Phi 270 F08 test 5

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. Dave found a coin. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]
answer

2. There is an elf who neglects no one. [Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]
answer

3. Everyone watched a movie. [On one way of understanding this sentence, it would not be true unless everyone watched the same movie. Analyze it according to that interpretation.]
answer

4. Someone sang to someone else.
answer

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator to analyze the definite description.

5. Rudolph guided the sleigh that flew.
answer

Use a derivation to show that the following argument is valid. You may use any rules.

6.
∃x Gx
∀x Fx
———
∃x (Fx ∧ Gx)

answer

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that divides an open gap of your derivation.

7.
∃x ∀y Rxy
———
∀x ∃y Rxy

answer

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

8. A pair of sentences φ and ψ entails a sentence χ (in symbols, φ, ψ ⊨ χ) if and only if ...
answer

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). A letter standing for an individual term should appear in your analysis only as often as the individual term appears in the original sentence.

9. Bill called Carol and mentioned his father to her.
answer

Phi 270 F08 test 5 answers

1. Dave found a coin
A coin is such that (Dave found it)
(∃x: x is a coin) Dave found x
(∃x: Cx) Fdx
∃x (Cx ∧ Fdx)

C: [_ is a coin]; F: [_ found _]; d: Dave

2. There is an elf who neglects no one
Something is an elf who neglects no one
∃x x is an elf who neglects no one
∃x (x is an elf ∧ x neglects no one)
∃x (x is an elf ∧ ¬ x neglects someone)
∃x (Ex ∧ ¬ someone is such that (x neglects him or her))
∃x (Ex ∧ ¬ (∃y: y is a person) x neglects y)
∃x (Ex ∧ ¬ (∃y: Py) Nxy)

E: [_ is an elf]; N: [_ neglects _]; P: [_ is a person]

3. Everyone watched a movie
some movie is such that (everyone watched it)
(∃x: x is a movie) everyone watched x
(∃x: Mx) everyone is such that (he or she watched x)
(∃x: Mx) (∀y: y is a person) y watched x
(∃x: Mx) (∀y: Py) Wyx

M: [_ is a movie]; P: [_ is a person]; W: [_ watched _]

The alternative interpretation Everyone is such that (he or she watched a movie) could be true even if there was no one movie that everyone watched

4. Someone sang to someone else

Someone is such that (he or she sang to someone else)

(∃x: x is a person) x sang to someone else

(∃x: Px) someone other than x is such that (x sang to him or her)

(∃x: Px) (∃y: y is a person ∧ ¬ y = x) x sang to y

$$(\exists x\colon Px)\ (\exists y\colon Py \land \neg\, y = x)\ Sxy$$

P: [_ is a person]; S: [_ sang to _]

5. *Using Russell's analysis:*

Rudolph guided the sleigh that flew

the sleigh that flew is such that (Rudolph guided it)

(∃x: x is a sleigh that flew ∧ only x is a sleigh that flew) Rudolph guided x

(∃x: (x is a sleigh ∧ x flew) ∧ (∀y: ¬ y = x) ¬ (y is a sleigh ∧ y flew)) Grx

$$(\exists x\colon (Sx \land Fx) \land (\forall y\colon \neg\, y = x)\ \neg (Sy \land Fy))\ Grx$$

also correct: $(\exists x\colon (Sx \land Fx) \land \neg (\exists y\colon \neg\, y = x)\ (Sy \land Fy))\ Grx$

also correct: $(\exists x\colon (Sx \land Fx) \land (\forall y\colon Sy \land Fy)\ x = y)\ Grx$

Using the description operator:

Rudolph guided the sleigh that flew

[_ guided _] Rudolph the sleigh that flew

Gr(Ix x is a sleigh that flew)

Gr(Ix (x is a sleigh ∧ x flew))

$$Gr(\text{I}x\ (Sx \land Fx))$$

F: [_ flew]; G: [_ guided _]; S: [_ is a sleigh]; r: Rudoph

6.

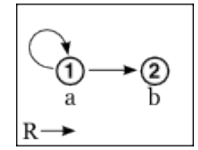
	∃x Gx	1
	∀x Fx	a:2
	ⓐ Ga	(6)
2 UI	Fa	(5)
	∀x ¬ (Fx ∧ Gx)	a:4
4 UI	¬ (Fa ∧ Ga)	5
5 MPT	¬ Ga	(6)
	●	
6 Nc	⊥	3
3 NCP	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	

or

	∃x Gx	1
	∀x Fx	a:2
	ⓐ Ga	(3)
2 UI	Fa	(3)
3 Adj	Fa ∧ Ga	X, (4)
4 EG	∃x (Fx ∧ Gx)	X, (5)
	●	
5 QED	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	

7.

	∃x ∀y Rxy	1
	ⓐ ∀y Ray	a:4, b:5
	ⓑ ∀y ¬ Rby	a:6, b:7
4 UI	Raa	
5 UI	Rab	
6 UI	¬ Rba	
7 UI	¬ Rbb	
	○	Raa, Rab, ¬ Rba, ¬ Rbb ⊭ ⊥
	⊥	3
3 NCP	∃y Rby	2
2 UG	∀x ∃y Rxy	1
1 PCh	∀x ∃y Rxy	



8. A pair of sentences φ and ψ entails a sentence χ if and only if there is no possible world in which both φ and ψ are true and χ is false

or

A pair of sentences φ and ψ entails a sentence χ if and only if χ is true in every possible world in which both φ and ψ are true

9. Bill called Carol and mentioned his father to her

Bill and Carol are such that (he called her and mentioned his father to her)

[x called y and mentioned x's father to y]$_{xy}$ Bill Carol

[x called y ∧ x mentioned x's father to y]$_{xy}$bc

[Cxy ∧ Mx(x's father)y]$_{xy}$bc

$$[Cxy \land Mx(fx)y]_{xy}bc$$

C: [_ called _]; M: [_ mentioned _ to _]; b: Bill; c: Carol; f: [_'s father]

Phi 270 F06 test 5

Analyze the following sentences in as much detail as possible, providing a key to the items of non-logical vocabulary (upper and lower case letters apart from variables) that appear in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. Someone called Tom. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]
answer

2. Not a crumb was left, but there was a note from Santa. [Do not use ∀ in your analysis of this; that is, use ∃ in your analysis of any quantifier phrases.]
answer

3. A card was sent to each customer. [On one way of understanding this sentence, it would be true even if no two customers were sent the same card. Analyze it according to that interpretation.]
answer

4. At most one size was left.
answer

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator.

5. Ann found the note that Bill left.
answer

Use a derivation to show that the following argument is valid. You may use any rules.

6. ∃x (Fx ∧ Gx)
∀x (Gx → Hx)
———
∃x Hx

answer

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that divides an open gap of your derivation.

7. ∃x ∃y (Rxa ∧ Ray)
———
∃x Rxx

answer

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

8. A pair of sentences φ and ψ are logically equivalent (in symbols, φ ≃ ψ) if and only if ...
answer

Analyze the sentence below using abstracts and variables to represent pronominal cross reference to individual terms (instead of replacing pronouns by such antecedents). An individual term should appear in your analysis only as often as it appears in the original sentence.

9. Ann wrote to Bill and he called her.
answer

Phi 270 F06 test 5 answers

1. Someone called Tom
Someone is such that (he or she called Tom)
(∃x: x is a person) x called Tom
(∃x: Px) Cxt
∃x (Px ∧ Cxt)
C: [_ called _]; P: [_ is a person]; t: Tom

2. Not a crumb was left, but there was a note from Santa
Not a crumb was left ∧ there was a note from Santa
¬ a crumb was left ∧ something was a note from Santa
¬ some crumb is such that (it was left) ∧ something is such that (it was a note from Santa)
¬ (∃x: x is a crumb) x was left ∧ ∃y (y was a note from Santa)
¬ (∃x: Cx) Lx ∧ ∃y (y was a note ∧ y was from Santa)
¬ (∃x: Cx) Lx ∧ ∃y (Ny ∧ Fys)
C: [_ is a crumb]; F: [_ was from _]; L: [_ was left]; N: [_ was a note]; s: Santa

3. A card was sent to each customer
each customer is such that (a card was sent to him or her)
(∀x: x is a customer) a card was sent to x
(∀x: Cx) some card is such that (it was sent to x)
(∀x: Cx) (∃y: y is a card) y was sent to x
(∀x: Cx) (∃y: Dy) Syx
C: [_ is a customer]; D: [_ is a card]; S: [_ was sent to _]

Some card is such that (it was sent to each customer) would be true only if there was at least one card that was sent to all customers, so an analysis of it would not be a correct answer

4. At most one size was left

¬ at least two sizes were left

¬ at least two sizes are such that (they were left)

¬ (∃x: x is a size) (∃y: y is a size ∧ ¬ y = x) (x was left ∧ y was left)

¬ (∃x: Sx) (∃y: Sy ∧ ¬ y = x) (Lx ∧ Ly)

