

Phi 270 F08 test 1

1. Define the idea of two sentences being mutually exclusive by completing the following with a definition in terms of truth values and possible worlds:

ϕ and ψ are mutually exclusive if and only if ...

answer

2. Suppose you are told about some sentences ϕ and ψ and the tautology T that $\phi \models T$ and $T \models \psi$. (i) What does this tell you about the possible truth values of ϕ ? And (ii) what does it tell you about the possible truth values of ψ ? In each case, explain your answer by reference to the definitions of a tautology and of implication (i.e., entailment).

answer

3. Give an example of three sentences where the first implies (i.e., entails) the second, the second implicates the third (i.e., has the third as an implicature), but the first does not implicate the third. (It may be easiest to choose the second and third sentences as an example of an implicature—and it doesn't have to be a new example—and then look for a sentence that implies one without implicating the other.) Be sure to say enough about the context of your sentences for me to be able to see that what you claim about them is so.

answer

4. Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and also English notation (i.e., using **both ... and**). Be sure that the unanalyzed components of your answer are complete and independent sentences (and give a key to the abbreviations you use for them); also try to respect any grouping in the English.

Although Al took the first turn, he missed the second; but he found his way to the meeting.

answer

5. Synthesize an English sentence that has the analysis below. Choose a simple and natural sentence whose organization reflects the grouping of the logical form.

$B \wedge (C \wedge D)$

B: Al wrote to Bob; C: Al spoke to Carol; D: Al spoke to Dave

answer

- Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, present a counterexample by providing a table in which

you calculate the truth values of the premises and conclusion on an assignment of truth values that divides an open gap. (Your table should indicate the value of any compound component by writing this value under the main connective of the component.) *Do not use the rule Adj in the first derivation*, but you may use it in the second.

6. $A \wedge (B \wedge C), D \models C \wedge D$

answer

7. $A \wedge B, B \wedge D \models A \wedge (C \wedge D)$

answer

Phi 270 F08 test 1 answers

1. ϕ and ψ are mutually exclusive if and only if there is no possible world in which ϕ and ψ are both true (*or*: if and only if, in each possible world, at least one of ϕ and ψ is false)
2. (i) It tells you nothing about the possible truth values of ϕ . The fact that $\phi \models T$ tells you that ϕ is not true in any world in which T is false; but, since T is a tautology, there are no worlds in which it is false, so you are told nothing about the truth value of ϕ in any possible world.

(ii) It tells you that ψ is true in every possible world—i.e., it, too, is a tautology. The fact that $T \models \psi$ tells you that ψ is true in any world in which T is true; but, since T is a tautology, it is true in every possible world.

3. Although it isn't the only way to find an example, an easy way of doing so is to think about a **yes-but** answer in the case of a true sentence with a false implicature. Such an answer will imply the sentence (since it says it is true) but not have the implicature (because it explicitly rejects it). For example, in a context where **There's a cooler in the trunk** implicates **There's beer in the cooler**, the sentence **There's a cooler in the trunk but there's no beer in it** will imply the first but won't implicate the second.

4. **Although Al took the first turn, he missed the second; but he found his way to the meeting**
Although Al took the first turn, he missed the second \wedge Al found his way to the meeting
(Al took the first turn \wedge Al missed the second turn) \wedge Al found his way to the meeting

$(F \wedge S) \wedge W$

both both F and S and W

F: Al took the first turn; S: Al missed the second turn; W: Al found

his way to the meeting

5. Al wrote to Bob \wedge (Al spoke to Carol \wedge Al spoke to Dave)

Al wrote to Bob \wedge Al spoke to Carol and Dave

Al wrote to Bob, and [or: but] he Al spoke to Carol and Dave

[Notice that, while reporting the same facts, you might choose **and** if you were asked *whether* Al reached these people and **but** if you were asked *how* he reached them and were asked this in a context where the difference between writing and speech was important.]

6.

