term which is equivalent to, rather than in contrast with, a use of every. Indeed, a appears in a location where any would not contrast with every, so the force of any in this context could not be captured by a use of some.

g. Everyone who has seen a rainbow has seen a rainstorm
Everyone who has seen a rainbow is such that (he or she has seen a rainstorm)
($\forall$x: x is person who has seen a rainbow) x has seen a rainstorm
($\forall$x: x is person $\land$ x has seen a rainbow) a rainstorm is such that (x has seen it)

(∀x: x is person ∧ a rainbow is such that (x has seen it)) (∃z: z is a rainstorm) x has seen z

(∀x: x is person ∧ (∃y: y is a rainbow) x has seen y) (∃z: Rz) Sxz

(∀x: Px ∧ (∃y: Ry) Sxy) (∃z: Tz) Sxz

∀x ((Px ∧ ∃y (Ry ∧ Sxy)) → ∃z (Tz ∧ Sxz))

P: [_ is a person]; R: [_ is a rainbow]; S: [_ has seen _]; T: [_ is a rainstorm]

h. Every child was given a toy by each Santa

Every child is such that (he or she was given a toy by each Santa)

(∀x: x is a child) x was given a toy by each Santa

(∀x: Cx) each Santa is such that (x was given a toy by him or her)

(∀x: Cx) (∀y: y is a Santa) x was given a toy by y

(∀x: Cx) (∀y: Sy) a toy is such that (x was given it by y)

(∀x: Cx) (∀y: Sy) (∃z: z is a toy) x was given z by y

(∀x: Cx) (∀y: Sy) (∃z: Tz) Gxzy

∀x (Cx → ∀y (Sy → ∃z (Tz ∧ Gxzy)))

C: [_ is a child]; G: [_ was given _ by _]; S: [_ is a Santa]; T: [_ is a toy]

Notice that, in spite of the capitalization, Santa is not used here as a proper name but instead as a sort of job title. As a result it is represented not by an individual term but instead by a predicate. For representation by an individual term to be appropriate, it would have to be possible to paraphrase the sentence using each thing that is Santa rather than each thing that is a Santa.

i. There is a toy that every child was given by each Santa

Something is a toy that every child was given by each Santa

∃x x is a toy that every child was given by each Santa

∃x (x is a toy ∧ every child was given x by each Santa)

∃x (Tx ∧ every child is such that (he or she was given x by each Santa))

∃x (Tx ∧ (∀y: y is a child) y was given x by each Santa)

∃x (Tx ∧ (∀y: Cy) each Santa is such that (y was given x by him or her))

∃x (Tx ∧ (∀y: Cy) (∀z: z is a Santa) y was given x by z)

∃x (Tx ∧ (∀y: Cy) (∀z: Sz) Gyxz)

∃x (Tx ∧ ∀y (Cy → ∀z (Sz → Gyxz)))

C: [_ is a child]; G: [_ was given _ by _]; S: [_ is a Santa]; T:

[_ is a toy]

2. **a.** ∀x ∃y x depends on y

∀x something is such that (x depends on it)

∀x x depends on something

Everything is such that (it depends on something)

Everything depends on something

b. ∃x ∀y x depends on y

∃x everything is such that (x depends on it)

∃x x depends on everything

Something is such that (it depends on everything)

Something depends on everything

c. ∀x ∃y y depends on x

∀x something is such that (it depends on x)

∀x something depends on x

Everything is such that something depends on it

or: Everything has something depending on it

or (perhaps): Something or other depends on each thing

d. ∃x ∀y y depends on x

∃x everything is such that (it depends on x)

∃x everything depends on x

Something is such that everything depends on it

or: Something has everything depending on it

or: There is something that everything depends on

or (perhaps): All things depend on a certain thing

e. (∀x: x is a person ∧ x is humble) (∃y: y is a person) x admires y

(∀x: x is a humble person) someone is such that (x admires him or her)

(∀x: x is a humble person) x admires someone

Every humble person is such that (he or she admires someone)

Every humble person admires someone

or: Everyone who is humble admires someone

f. (∃y: y is a person) (∀x: x is a person ∧ x is humble) x admires y

(∃y: y is a person) (∀x: x is a humble person) x admires y

(∃y: y is a person) every humble person is such that (he or she admires y)

(∃y: y is a person) every humble person admires y

Someone is such that every humble person [*or:* everyone who is humble] admires him or her

or: Someone has every humble person admiring him or her

or: There is someone [*or:* a person] who every humble person admires

or (perhaps): All who are humble admire a certain person

g. ¬ (∀x: x is a person ∧ (∃y: y is a person) x admires y) x is humble

¬ (∀x: x is a person ∧ someone is such that (x admires him or her)) x is humble

¬ (∀x: x is a person ∧ x admires someone) x is humble

¬ (∀x: x is a person who admires someone) x is humble

¬ everyone who admires someone is such that (he or she is humble)

¬ everyone who admires someone is humble

Not everyone who admires someone is humble

or: Not everyone who admires anyone is humble

h. ¬ (∃x: x is a person) (∀y: y is a person ∧ y has seen x) x has seen y

¬ (∃x: x is a person) (∀y: y is a person who has seen x) x has seen y

¬ (∃x: x is a person) everyone who has seen x is such that (x has seen him or her)

¬ (∃x: x is a person) x has seen everyone who has seen x

¬ someone is such that (he or she has seen everyone who has seen him or her)

¬ someone has seen everyone who has seen him or her

No one has seen everyone who has seen him or her

i. ¬ (∃x: x is a person ∧ (∀y: ¬ (y is a person ∧ y has spoken to x)) ¬ x has spoken to y) x is an extrovert

¬ (∃x: x is a person ∧ (∀y: ¬ y is a person who has spoken to x) ¬ x has spoken to y) x is an extrovert

¬ (∃x: x is a person ∧ only people who have spoken to x are such that(x has spoken to them)) x is an extrovert

¬ (∃x: x is a person ∧ x has spoken only to people who have spoken to x) x is an extrovert

¬ (∃x: x is a person who has spoken only to people who have spoken to him or her) x is an extrovert

¬ someone who has spoken only to people who have spoken to him or her is such that (he or she is an extrovert)

¬ someone who has spoken only to people who have spoken to him or her is an extrovert

No one who has spoken only to people who have spoken to him or her is an extrovert