S: [_ is a size]; L: [_ was left]

also correct: (∀x: Sx) (∀y: Sy ∧ ¬ y = x) ¬ (Lx ∧ Ly)

also correct: (∀x: Sx ∧ Lx) (∀y: Sy ∧ Ly) x = y

5. *Using Russell's analysis:*

Ann found the note that Bill left

the note that Bill left is such that (Ann found it)

(∃x: x is a note that Bill left ∧ only x is a note that Bill left) Ann found x

(∃x: (x is a note ∧ Bill left x) ∧ (∀y: ¬ y = x) ¬ (y is a note ∧ Bill left x)) Fax

(∃x: (Nx ∧ Lbx) ∧ (∀y: ¬ y = x) ¬ (Ny ∧ Lby)) Fax

also correct: (∃x: (Nx ∧ Lbx) ∧ ¬ (∃y: ¬ y = x) (Ny ∧ Lby)) Fax

also correct: (∃x: (Nx ∧ Lbx) ∧ (∀y: Ny ∧ Lby) x = y) Fax

Using the description operator:

Ann found the note that Bill left

[_ found _] Ann (the note that Bill left)

Fa(Ix x is note that Bill left)

Fa(Ix (x is a note ∧ Bill left x))

Fa(Ix (Nx ∧ Lbx))

F: [_ found _]; L: [_ left _]; N: [_ is a note]; a: Ann; b: Bill

6.

```
        | ∃x (Fx ∧ Gx)      1
        | ∀x (Gx → Hx)      a: 3
        |---
        | (a)
        | | Fa ∧ Ga          2
        | |---
2 Ext   | | Fa
2 Ext   | | Ga               (4)
3 UI    | | Ga → Ha          4
4 MPP   | | Ha               (5)
5 EG    | | ∃x Hx            X,6
        | | ●
6 QED   | | ∃x Hx            1
        |---
1 Pch   | ∃x Hx
```

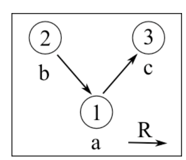
or

```
        | ∃x (Fx ∧ Gx)      1
        | ∀x (Gx → Hx)      a: 3
        |---
        | (a)
        | | Fa ∧ Ga          2
        | |---
2 Ext   | | Fa
2 Ext   | | Ga               (4)
3 UI    | | Ga → Ha          4
4 MPP   | | Ha               (7)
        | | | ∀x ¬ Hx        a: 6
        | | |---
6 UI    | | | ¬ Ha         (7)
        | | | ●
        | | |---
7 Nc    | | | ⊥            5
        | |---
5 NcP   | | ∃x Hx            1
        |---
1 PCh   | ∃x Hx
```

Many different orders are possible for the rules used. In particular, NcP could be used before PCh in the second.

7.

```
        | ∃x ∃y (Rxa ∧ Ray)        1
        |---
        | (b)
        | | ∃y (Rba ∧ Ray)         2
        | |---
        | | (c)
        | | | Rba ∧ Rac            3
        | | |---
3 Ext   | | | Rba
3 Ext   | | | Rac
        | | | | ∀x ¬ Rxx          a:5, b:6, c:7
        | | | |---
5 UI    | | | | ¬ Raa
6 UI    | | | | ¬ Rbb
7 UI    | | | | ¬ Rcc
        | | | | ○               Rba, Rac, ¬ Raa, ¬ Rbb, ¬ Rcc ⊭ ⊥
        | | | |---
        | | | | ⊥               4
        | | |---
4 NcP   | | | ∃x Rxx             2
        | |---
2 PCh   | | ∃x Rxx               1
        |---
1 PCh   | ∃x Rxx
```



range: 1, 2, 3

a	b	c
1	2	3

R	1	2	3
1	F	F	T
2	T	F	F
3	F	F	F

8. A pair of sentences φ and ψ are logically equivalent if and only if there is no possible world in which φ and ψ have different truth values

or

A pair of sentences φ and ψ are logically equivalent if and only if φ and ψ have the same truth value as each other in every possible world

9. Ann wrote to Bill and he called her

Ann and Bill are such that (she wrote to him and he called her)

[x wrote to y and y called x]$_{xy}$ Ann Bill

[x wrote to y ∧ y called x]$_{xy}$ab

[Wxy ∧ Cyx]$_{xy}$ab

C: [_ called _]; W: [_ wrote to _]; a: Ann; b: Bill

Phi 270 F05 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the special instructions given for each of **1**, **2**, and **3**.

1. A bell rang. [Give an analysis using a restricted quantifier, and restate it using an unrestricted quantifier.]

answer

2. There was a storm but no flight was delayed. [Avoid using ∀ in your analysis of any quantifier phrases in this sentence.]

answer

3. Everyone was humming a tune. [On one way of understanding this sentence, it would be false if people were humming different tunes. Analyze it according to that interpretation.]

answer

4. Tom saw at least two snowflakes.

answer

Analyze the sentence below using each of the two ways of analyzing the definite description. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases as well as one that uses the description operator.

5. Ann saw the play.

answer

Use a derivation to show that the following argument is valid. You may use any rules.

6. ∃x (Fa → Gx)

———————

Fa → ∃x Gx

answer

Use a derivation to show that the following argument is not valid, and use either a diagram or tables to present a counterexample that divides an open gap of your derivation.

7. ∃x Fx

∃x Rxa

———————

∃x (Fx ∧ Rxa)

answer

Complete the following to give a definition of inconsistency in terms of truth values and possible worlds:

8. A set Γ of sentences is inconsistent (in symbols, Γ ⊨ or, equivalently, Γ ⊨ ⊥) if and only if …

answer

Complete the following truth table for the two rows shown. In each row, indicate the value of each compound component of the sentence on the right by writing the value under the main connective of that component (so, in each row, every connective should have a value under it); also circle the value that is under the main connective of the whole sentence.

9.

A	B	C	D	$(A \to \neg C) \wedge \neg (B \vee D)$
T	F	F	F	
F	F	T	T	

answer

Phi 270 F05 test 5 answers

1. A bell rang

Some bell is such that (it rang)

(∃x: x is a bell) x rang

$(\exists x{:}\ Bx)\ Rx$

$\exists x\ (Bx \wedge Rx)$

B: [_ is a bell]; R: [_ rang]

2. There was a storm but no flight was delayed

There was a storm ∧ no flight was delayed

Something was a storm ∧ ¬ some flight was delayed

Something is such that (it was a storm) ∧ ¬ some flight is such that (it was delayed)

∃x x was a storm ∧ ¬ (∃x: x is a flight) x was delayed

$\exists x\ Sx \wedge \neg (\exists x{:}\ Fx)\ Dx$

D: [_ was delayed]; F: [_ is a flight]; S: [_ was a storm]

3. Everyone was humming a tune

Some tune is such that (everyone was humming it)

(∃x: x is a tune) everyone was humming x

(∃x: Tx) everyone is such that (he or she was humming x)

(∃x: Tx) (∀y: y is a person) (y was humming x)

$(\exists x{:}\ Tx)\ (\forall y{:}\ Py)\ Hyx$

H: [_ was humming _]; P: [_ is a person]; T: [_ is a tune]

Everyone is such that (he or she was humming a tune) could be true even though people were humming different tunes, so an analysis of it would not be a correct answer.

4. Tom saw at least two snowflakes

At least two snowflakes are such that (Tom saw them)

(∃x: x is a snowflake) (∃y: y is a snowflake ∧ ¬ y = x) (Tom saw x ∧ Tom saw y)

$(\exists x{:}\ Fx)\ (\exists y{:}\ Fy \wedge \neg\, y = x)\ (Stx \wedge Sty)$

F: [_ is a snowflake]; S: [_ saw _]; t: Tom

5. *Using Russell's analysis:*

Ann saw the play

The play is such that (Ann saw it)

(∃x: x is a play ∧ (∀y: ¬ y = x) ¬ y is a play) Ann saw x

$(\exists x{:}\ Px \wedge (\forall y{:}\ \neg\, y = x)\ \neg Py)\ Sax$

also correct:

$(\exists x{:}\ Px \wedge \neg (\exists y{:}\ \neg\, y = x)\ Py)\ Sax$

or:

$(\exists x{:}\ Px \wedge (\forall y{:}\ Py)\ x = y)\ Sax$

Using the description operator:

Ann saw the play

S Ann the play

Sa (Ix x is a play)

$Sa(\text{I}x\ Px)$

P: [_ is a play]; S: [_ saw _]; a: Ann

6.

	∃x (Fa → Gx)	2
	Fa	(3)
	ⓑ Fa → Gb	3
3 MPP	Gb	(4)
4 EG	∃x Gx	X,(5)
	●	
5 QED	∃x Gx	2
2 PCh	∃x Gx	1
1 CP	Fa → ∃x Gx	

or

	∃x (Fa → Gx)	2
	Fa	(3)
	ⓑ Fa → Gb	3
3 MPP	Gb	(6)
	∀x ¬ Gx	b:5
5 UI	¬ Gb	(6)
	●	
6 Nc	⊥	4
4 NcP	∃x Gx	2
2 PCh	∃x Gx	1
1 CP	Fa → ∃x Gx	

The order of CP and PCh can be reversed in these and the use of MPP in the second could come after NcP and UI.

7.