A \wedge (B \wedge C)	1
D	(5)
A	2
B \wedge C	2
B	2
C	(4)

4 QED

●	3
C	3

5 QED

●	3
D	3

3 Cnj C \wedge D

7.

A \wedge B	1
B \wedge D	2

1 Ext A (4)

1 Ext B

2 Ext B (6)

2 Ext D

4 QED

●	3
A	3

○	A, B, D \neq C
C	5

5 QED

●	5
D	5

6 QED

●	5
D	5

5 Cnj C \wedge D 3

3 Cnj A \wedge (C \wedge D)

A B C D | A \wedge B B \wedge D / A \wedge (C \wedge D)

T T F T | ⊕ ⊕ ⊕ F

Phi 270 F06 test 1

1. Define tautologousness by completing the following with a definition in terms of truth values and possible worlds:

ϕ is a tautology if and only if ...

answer

2. Explain what truth values are possible for sentences ϕ and ψ that are both mutually exclusive (i.e., $\phi, \psi \models$) and jointly exhaustive (i.e., $\models \phi, \psi$).

answer

The nursery rhyme “Jack and Jill” contains the line

Jack fell down and broke his crown

Even when this is taken out of context, it is natural to suppose that Jack broke his crown as a result of falling down rather than that falling down and the injury were simply two things that happened to him. I would claim that this tie between the two events is an implicature rather than an implication of the sentence.

3. Explain what I mean when I make that claim in a way that shows you understand the definition of implicature. (You need not support or reject my claim; I’m asking you only to explain what it means.)

answer

4. If the line did imply (rather than merely implicate) that Jack’s broken crown was the result of the fall, the sentence would not be a conjunction of Jack fell down and Jack broke his crown. Explain why this is so in a way that shows you understanding the meaning of implication and the conditions under which conjunctions are true.

answer

Analyze the sentence below in as much detail as possible, presenting the result using symbolic notation and also English notation (i.e., using both ... and). Be sure that the unanalyzed components of your answer are complete and independent sentences (and give a key to the abbreviations you use for them); also try to respect any grouping in the English.

5. The building was completed on time and with no cost overruns, but not everyone was satisfied with it.

answer

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, present a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that divides an open gap. Your table should show the value of any component of any component of the premises and conclusion that is also

compound by writing this value under the main connective of the component.
 Do not use the rule Adj in the first derivation, but you may use it in the second.

6. $A \wedge C, B \wedge D \vDash B \wedge C$

answer

7. $A \wedge (D \wedge E), B \wedge F \vDash (A \wedge B) \wedge C$

answer

Phi 270 F06 test 1 answers

- ϕ is a tautology if and only if there is no possible world in which ϕ is false (or: if and only if ϕ is true in every possible world)
- Sentences that are both mutually exclusive and jointly exhaustive can only have different truth values because mutually exclusive sentences cannot be both true and jointly exhaustive sentences cannot be both false.
- To say that **Jack fell down and broke his crown** implicates but does not imply that breaking his crown is the result of falling down is to say that the sentence would be inappropriate if the two events were not connected in this way but the sentence doesn't actually say that they were so connected.
- The conjunction **Jack fell down \wedge Jack broke his crown** is true whenever both components are true whether or not one led to the other, so **Jack fell down \wedge Jack broke his crown** does not imply that one led to the other because it can be true even if the sentence **Jack's broken crown is the result of his fall** is false. So, if **Jack fell down and broke his crown** did imply this connection, it would not say the same thing as the conjunction.
- The building was completed on time and with no cost overruns, but not everyone was satisfied with it**
The building was completed on time and with no cost overruns \wedge not everyone was satisfied with the building
(the building was completed on time \wedge the building was completed with no cost overruns) \wedge not everyone was satisfied with the building

$$(T \wedge C) \wedge S$$

both both T and C and S

C: the building was completed with no cost overruns; S: not everyone was satisfied with the building; T: the building was completed on time

6.

	$A \wedge C$	1
	$B \wedge D$	2
1 Ext	A	
1 Ext	C	(5)
2 Ext	B	(4)
2 Ext	D	
	●	
4 QED	B	3
	●	
5 QED	C	3
3 Cnj	B \wedge C	

7.