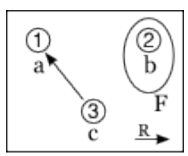
	∃x Fx	1
	∃x Rxa	2
	ⓑ Fb	(5)
	ⓒ Rca	(7)
	∀x ¬ (Fx ∧ Rxa)	b:4, c:6, a:8
4 UI	¬ (Fb ∧ Rba)	5
5 MPT	¬ Rba	
6 UI	¬ (Fc ∧ Rca)	7
7 MPT	¬ Fc	
8 UI	¬ (Fa ∧ Raa)	9
	¬ Fa	
	○	Fb,Rca,¬Rba,¬Fc,¬Fa ⊭ ⊥
	⊥	11
11 IP	Fa	10
	¬ Raa	
	○	Fb,Rca,¬Rba,¬Fc,¬Raa ⊭ ⊥
	⊥	12
12 IP	Raa	10
10 Cnj	Fa ∧ Raa	9
9 CR	⊥	3
3 NcP	∃x (Fx ∧ Rxa)	2
2 PCh	∃x (Fx ∧ Rxa)	1
1 PCh	∃x (Fx ∧ Rxa)	

range: 1, 2, 3

a	b	c
1	2	3

τ	Fτ
1	F
2	T
3	F

R	1	2	3
1	F	F	F
2	F	F	F
3	T	F	F



This interpretation divides both gaps; the value for F1 is needed only for the first gap and the value for R11 is needed only for the second.

8. A set Γ of sentences is inconsistent if and only if there is no possible world in which all members of Γ are true

or

A set Γ of sentences is inconsistent if and only if, in each possible world, at least one member of Γ is false

9.

A	B	C	D	(A → ¬ C) ∧ ¬ (B ∨ D)
T	F	F	F	T T Ⓣ T F
F	F	T	T	T F Ⓕ F T

Phi 270 F04 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the special instructions given for **1** and **3**.

1. Someone was singing [Present your analysis also using an unrestricted quantifier.]
answer

2. There is a package that isn't addressed to anyone.
answer

3. An airline served each airport. [This sentence is ambiguous. On one way of interpreting it, it could be true even if no one airline served all airports. Analyze the sentence according to that interpretation of it.]
answer

4. At least two people called.
answer

Analyze the sentence below using each of the two ways of analyzing the definite description the sleigh Santa drove. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases and another analysis that uses the description operator.

5. The sleigh Santa drove was red.
answer

Use derivations to show that the following arguments are valid. You may use any rules.

6. $\exists x\,(Fx \wedge Gx)$
$\overline{\qquad\qquad\qquad}$
$\exists x\, Gx$
answer

7. $\exists x\,(Fx \wedge \exists y\, Rxy)$
$\overline{\qquad\qquad\qquad}$
$\exists x\, \exists y\,(Fy \wedge Ryx)$
answer

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

8. A sentence φ is entailed by a set Γ (i.e., Γ ⊨ φ) if and only if …
answer

Complete the following truth table for the two rows shown. Indicate the value of each component of the sentence on the right by writing the value under the main connective of that component.

9.

A B C D	¬ (A ∧ B) → (¬ C ∨ D)
T T F F	
F F T F	

answer

Use either tables or a diagram to describe a structure in which the following sentences are true. (That is, do what would be required to present a counterexample when a dead-end gap of a derivation had these sentences as its active resources.)

10. a = c, fa = fb, ¬ Ga, Gb, G(fc), Ra(fb), Rb(fa)
answer

Phi 270 F04 test 5 answers

1. Someone was singing
Someone is such that (he or she was singing)
(∃x: x is a person) x was singing
(∃x: Px) Sx
∃x (Px ∧ Sx)
P: [_ is a person]; S: [_ was singing]

2. There is a package that isn't addressed to anyone
Something is a package that isn't addressed to anyone
∃x x is a package that isn't addressed to anyone
∃x (x is a package ∧ x isn't addressed to anyone)
∃x (Kx ∧ ¬ x is addressed to someone)
∃x (Kx ∧ ¬ someone is such that (x is addressed to him or her))
∃x (Kx ∧ ¬ (∃y: y is a person) x is addressed to y)
∃x (Kx ∧ ¬ (∃y: Py) Axy)
or: ∃x (Kx ∧ (∀y: Py) ¬ Axy)
A: [_ is addressed to _]; K: [_ is a package]; P: [_ is a person]

3. An airline served each airport
Every airport is such that (an airline served it)
(∀x: x is an airport) an airline served x
(∀x: Ax) some airline is such that (it served x)
(∀x: Ax) (∃y: y is an airline) y served x
(∀x: Ax) (∃y: Ly) Syx
P: [_ is an airport]; L: [_ is an airline]; S: [_ served _]
(∃x: Lx) (∀y: Ay) Sxy would be incorrect since it is true only if there is a single airline that serves all airports

4. At least two people called
At least two people are such that (they called)
(∃x: x is a person) (∃y: y is a person ∧ ¬ y = x) (x called ∧ y called)
(∃x: Px) (∃y: Py ∧ ¬ y = x) (Cx ∧ Cy)
C: [_ called]; P: [_ is a person]

5. Using Russell's analysis:

The sleigh Santa drove was red

The sleigh Santa drove is such that (it was red)

(∃x: x is a sleigh Santa drove ∧ (∀y: ¬ y = x) ¬ y is a sleigh Santa drove) x was red

(∃x: (x is a sleigh ∧ Santa drove x) ∧ (∀y: ¬ y = x) ¬ (y is a sleigh ∧ Santa drove y)) x was red

(∃x: (Sx ∧ Dsx) ∧ (∀y: ¬ y = x) ¬ (Sy ∧ Dsy)) Rx

Using the description operator:

The sleigh Santa drove was red

R (the thing such that (it is a sleigh Santa drove))

R (lx x is a sleigh Santa drove)

R (lx (x is a sleigh ∧ Santa drove x))

R(lx (Sx ∧ Dsx))

D: [_ drove _]; R: [_ was red]; S: [_ is a sleigh]; s: Santa

6.

	∃x (Fx ∧ Gx)	1
	ⓐ Fa ∧ Ga	2
2 Ext	Fa	
2 Ext	Ga	(3)
3 EG	∃x Gx	X, (4)
	●	
4 QED	∃x Gx	1
1 PCh	∃x Gx	

or

	∃x (Fx ∧ Gx)	1
	ⓐ Fa ∧ Ga	2
2 Ext	Fa	
2 Ext	Ga	(5)
	∀x ¬ Gx	a: 4
4 UI	¬ Ga	(5)
	●	
5 Nc	⊥	3
3 NcP	∃x Gx	1
1 PCh	∃x Gx	

7.

	∃x (Fx ∧ ∃y Rxy)	1
	ⓐ Fa ∧ ∃y Ray	2
2 Ext	Fa	(4)
2 Ext	∃y Ray	3
	ⓑ Rab	(4)
4 Adj	Fa ∧ Rab	X, (5)
5 EG	∃y (Fy ∧ Ryb)	X, (6)
6 EG	∃x ∃y (Fy ∧ Ryx)	X, (7)
	●	
7 QED	∃x ∃y (Fy ∧ Ryx)	3
3 PCh	∃x ∃y (Fy ∧ Ryx)	1
1 PCh	∃x ∃y (Fy ∧ Ryx)	

or

	∃x (Fx ∧ ∃y Rxy)	1
	ⓐ Fa ∧ ∃y Ray	2
2 Ext	Fa	(9)
2 Ext	∃y Ray	
	ⓑ Rab	(10)
	∀x ¬ ∃y (Fy ∧ Ryx)	b: 5
5 UI	¬ ∃y (Fy ∧ Ryb)	6
	∀y ¬ (Fy ∧ Ryb)	a :8
8 UI	¬ (Fa ∧ Rab)	9
9 MPT	¬ Rab	(10)
	●	
10 Nc	⊥	7
7 NcP	∃y (Fy ∧ Ryb)	6
6 CR	⊥	4
4 NcP	∃x ∃y (Fy ∧ Ryx)	3
3 PCh	∃x ∃y (Fy ∧ Ryx)	1
1 PCh	∃x ∃y (Fy ∧ Ryx)	

8. A sentence φ is entailed by a set Γ if and only if there is no possible world in which φ is false while all members of Γ are true

or

A sentence φ is entailed by a set Γ if and only φ is true in every possible world in which all members of Γ are true

9.

A	B	C	D	¬	(A ∧ B)	→	(¬ C	∨ D)
T	T	F	F	F	T	Ⓣ	T	T
F	F	T	F	T	F	Ⓕ	F	F

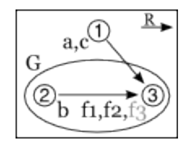
10. range: 1, 2, 3

a	b	c
1	2	1

τ	fτ
1	3
2	3
3	3

τ	Gτ
1	F
2	T
3	T

R	1	2	3
1	F	F	T
2	F	F	T
3	F	F	F



(The diagram provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

alias sets	*IDs*	*values*
a	1	a: 1
c		c: 1
b	2	b: 2
fa	3	f1: 3
fb		f2: 3
fc		f1: 3

resources	*values*
¬ Ga	G1: **F**
Gb	G2: **T**
G(fc)	G3: **T**
Ra(fb)	R13: **T**
Rb(fa)	R23: **T**

Phi 270 F03 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice theadditional instructions given for the first.

1. Tom sent something to Sue
answer

2. Everyone heard a sound. [This is ambiguous but you need only analyze one interpretation; justchoose the one that seems most natural to you.]
answer

3. There is someone who knows just one other person.
answer

Analyze the sentence below using each of the two ways of analyzing the definite description the package. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The package rattled.
answer

Use derivations to show that the following argument is valid. You may use any rules.

5. $\exists x\, Fx$
$\forall x\, Gx$
―――――
$\exists x\, (Fx \land Gx)$
answer

Use a derivation to show that the following argument is not valid and use either tables or a diagram to describe a structure dividing an open gap.

6. $\exists x\, \forall y\, Rxy$
―――――
$\exists x\, Rax$
answer

Complete the following to give a definition of equivalence in terms of truth values and possible worlds:

7. A sentence φ is equivalent to a sentence ψ (i.e., $\varphi \simeq \psi$) if and only if …
answer

Answer the following question and explain your answer in terms of the definitions of the basic concepts it involves.

8. Suppose you are told that (i) $\varphi \vDash \psi$ and (ii) ψ is inconsistent with χ (i.e., the set formed of the two is inconsistent). What can you conclude about the relation between of φ and χ? That is, what patterns of truth values for the two are ruled out (if any are); and, if any are ruled out, what logical relation or relations holds as a result.
answer

Complete the following truth table by calculating the truth value of the sentence on each of the given assignments. In each row, write under each connective the value of the component of which it is the main connective and circle the truth value of the sentence as a whole.

9.

A	B	C	D	$(A \land \neg B) \lor \neg (C \to D)$
T	T	T	T	
F	F	T	F	

answer

Phi 270 F03 test 5 answers

1. Tom sent something to Sue
$\exists x$ Tom sent x to Sue

$$\exists x\, Ntxs$$

C: [_ sent _ to _]; s: Sue; t: Tom

2. Everyone heard a sound
($\exists x$: x is a sound) everyone heard x
($\exists x$: x is a sound) ($\forall y$: y is a person) y heard x

$$(\exists x: Sx)\ (\forall y: Py)\ Hyx$$

H: [_ heard _]; P: [_ is a person]; S: [_ is a sound]

3. There is someone who knows just one other person
$\exists x$ x is a person who knows just one other person
$\exists x$ (x is a person $\land$ x knows just one other person)
$\exists x\, (Px \land (\exists y: Py \land \neg\, y = x)$ x knows y and no other person besides y)
$\exists x\, (Px \land (\exists y: Py \land \neg\, y = x)\, (Kxy \land$ x knows no other person besides y))

$$\exists x\, (Px \land (\exists y: Py \land \neg\, y = x)\, (Kxy \land (\forall z: Pz \land \neg\, z = x \land \neg\, z = y)\, \neg\, Kxz))$$

or:

$$\exists x\, (Px \land (\exists y: Py \land \neg\, y = x)\, (Kxy \land (\forall z: Pz \land \neg\, z = x \land Kxz)\, y = z))$$

K: [_ knows _]; P: [_ is a person]

4. *using Russell's analysis:*

The package rattled

(∃x: x and only x is a package) x rattled

(∃x: x is a package ∧ (∀y: ¬ y = x) ¬ y is a package) Rx

$$(\exists x: Px \land (\forall y: \neg\, y = x)\ \neg\, Py)\ Rx$$

or:

$$(\exists x: Px \land (\forall y: Py)\ x = y)\ Rx$$

using the description operator:

The package rattled

R(the package)

R (Ix x is a package)

$$R(Ix\ Px)$$

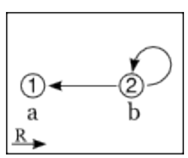
P: [_ is a package]; r: [_ rattled]

5.

	∃x Fx	1
	∀x Gx	a: 2
	ⓐ Fa	(3)
2 UI	Ga	(3)
3 Adj	Fa ∧ Ga	X, (4)
4 EG	∃x (Fx ∧ Gx)	X, (5)
	●	
5 QED	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	

6.

	∃x ∀y Rxy	1
	ⓑ ∀y Rby	a:3, b:4
	∀x ¬ Rax	a:5, b:6
3 UI	Rba	
4 UI	Rbb	
5 UI	¬ Raa	
6 UI	¬ Rab	
	○	Rba,Rbb,¬Raa,¬Rab ⊭ ⊥
	⊥	2
2 NcP	∃x Rax	1
1 PCh	∃x Rax	



7. φ and ψ are equivalent if and only if there is no possible world in which they have different truth values (or: if and only, in every possible world, each has the same value as the other)

8. φ and χ are inconsistent. That is, φ and χ cannot be both true because ψ will be true when φ is, and ψ and χ cannot be both true. Other patterns of values for φ and χ are possible because they are not ruled out for ψ and χ by the fact that they are inconsistent and, for all weknow, φ and ψ may be equivalent.

9.

A	B	C	D	(A ∧	¬ B)	∨	¬	(C → D)
T	T	T	T	F	F	Ⓕ	F	T
F	F	T	F	F	T	Ⓣ	T	F

Phi 270 F02 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for the first.

1. Al received a card that made him laugh [Give this analysis also using an unrestricted quantifier.]
answer

2. There is a toy that every child wanted
answer

3. Santa left at least two packages
answer

Analyze the sentence below using each of the two ways of analyzing the definite description the battery. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The battery is dead
answer

Use derivations to show that the following argument is valid. You may use any rules.

5. $\exists x\,(Fx \land Gx)$
$\overline{\exists x\,(Gx \land Fx)}$
answer

Use a derivation to show that the following argument is not valid and use either tables or a diagram to describe a structure dividing an open gap.

6. $\exists x\,\exists y\,Rxy$
$\overline{\exists x\,Rax}$
answer

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

7. A set Γ entails a sentence φ (i.e., $\Gamma \vDash \varphi$) if and only if …
answer

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component, and circle the truth value of the sentence as a whole.

8.

A	B	C	D	$(A \to B) \land \neg (C \lor \neg D)$
T	F	F	T	

answer

Give at least two restatements of the following sentence as an expansion on a term appearing in it (i.e., as an abstract applied to such a term):

9. Raba
answer

Phi 270 F02 test 5 answers

1. Al received a card that made him laugh
some card that made Al laugh is such that (Al received it)
($\exists x$: x is a card that made Al laugh) Al received x
($\exists x$: x is a card $\land$ x made Al laugh) Rax

$$(\exists x: Cx \land Lxa)\ Rax$$
$$\exists x\,((Cx \land Lxa) \land Rax)$$

C: [_ is a card]; L: [_ made _ laugh]; R: [_ received _]; a: Al

2. There is a toy that every child wanted
Something is a toy that every child wanted
Something is such that (it is a toy that every child wanted)
$\exists x$ x is a toy that every child wanted
$\exists x$ (x is a toy $\land$ every child wanted x)
$\exists x$ (Tx $\land$ every child is such that (he or she wanted x))
$\exists x$ (Tx $\land$ ($\forall y$: y is a child) y wanted x)

$$\exists x\,(Tx \land (\forall y: Cy)\ Wyx)$$

C: [_ is a child]; T: [_ is a toy]; W: [_ wanted _]

3. Santa left at least two packages
at least two packages are such that (Santa left them)
($\exists x$: x is a package) ($\exists y$: y is a package $\land \neg$ y = x) (Santa left x $\land$ Santa left y)

$$(\exists x: Px)\ (\exists y: Py \land \neg\, y = x)\ (Lsx \land Lsy)$$

L: [_ left _]; P: [_ is a package]; s: Santa

4. *using Russell's analysis:*

The battery is dead

The battery is such that (it is dead)

(∃x: x and only x is a battery) x is dead

(∃x: x is a battery ∧ (∀y: ¬ y = x) ¬ y is a battery) x is dead

$$(\exists x\!: Bx \land (\forall y\!: \neg\, y = x)\ \neg By)\ Dx$$

or:

$$(\exists x\!: Bx \land (\forall y\!: By)\ x = y)\ Dx$$

B: [_ is a battery]; D: [_ is dead]

using the description operator:

The battery is dead

D the battery

D(Ix x is a battery)

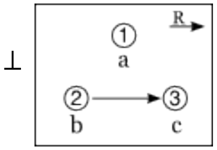
$$D(\mathrm{I}x\ Bx)$$

5.

	∃x (Fx ∧ Gx)	1
	ⓐ Fa ∧ Ga	2
2 Ext	Fa	(6)
2 Ext	Ga	(5)
	∀x ¬ (Gx ∧ Fx)	a:4
4 UI	¬ (Ga ∧ Fa)	5
5 MPT	¬ Fa	(6)
	●	
6 Nc	⊥	3
3 NcP	∃x (Gx ∧ Fx)	1
1 PCh	∃x (Gx ∧ Fx)	

6.