	$A \wedge (D \wedge E)$	1
	$B \wedge F$	3
1 Ext	A	(6)
1 Ext	$D \wedge E$	2
2 Ext	D	
2 Ext	E	
3 Ext	B	(7)
3 Ext	F	
	●	
6 QED	A	5
	●	
7 QED	B	5
5 Cnj	A \wedge B	4
	O	A, B, D, E, F \neq C
	C	4
4 Cnj	(A \wedge B) \wedge C	
	A B C D E F	A \wedge (D \wedge E) B \wedge F / (A \wedge B) \wedge C
	T T F T T T	ⓐ T ⓑ T T ⓓ T ⓔ

Phi 270 F05 test 1

1. Define entailment by completing the following: Γ entails ϕ (i.e., $\Gamma \models \phi$) if and only if (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of the ideas of truth values and possible worlds. Remember that Γ is a set, not a sentence, so it does not have a truth value; but any members of it are sentences and have truth values.)

answer

2. Suppose you know that (i) the set containing ϕ and ψ is inconsistent (i.e., $\phi, \psi \models$) and (ii) the set containing ψ and χ is inconsistent (i.e., $\psi, \chi \models$). What, if anything, can you conclude about the consistency or inconsistency of the set containing ϕ and χ ? That is, what can you conclude about the truth of a claim that $\phi, \chi \models$? Be sure to explain your answers in terms of the definition of inconsistency.

answer

3. Consider the following exchange:

Al: I'm going to the restaurant Chuck told us about.

Bob: I was there yesterday. Do you have health insurance?

Bob could be said to convey information about the restaurant not only through his assertion but also through the question that follows it. Use the idea of *implicature* to explain how this might work. (Just what information you think might be conveyed by the question is less important than your explanation of how that information would be conveyed.)

answer

Analyze the sentences below in as much detail as possible, presenting the result in both symbolic and English notation (i.e., using **both ... and**). Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

4. The water was cool and clear.

answer

5. Adam found Barb's number and called her, but she was out; nevertheless, he went to the party.

answer

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, present a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values that divides an open gap. *Do not use the rule Adj in the first derivation*, but you may use it in the second.

6. $(A \wedge B) \wedge C \models C \wedge A$

answer

7. $F \wedge C, A \wedge (D \wedge E) \models E \wedge (B \wedge C)$

answer

Phi 270 F05 test 1 answers

1. Γ entails ϕ (i.e., $\Gamma \models \phi$) if and only if there is no possible world in which all members of Γ are true but ϕ is false. (*Or*: ... if and only if, in every possible world in all members of Γ are true, ϕ is true as well.)
2. You know, from (i), that ϕ and ψ cannot be true together and, from (ii), that ψ and χ cannot be true together. But it may still be possible for ϕ and χ to be true together (provided ψ is false). Therefore, (i) and (ii) do not provide enough information for you to be able to conclude that the set containing ϕ and χ is inconsistent.
3. The specific information conveyed will depend on the circumstances; but Bob's question fits into the conversation—i.e., is appropriate—only if eating at the restaurant makes having health insurance somehow relevant. So it would be an implicature of the question that eating at the restaurant makes whether you have health insurance in some way significant. (In most contexts, the specific suggestion would probably be that eating at the restaurant puts one's health in danger; but, in certain contexts, the suggestion might be something different—for example, that the restaurant is very expensive.)
4. The water was cool and clear
The water was cool \wedge the water was clear
 $C \wedge R$
both C and R
C: the water was cool; R: the water was clear
5. Adam found Barb's number and called her, but she was out; nevertheless, he went to the party
Adam found Barb's number and called her, but she was out \wedge Adam went to the party
(Adam found Barb's number and called her \wedge Barb was out) \wedge Adam went to the party
((Adam found Barb's number \wedge Adam called Barb) \wedge Barb was out) \wedge Adam went to the party
 $((F \wedge C) \wedge O) \wedge W$
both both both F and C and O and W
C: Adam called Barb; F: Adam found Barb's number; O: Barb was out; W: Adam went to the party