	∃x ∃y Rxy	1
	ⓑ ∃y Rby	2
	ⓒ Rbc	
	∀x ¬ Rax	a:4, b:5, c:6
4 UI	¬ Raa	
5 UI	¬ Rab	
6 UI	¬ Rac	
	○	Rbc,¬Raa, ¬Rab,¬Rac ⊭ ⊥
	⊥	3
3 NcP	∃x Rax	2
2 PCh	∃x Rax	1
1 PCh	∃x Rax	



7. A set Γ entails a sentence φ if and only if there is no possible world in which every member of Γ is true but φ is false (*or:* if and only if φ is true in every possible world in which all members of Γ are true)

8.

A	B	C	D	(A → B) ∧ ¬ (C ∨ ¬ D)
T	F	F	T	F Ⓕ T F F

9. Up to the choice of variables, the possibilities are the following:

$$[Rabx]_x a,\ [Rxba]_x a,\ [Rxbx]_x a,\ [Raxa]_x b$$

Phi 270 F00 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for each of the first two.

1. There is a yak that someone yoked. [Give this analysis also using an unrestricted quantifier.]

answer

2. Each explorer mapped a route. [This sentence is ambiguous. Analyze it in two nonequivalent ways, and describe a situation in which the sentence is true on one of your analyses and false on the other.]

answer

3. Exactly one reindeer was red nosed. [You may leave the predicate _ was red nosed unanalyzed.]

answer

Analyze the sentence below using each of the two ways of analyzing the definite description the fireplace. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. Santa gained entry through the fireplace.

answer

Use derivations to show that the following argument is valid. You may use any rules.

5.

$$\frac{\exists x\, \forall y\, (Fy \to Rxy)}{\forall x\, (Fx \to \exists y\, Ryx)}$$

answer

That is: Something is relevant to all findings ⊨ Each finding has something relevant to it

[Don't hesitate to ignore this English reading if it doesn't help you think about the argument.]

Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

6.

$$\frac{\exists x\, \exists y\, (\neg\, y = x \land Rxy)}{\exists x\, \neg\, Rxx}$$

answer

Complete the following to give a definition of inconsistency in terms of truth values and possible worlds:

7. A set Γ is inconsistent if and only if …

answer

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component.

8.

A	B	C	D	$(A \lor \neg B) \land \neg (C \to D)$
T	F	T	F	

answer

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the sentences below all true. (You may use either tables or a diagram.)

9. $a = c$, $fc = b$, $d = e$, Fc, Fd, $\neg\, Fb$, Rab, Rea, $R(fa)b$, $\neg\, Re(fc)$

answer

Phi 270 F00 test 5 answers

1. There is a yak that someone yoked

something is a yak that someone yoked

something is such that (it is a yak that someone yoked)

$\exists x$ x is a yak that someone yoked

$\exists x$ (x is a yak $\land$ someone yoked x)

$\exists x$ (Yx $\land$ someone is such that (he or she yoked x))

$\exists x$ (Yx $\land$ ($\exists y$: y is a person) y yoked x)

$$\exists x\, (Yx \land (\exists y{:}\ Py)\, Kyx)$$
$$\exists x\, (Yx \land \exists y\, (Py \land Kyx))$$

K: [_ yoked _]; P: [_ is a person]; Y: [_ is a yak]

2. *first analysis:*

Each explorer mapped a route

each explorer is such (he or she mapped a route)

($\forall x$: x is an explorer) x mapped a route

($\forall x$: Ex) some route is such that (x mapped it)

($\forall x$: Ex) ($\exists y$: y is a route) x mapped y

$$(\forall x{:}\ Ex)\, (\exists y{:}\ Ry)\, Mxy$$

second analysis:

Each explorer mapped a route

some route is st (each explorer mapped it)

(∃x: x is a route) each explorer mapped x

(∃x: Rx) each explorer is such that (he or she mapped x)

(∃x: Rx) (∀y: y is an explorer) y mapped x

$$(\exists x: Rx)\ (\forall y: Ey)\ Myx$$

P: [_ is an explorer]; M: [_ mapped _]; R: [_ is a route]

The first is true and the second false if every explorer mapped some route or other but no one route was mapped by all explorers

3. Exactly one reindeer was red nosed

at least one reindeer was red nosed ∧ ¬ at least two reindeer were red nosed

some reindeer is such that (it was red nosed) ∧ ¬ at least two reindeer were such that (they were red nosed)

(∃x: x is a reindeer) x was red nosed ∧ ¬ (∃x: x is a reindeer) (∃y: y is a reindeer ∧ ¬ y = x) (x was red nosed ∧ y was red nosed)

$$(\exists\ x: Rx)\ Nx \land \neg\ (\exists\ x: Rx)\ (\exists\ y: Ry \land \neg\ y = x)\ (Nx \land Ny)$$

or:

Exactly one reindeer was red nosed

some reindeer is such that (it was red nosed and no other reindeer was red nosed)

(∃x: x is a reindeer) (x was red nosed and no other reindeer was red nosed)

(∃x: Rx) (Nx ∧ no reindeer other than x was red nosed)

(∃x: Rx) (Nx ∧ no reindeer other than x is such that (it was red nosed)

(∃x: Rx) (Nx ∧ (∀y: y is a reindeer ∧ ¬ y = x) ¬ y was red nosed)

$$(\exists\ x: Rx)\ (Nx \land (\forall y: Ry \land \neg\ y = x)\ \neg\ Ny)$$

or:

$$(\exists\ x: Rx)\ (Nx \land (\forall y: Ry \land Ny)\ x = y)$$

N: [_ was red nosed]; R: [_ is a reindeer]

The generalization using the variable y must be resricted to reindeer or else the sentence will say that some reindeer is the only and only thing that is red nosed—i.e., that there is exactly one red-nosed thing and it is a reindeer.

4. *using Russell's analysis:*

Santa gained entry through the fireplace

the fireplace is such that (Santa gained entry through it)

(∃x: x and only x is a fireplace) Santa gained entry through x

(∃x: x is a fireplace ∧ (∀y: ¬ y = x) ¬ y is a fireplace) Gsx

$$(\exists x: Fx \land (\forall y: \neg\ y = x)\ \neg\ Fy)\ Gsx$$

or:

$$(\exists x: Fx \land (\forall y: Fy)\ x = y)\ Gsx$$

using the description operator:

Santa gained entry through the fireplace

G s (the fireplace)

G s (Ix x is a fireplace)

$$Gs(\text{I}x\ Fx)$$

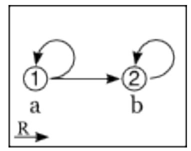
F: [_ is a fireplace]; G: [_ gained entry through _]; s: Santa

5.

	∃x ∀y (Fy → Rxy)	
	ⓐ ∀y (Fy → Ray)	b:4
	ⓑ Fb	(5)
4 UI	Fb → Rab	5
5 MPP	Rab	(6)
6 EG	∃y Ryb	X, (7)
	●	
7 QED	∃y Ryb	3
3 CP	Fb → ∃y Ryb	2
2 UG	∀x (Fx → ∃y Ryx)	1
1 PCh	∀x (Fx → ∃y Ryx)	

6.

	∃x ∃y (¬ y = x ∧ Rxy)	1
	ⓐ ∃y (¬ y = a ∧ Ray)	2
	ⓑ ¬ b = a ∧ Rab	3
3 Ext	¬ b = a	
3 Ext	Rab	
	∀x Rxx	a:5, b:6
5 UI	Raa	
6 UI	Rbb	
	○	¬ b = a, Rab, Raa, Rbb ⊭ ⊥
	⊥	4
4 NcP	∃x ¬ Rxx	2
2 PCh	∃x ¬ Rxx	1
1 PCh	∃x ¬ Rxx	



7. A set Γ is inconsistent if and only if there is no possible world in which every member of Γ is true

8.

A B C D	(A ∨ ¬ B) ∧ ¬ (C → D)
T F T F	T T Ⓣ T F

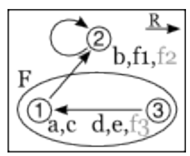
9. range: 1, 2, 3

a	b	c	d	e
1	2	1	3	3

τ	fτ
1	2
2	2
3	3

τ	Fτ
1	T
2	F
3	T

R	1	2	3
1	F	T	F
2	F	T	F
3	T	F	F



(The diagram above provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

alias sets	*IDs*	*values*
a	1	a: 1
c		c: 1
b	2	b: 2
fa		f1: 2
fc		f1: 2
d	3	d: 3
e		e: 3

resources	*values*
Fc	F1: **T**
Fd	F3: **T**
¬ Fb	F2: **F**
Rab	R12: **T**
Rea	R31: **T**
R(fa)b	R22: **T**
¬ Re(fc)	R32: **F**

Phi 270 F99 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for each of the first two.

1. Sam mentioned someone Tina didn't know. [Give this analysis also using an unrestricted quantifier.]
answer

2. Every shoe fit someone. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]
answer

3. Sam found at least two pieces.
answer

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The elephant standing on Sam sighed.
answer

[The following question was on a topic not covered in F08] Put the following sentence into prenex normal form (i.e., into a form which contains no restricted quantifiers and in which no quantifier is in the scope of a connective). Show each step where you move a quantifier past a connective separately.

5. ¬ ∀x ((Px ∧ ∃y Rxy) → ∃z Sxz)
answer

Use derivations to show that the following argument is valid. You may use attachment rules (but not replacement by equivalence).

6. ∀x ∀y (Rxy → (Ryx → Rxx))
∃x ∃y (Rxy ∧ Ryx)
―――――――――――――
∃x Fxx
answer

Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

7. ∃x Fx
∃x (Gx ∧ Hx)
―――――――――――――
∃x (Fx ∧ Hx)
answer

Complete the following to give a definition of entailment by a single sentence (i.e., implication) in terms of truth values and possible worlds:

8. A sentence φ entails a sentence ψ if and only if …

answer

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component.

9.

A	B	C	D	¬ (A ∧ B) → (C ∨ ¬ D)
T	F	F	T	

answer

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the sentences below all true. (You may use either tables or a diagram.)

10. $a = fb,\ fb = fc,\ fa = c,\ Pa,\ Pb,\ \neg Pc,\ Rab,\ Rbc,\ Rc(fb)$

answer

Phi 270 F99 test 5 answers

1. Sam mentioned someone Tina didn't know

someone Tina didn't know is such that (Sam mentioned him or her)

(∃x: x is a person Tina didn't know) Sam mentioned x

(∃x: x is a person ∧ ¬ Tina knew x) Sam mentioned x

$$(\exists x: Px \land \neg Ktx)\ Msx$$
$$\exists x\ ((Px \land \neg Ktx) \land Msx)$$

K: [_ knew _]; M: [_ mentioned _]; P: [_ is a person]; s: Sam; t: Tina

2. *first analysis:*

Every shoe fit someone

every shoe is such that (it fit someone)

(∀x: x is a shoe) x fit someone

(∀x: Sx) someone is such that (x fit him or her)

(∀x: Sx) (∃y: y is a person) x fit y

$$(\forall x: Sx)\ (\exists y: Py)\ Fxy$$

second analysis:

Every shoe fit someone

someone is such that (every shoe fit him or her)

(∃x: x is a person) every shoe fit x

(∃x: Px) every shoe is such that (it fit x)

(∃x: Px) (∀y: y is a shoe) y fit x

$$(\exists x: Px)\ (\forall y: Sy)\ Fyx$$

F: [_ fit _]; P: [_ is a person]; S: [_ is a shoe]

The first is true and the second false if every shoe could be worn but not all by the same person

3. Sam found at least two pieces

at least two pieces are such that (Sam found them)

(∃x: x is a piece) (∃y: y is a piece ∧ ¬ y = x) (Sam found x ∧ Sam found y)

$$(\exists\ x: Px)\ (\exists\ y: Py \land \neg\ y = x)\ (Fsx \land Fsy)$$

F: [_ found _]; P: [_ is a piece]; s: Sam

4. *using Russell's analysis:*

The elephant standing on Sam sighed

The elephant standing on Sam is such that (it sighed)

(∃x: x and only x is an elephant standing on Sam) x sighed

(∃x: x is an elephant standing on Sam ∧ (∀y: ¬ y = x) ¬ y is an elephant standing on Sam) Sx

(∃x: (x is an elephant ∧ x is standing on Sam) ∧ (∀y: ¬ y = x) ¬ (y is an elephant ∧ y is standing on Sam)) Sx

$$(\exists x: (Ex \land Txs) \land (\forall y: \neg\ y = x)\ \neg\ (Ey \land Tys))\ Sx$$

or:

$$(\exists x: (Ex \land Txs) \land (\forall y: Ey \land Tys)\ x = y)\ Sx$$

using the description operator:

The elephant standing on Sam sighed

S (the elephant standing on Sam)

S (Ix x is an elephant standing on Sam)

S (Ix (x is an elephant ∧ x is standing on Sam))

$$S(\text{I}x\ (Ex \land Txs))$$

E: [_ is an elephant]; S: [_ sighed]; T: [_ is standing on _]; s: Sam

5. [The following question was on a topic not covered in F08]

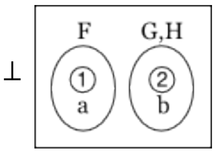
¬ ∀x ((Px ∧ ∃y Rxy) → ∃z Sxz)
∃x ¬ ((Px ∧ ∃y Rxy) → ∃z Sxz)
∃x ¬ (∃y (Px ∧ Rxy) → ∃z Sxz)
∃x ¬ ∀y ((Px ∧ Rxy) → ∃z Sxz)
∃x ∃y ¬ ((Px ∧ Rxy) → ∃z Sxz)
∃x ∃y ¬ ∃z ((Px ∧ Rxy) → Sxz)
∃x ∃y ∀z ¬ ((Px ∧ Rxy) → Sxz)

6.

	∀x ∀y (Rxy → (Ryx → Rxx))	a:4
	∃x ∃y (Rxy ∧ Ryx)	1
	ⓐ	
	∃y (Ray ∧ Rya)	2
	ⓑ	
	Rab ∧ Rba	3
3 Ext	Rab	(6)
3 Ext	Rba	(7)
4 UI	∀y (Ray → (Rya → Raa))	b:5
5 UI	Rab → (Rba → Raa)	6
6 MPP	Rba → Raa	7
7 MPP	Raa	(8)
8 EG	∃x Rxx	X, (9)
	●	
9 QED	∃x Rxx	2
2 PCh	∃x Rxx	1
1 PCh	∃x Rxx	

7.

	∃x Fx	1
	∃x (Gx ∧ Hx)	2
	ⓐ	
	Fa	(7)
	ⓑ	
	Gb ∧ Hb	3
3 Ext	Gb	
3 Ext	Hb	(8)
	∀x ¬ (Fx ∧ Hx)	a:5, b:6
5 UI	¬ (Fa ∧ Ha)	7
6 UI	¬ (Fb ∧ Hb)	8
7 MPT	¬ Ha	
8 MPT	¬ Fb	
	○	Fa,Gb,Hb,¬Ha,¬Fb ⊭ ⊥
	⊥	4
4 NcP	∃x (Fx ∧ Hx)	2
2 PCh	∃x (Fx ∧ Hx)	1
1 PCh	∃x (Fx ∧ Hx)	



8. A sentence φ entails a sentence ψ if and only if there is no possible world in which φ is true but ψ is false (*or:* if and only if ψ is true in every possible world in which φ is true)

9.

A B C D	¬ (A ∧ B) → (C ∨ ¬ D)
T F F T	T F Ⓕ F F

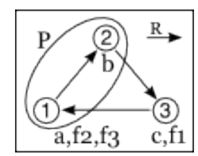
10. range: 1, 2, 3

a	b	c
1	2	3

τ	fτ
1	3
2	1
3	1

τ	Pτ
1	T
2	T
3	F

R	1	2	3
1	F	T	F
2	F	F	T
3	T	F	T



The diagram above provides a complete answer, as do the tables to its left. The tables below illustrate a way of finding this structure.

alias sets	*IDs*	*values*
a	1	a: 1
fb		f2: 1
fc		f3: 1
b	2	b: 2
c	3	c: 3
fa		f1: 3

resources	*values*
Pa	P1: **T**
Pb	P2: **T**
¬ Pc	P3: **F**
Rab	R12: **T**
Rbc	R23: **T**
Rc(fb)	R31: **T**

Phi 270 F98 test 5

(questions from the last of 6 quizzes)

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. George traveled to LA by way of some town in Wyoming. [Give this analysis also using an unrestricted quantifier.]
answer

2. Everyone is afraid of something. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]
answer

3. Spot knew exactly one trick.
answer

4. Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

Tom opened the letter from Bulgaria

answer

5. Use derivations to show that the following argument is valid. You may use any rules.

$$\frac{\exists x\,(Fx \land \exists y\, \neg\, x = y)}{\exists x\, \exists y\,(\neg\, y = x \land Fy)}$$

That is: Some finding is different from something ⊨ Something is such that something different from it is a finding [but don't hesitate to ignore the English if it doesn't help].
answer

6. Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

$$\frac{\exists x\, \exists y\, Rxy}{\exists x\, Rxx}$$

answer

7. Complete the following to give a definition of equivalence in terms of truth values and possible worlds:
A sentence φ is equivalent to a sentence ψ if and only if …
answer

8. Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the 8 sentences at the left below all true.

$fab = fba,\ ga = fab,\ fba = c,\ Fb,\ F(ga),\ Rab,\ \neg\, Rba,\ R(ga)c$

answer

9. [This question was on a topic not covered in F08] Use replacement by equivalence to put the following sentence into disjunctive normal form. Show how you reach your result; you may combine uses of associativity and commutativity with other principles in a single step but there should be no more than one use of De Morgan's laws or distributivity in each step.

$$\neg\,((A \land B) \lor (C \lor D))$$

answer

Phi 270 F98 test 5 answers

1. George traveled to LA by way of some town in Wyoming
some town in Wyoming is such that (George traveled to LA by way of it)
($\exists$x: x is a town in Wyoming) George traveled to LA by way of x
($\exists$x: x is a town ∧ x is in Wyoming) George traveled to LA by way of x

$$(\exists x: Tx \land Nxm)\ Rglx$$
$$\exists x\,((Tx \land Nxm) \land Rglx)$$

N: [_ is in _]; R: [_ traveled to _ by way of _]; T: [_ is a town]; g: George; l: LA; m: Wyoming

2. *first analysis:*

Everyone is afraid of something
everyone is such that (he or she is afraid of something)
($\forall$x: x is a person) x is afraid of something
($\forall$x: Px) something is such that (x is afraid of it)
($\forall$x: Px) $\exists$y x is afraid of y

$$(\forall x: Px)\ \exists y\ Axy$$

second analysis:

Everyone is afraid of something
something is such that (everyone is afraid of it)
$\exists$x everyone is afraid of x
$\exists$x everyone is such that (he or she is afraid of x)
$\exists$x ($\forall$y: y is a person) y is afraid of x

$$\exists x\ (\forall y: Py)\ Ayx$$

A: [_ is afraid of _]; P: [_ is a person]
The first is true and the second false if all people are fearful but not all fearful of the same thing

3. Spot knew exactly one trick
Spot knew a trick ∧ ¬ Spot knew at least two tricks
(∃ x: x is a trick) Spot knew x ∧ ¬ (∃ x: x is a trick) (∃ y: y is a trick ∧ ¬ y = x) (Spot knew x ∧ Spot knew y)

$$(\exists\, x{:}\ Tx)\ Ksx \land \neg (\exists\, x{:}\ Tx)\ (\exists\, y{:}\ Ty \land \neg\, y = x)\ (Ksx \land Ksy)$$

or:

$$(\exists\, x{:}\ Tx)\ (Ksx \land (\forall\, y{:}\ Ty \land \neg\, y = x)\ \neg Ksy)$$

or:

$$(\exists\, x{:}\ Tx)\ (Ksx \land (\forall\, y{:}\ Ty \land Ksy)\ x = y)$$

K: [_ knew _]; T: [_ is a trick]; s: Spot

4. *using Russell's analysis:*
Tom opened the letter from Bulgaria
the letter from Bulgaria is such that (Tom opened it)
(∃x: x and only x is a letter from Bulgaria) Tom opened x
(∃x: x is a letter from Bulgaria ∧ (∀y: ¬ y = x) ¬ y is a letter from Bulgaria) Otx
(∃x: x is a letter ∧ x is from Bulgaria ∧ (∀y: ¬ y = x) ¬ y is a letter ∧ y is from Bulgaria) Otx

$$(\exists x{:}\ (Lx \land Fxb) \land (\forall y{:}\ \neg\, y = x)\ \neg (Ly \land Fyb))\ Otx$$

or:

$$(\exists x{:}\ (Lx \land Fxb) \land (\forall y{:}\ Ly \land Fyb)\ x = y)\ Otx$$

using the description operator:
Tom opened the letter from Bulgaria
Ot(the letter from Bulgaria)
Ot(Ix x is a letter from Bulgaria)
Ot(Ix (x is a letter ∧ x is from Bulgaria))

$$Ot(\text{I}x\ (Lx \land Fxb))$$

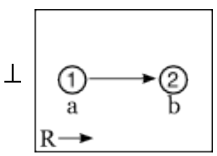
F: [_ is from _]; L: [_ is a letter]; O: [_ opened _]; b: Bulgaria; t: Tom

5.

	∃x (Fx ∧ ∃y ¬ x = y)	1
	ⓐ Fa ∧ ∃y ¬ a = y	2
2 Ext	Fa	(4)
2 Ext	∃y ¬ a = y	3
	ⓑ ¬ a = b	(4)
4 Adj	¬ a = b ∧ Fa	X, (5)
5 EG	∃y (¬ y = b ∧ Fy)	X, (6)
6 EG	∃x ∃y (¬ y = x ∧ Fy)	X, (7)
	●	
7 QED	∃x ∃y (¬ y = x ∧ Fy)	3
3 PCh	∃x ∃y (¬ y = x ∧ Fy)	1
1 PCh	∃x ∃y (¬ y = x ∧ Fy)	

6.

	∃x ∃y Rxy	1
	ⓐ ∃y Ray	2
	ⓑ Rab	
	∀x ¬ Rxx	a:4,b:5
4 UI	¬ Raa	
5 UI	¬ Rbb	
	○	Rab, ¬Raa, ¬Rbb ⊭ ⊥
	⊥	3
3 NcP	∃x Rxx	2
2 PCh	∃x Rxx	1
1 PCh	∃x Rxx	



7. A sentence φ is equivalent to a sentence ψ if and only if there is no possible world in which φ and ψ have different truth values

8. range: 1, 2, 3

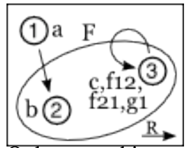
a	b	c
1	2	3

f	1	2	3
1	1	3	1
2	3	1	1
3	1	1	1

τ	gτ
1	3
2	1
3	1

τ	Fτ
1	F
2	T
3	T

R	1	2	3
1	F	T	F
2	F	F	F
3	F	F	T



Only non-arbitrary values are shown for f and g

The diagram provides a complete answer, as do the tables to its left. The tables below are a way of finding this structure.

alias sets	*IDs*	*values*
a	1	a: 1
b	2	b: 2
c	3	c: 3
fab		f12: 3
fba		f21: 3
ga		g1: 3

resources	*values*
Fb	F2: **T**
F(ga)	F3: **T**
Rab	R12: **T**
¬ Rba	R21: **F**
R(ga)c	R33: **T**

9. [This question was on a topic not covered in F08]

$$\neg ((A \wedge B) \vee (C \vee \neg D))$$
$$\simeq$$
$$\neg (A \wedge B) \wedge \neg (C \vee \neg D)$$
$$\simeq$$
$$(\neg A \vee \neg B) \wedge (\neg C \wedge D)$$
$$\simeq$$
$$(\neg A \wedge \neg C \wedge D) \vee (\neg B \wedge \neg C \wedge D)$$

Phi 270 F97 test 5

(questions from the last of 6 quizzes)

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. Tom phoned someone who had left a message for him. [Give this analysis also using an unrestricted quantifier.]
answer

2. Santa said something to each child. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]
answer

3. Ron asked Santa for at least two things.
answer

4. Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

Bill lent the book Ann gave him to Carol

answer

5. Use derivations to show that the following argument is valid. You may use any rules.

∃x ∃y (Rxy ∧ Sxy)
―――――――――――
∃y ∃x (Sxy ∧ Rxy)

answer

6. Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

∃x Rax
――――――
∃x Rxa

answer

7. Complete the following to give a definition of inconsistency in terms of truth values and possible worlds:

A set Γ is inconsistent if and only if …

answer

8. Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the list of 5 sentences below all true and use it to calculate a truth value for the sentence that follows them. (You may present the structure using either tables or a diagram.)

make these true: b = ga, fa = f(ga), Rab, R(fa)a, ¬ R(fb)b

calculate the value: (b = gb ∨ Ra(ga)) → (R(fa)(ga) ∧ f(gb) = g(fb))

answer

9. Give two different restatements of the sentence below in expanded form as a complex predicate (i.e., an abstract) applied to a term.

∃y Rayb

answer

Phi 270 F97 test 5 answers

1. Tom phoned someone who had left a message for him

someone who had left a message for Tom is such that (Tom phoned him or her)

(∃x: x is a person who had left a message for Tom) Tom phoned x

(∃x: x is a person ∧ x had left a message for Tom) Htx

(∃x: Px ∧ some message is such (x had left it for Tom)) Htx

(∃x: Px ∧ (∃y: y is a message) x had left y for Tom) Htx

(∃x: Px ∧ (∃y: My) Lxyt) Htx

∃x ((Px ∧ ∃y (My ∧ Lxyt)) ∧ Htx)

H: [_ phoned _]; L: [_ had left _ for _]; M: [_ is a message]; P: [_ is a person]; t: Tom

2. *first analysis:*

each child is such that (Santa said something to him or her)

(∀x: x is a child) Santa said something to x

(∀x: Cx) something is such that (Santa said it to x)

(∀x: Cx) ∃y Santa said y to x

(∀x: Cx) ∃y Dsyx

second analysis:

something is such that (Santa said it to each child)

∃x Santa said x to each child

∃x each child is such that (Santa said x to him or her)

∃x (∀y: y is a child) Santa said x to y

∃x (∀y: Cy) Dsxy

C: [_ is a child]; D: [_ said _ to _]; s: Santa

The first is true and the second false if Santa spoke to each child but said different things to different children

3. Ron asked Santa for at least two things

∃x (∃y: ¬ y = x) (Ron asked Santa for x ∧ Ron asked Santa for y)

∃x (∃ y: ¬ y = x) (Arsx ∧ Arsy)

A: [_ asked _ for _]; r: Ron; s: Santa

4. *using Russell's analysis:*

Bill lent the book Ann gave him to Carol

the book Ann gave Bill is such that (Bill lent it to Carol)

(∃x: x and only x is a book Ann gave Bill) Bill lent x to Carol

(∃x: x is a book Ann gave Bill ∧ (∀y: ¬ y = x) ¬ y is a book Ann gave Bill) Lbxc

(∃x: (x is a book ∧ Ann gave Bill x) ∧ (∀y: ¬ y = x) ¬ (y is a book ∧ Ann gave Bill y)) Lbxc

(∃x: (Bx ∧ Gabx) ∧ (∀y: ¬ y = x) ¬ (By ∧ Gaby)) Lbxc

or:

(∃x: (Bx ∧ Gabx) ∧ (∀y: By ∧ Gaby) x = y) Lbxc

using the description operator:

Bill lent the book Ann gave him to Carol

Lb(the book Ann gave Bill)c

Lb(Ix x is a book Ann gave Bill)c

Lb(Ix (x is a book ∧ Ann gave Bill x))c

Lb(Ix (Bx ∧ Gabx))c

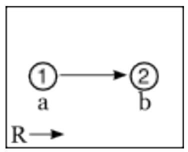
B: [_ is a book]; G: [_ gave _ _]; L: [_ lent _ to _]; a: Ann; b: Bill; c: Carol

5.

```
         │ ∃x ∃y (Rxy ∧ Sxy)              1
         ├─
         │ ⓐ
         │ │ ∃y (Ray ∧ Say)               2
         │ ├─
         │ │ ⓑ
         │ │ │ Rab ∧ Sab                  3
         │ │ ├─
3 Ext    │ │ │ Rab                        (4)
3 Ext    │ │ │ Sab                        (4)
4 Adj    │ │ │ Sab ∧ Rab                  X, (5)
5 EG     │ │ │ ∃x (Sxb ∧ Rxb)             X, (6)
6 EG     │ │ │ ∃y ∃x (Sxy ∧ Rxy)          X, (7)
         │ │ │ ●
         │ │ ├─
7 QED    │ │ │ ∃y ∃x (Sxy ∧ Rxy)          2
         │ ├─
2 PCh    │ │ ∃y ∃x (Sxy ∧ Rxy)            1
         ├─
1 PCh    │ ∃y ∃x (Sxy ∧ Rxy)
```

6.

```
         │ ∃x Rax
         ├─
         │ ⓑ
         │ │ Rab
         │ ├─
         │ │ │ ∀x ¬ Rxa            a:3,b:4
         │ │ ├─
3 UI     │ │ │ ¬ Raa
4 UI     │ │ │ ¬ Rba
         │ │ │ ○                   Rab, ¬Raa, ¬Rba ⊭ ⊥
         │ │ ├─
         │ │ │ ⊥                   2
         │ ├─
2 NcP    │ │ ∃x Rxa                1
         ├─
1 PCh    │ ∃x Rxa
```



7. A set Γ is inconsistent if and only if there is no possible world in which every member of Γ is true.

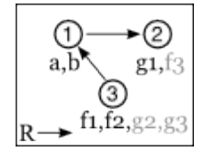
8. range: 1, 2, 3

a	b
1	2

τ	fτ
1	3
2	3
3	2

τ	gτ
1	2
2	3
3	3

R	1	2	3
1	F	T	F
2	F	F	F
3	T	F	F



Only non-arbitrary values of f and g are shown

(b = gb ∨ Ra(ga)) → (R(fa)(ga) ∧ f(gb) = g(fb))

2 F 3 2 T T 1 21 Ⓕ F 31 21 F 2 32 F 3 32

Your values for some of the compound terms and equations may differ from those shown here in gray, but your values for other predications and for truth-functional compounds should be the same as those shown.

The diagram above provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.

alias sets	*IDs*	*values*
a	1	a: 1
b	2	b: 2
ga		g1: 2
fa	3	f1: 3
fb		f2: 3
f(ga)		f2: 3

resources	*values*
Rab	R12: **T**
R(fa)a	R31: **T**
¬ R(fb)b	R32: **F**

9. The following are 3 possibilities (up to choice of the variable) from which your two might be chosen; in the last, τ may be any term:

$[\exists y\ Rxyb]_x a$, $[\exists y\ Rayx]_x b$, $[\exists y\ Rayb]_x \tau$

Phi 270 F96 test 5

(questions from the last of 6 quizzes)

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. Ned has visited a museum in Linden. [Give this analysis also using an unrestricted quantifier.]
answer

2. Something blocked each route. [This sentence is ambiguous. Analyze it in two ways, as making a claim of *general exemplification* and as making the stronger claim of *uniformly general exemplification*, and indicate which analysis is which.]
answer

3. At most one plan was implemented.
answer

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The scout you saw saw you.
answer

Use derivations to show that the following argument is valid. You may use any rules.

5.
∃x Rax
∀x (∃y Ryx → Fx)
―――――
∃x Fx

answer

Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

6.
∃x Fx
Ga
―――――
∃x (Fx ∧ Gx)

answer

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

7. A sentence φ is entailed by a set Γ if and only if …
answer

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the following sentences all true. (You may present the structure using either tables or a diagram.)

8. a = b, fb = fc, Pa, ¬ P(fa), Rab, ¬ Rbc, Rb(fb)
answer

Give two different restatements of the sentence below in expanded form as a complex predicate (i.e., an abstract) applied to a term.

9. Fa ∧ Ga
answer

Phi 270 F96 test 5 answers

1. Ned has visited a museum in Linden

(∃x: x is a museum in Linden) Ned has visited x

(∃x: x is a museum ∧ x is in Linden) Ned has visited x

(∃x: Mx ∧ Nxl) Vnx

∃x ((Mx ∧ Nxl) ∧ Vnx)

M: [_ is a museum]; N: [_ is in _]; V: [_ has visited _]; l: Linden; n: Ned

2. *general exemplification*

(∀x: x is a route) something blocked x

(∀x: Rx) ∃y y blocked x

(∀x: Rx) ∃y Byx

uniformly general exemplification

∃y y blocked each route

∃y (∀x: x is a route) y blocked x

∃y (∀x: Rx) Byx

B: [_ blocked _]; R: [_ is a route]

3. At most one plan was implemented

¬ at least two plans were implemented

¬ (∃x: x is a plan) (∃y: y is a plan ∧ ¬ y = x) (x was implemented ∧ y was implemented)

¬ (∃x: Px) (∃y: Py ∧ ¬ y = x) (Ix ∧ Iy)

I: [_ was implemented]; P: [_ is a plan]

4. *using Russell's analysis:*

the scout you saw is such that (he or she saw you)

(∃x: x and only x is a scout you saw) Sxo

(∃x: x is a scout you saw ∧ (∀y: ¬y = x) ¬ y is a scout you saw) Sxo

(∃x: (Tx ∧ Sox) ∧ (∀y: ¬y = x) ¬ (Ty ∧ Soy)) Sxo *using the description operator:*

the scout you saw saw you

S(the scout you saw)o

S(℩ x x is a scout you saw)o

S(℩ x (x is a scout ∧ you saw x))o

S(℩ x (Tx ∧ Sox))o

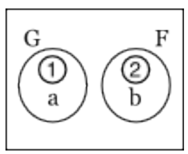
S: [_ saw _]; T: [_ is a scout]; o: you

5.

	∃x Rax	1
	∀x (∃y Ryx → Fx)	b:2
	ⓑ Rab	(3)
2 UI	∃y Ryb → Fb	4
3 EG	∃y Ryb	X, (4)
4 MPP	Fb	(4)
5 EG	∃x Fx	X, (6)
	●	
6 QED	∃x Fx	1
1 PCh	∃x Fx	

6.

	∃x Fx	1
	Ga	(4)
	ⓑ Fb	(6)
	∀x ¬ (Fx ∧ Gx)	a:3, b:5
3 UI	¬ (Fa ∧ Ga)	4
4 MPT	¬ Fa	
5 UI	¬ (Fb ∧ Gb)	6
6 MPT	¬ Gb	
	○	¬Fa, Fb, Ga, ¬Gb ⊭ ⊥
	⊥	2
2 NcP	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	



7. A sentence φ is entailed by a set Γ of sentences if and only if there is no possible world in which φ is false while each member of Γ is true.

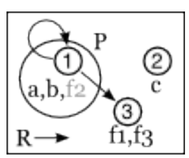
8. range: 1, 2, 3

a	b	c
1	1	3

τ	fτ
1	2
2	1
3	2

τ	Pτ
1	T
2	F
3	F

R	1	2	3
1	T	T	F
2	F	F	F
3	F	F	F



(The diagram provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

alias sets	*IDs*	*values*
a	1	a: 1
b		b: 1
fa	2	f1: 2
fb		f1: 2
fc		f3: 2
c	3	c: 3

resources	*values*
Pa	P1: **T**
¬ P(fa)	P2: **F**
Rab	R11: **T**
¬ Rbc	R13: **F**
Rb(fb)	R12: **T**

9. The following are 4 possibilities (up to choice of the variable) from which your two might be chosen; in the last, τ may be any term:

$[Fx \land Gx]_x a$

$[Fx \land Ga]_x a$

$[Fa \land Gx]_x a$

$[Fa \land Ga]_x \tau$