6.	(A ∧ B) ∧ C	1
1 Ext	A ∧ B	2
1 Ext	C	(4)
2 Ext	A	(5)
2 Ext	B	
	●	
4 QED	C	3
	●	
5 QED	A	3
3 Cnj	C ∧ A	

7.	F ∧ C	1
	A ∧ (D ∧ E)	2
1 Ext	F	
1 Ext	C	(7)
2 Ext	A	
2 Ext	D ∧ E	3
3 Ext	D	
3 Ext	E	(5)
	●	
5 QED	E	4
	○	A, C, D, E, F ≠ B
	B	6
	●	
7 QED	C	6
6 Cnj	B ∧ C	4
4 Cnj	E ∧ (B ∧ C)	
A B C D E F	F ∧ C	A ∧ (D ∧ E) / E ∧ (B ∧ C)
T F T T T T	Ⓢ	Ⓢ T Ⓢ F

Phi 270 F04 test 1

1. Define inconsistency by completing the following: Γ is inconsistent (i.e., $\Gamma \models$) if and only if (Your answer need not replicate the wording of the text's definitions, but it should define inconsistency in terms of the ideas of truth values and possible worlds. Remember that Γ is a set, not a sentence, so it does not have a truth value; but any members of it are sentences and have truth values.)

answer

2. Define equivalence by completing the following: $\phi \simeq \psi$ if and only if (Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of the ideas of truth values and possible worlds.)

answer

3. Suppose you know that (i) $\phi \models \psi$ (i.e., ϕ entails ψ), (ii) $\psi \models \chi$ (i.e., ψ entails χ), and (iii) ψ is true (in the actual world). What, if anything, can you conclude about the truth values of ϕ and χ (in the actual world)? Be sure to say what can be known about each of ϕ and χ and be sure to explain your answers in terms of the definition of entailment.

answer

4. Suppose that ϕ implies ψ and also that ϕ implicates χ . Which of the following patterns of truth values are ruled out and which are permitted by the cited relations among the three sentences? Explain your answer using the definitions of implication and implicature.

$\phi \quad \psi \quad \chi$

(a) T T F

(b) T F T

(c) T F F

answer

Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation (i.e., using **both ... and**). Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

5. **Ed tried the door, but it was locked; however, the window was open, and he climbed through it**

answer

Use derivations to check whether each of the claims of entailment below holds. If an entailment fails, present a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) that divides an open gap. *Do*

not use the rule Adj in the first derivation, but you may use it in the second.

6. $A \wedge C, B \wedge D \models B \wedge (C \wedge D)$

answer

7. $A \wedge (B \wedge C) \models (A \wedge B) \wedge (C \wedge D)$

answer

Phi 270 F04 test 1 answers

- Γ is inconsistent (i.e., $\Gamma \models$) if and only if there is no possible world in which all members of Γ are true. (Or: ... if and only if, in each possible world, at least one member of Γ is false.)
- $\phi \simeq \psi$ if and only if there is no possible world in which ϕ and ψ have different truth values. (Or: ... if and only if, in each possible world, ϕ has the same truth value as ψ .)
- You know that χ is true because you know that ψ is true and also that χ must be true in every possible world in which ψ is true (because $\psi \models \chi$). However, you know nothing about the truth value of ϕ because, while you know that ψ is true if ϕ is (because $\phi \models \psi$), it may be that ψ is true also in some cases in which ϕ is false.
- Patterns (b) and (c) are ruled out but (a) is not. Since ϕ implies ψ , it cannot be true when ψ is false; that rules out (b) and (c) but not (a). And, while a sentence with a false implicature is inappropriate and misleading, it may be true; therefore, the fact that ϕ implicates χ rules out no pattern of truth values for the two.
- Ed tried the door, but it was locked; however, the window was open, and he climbed through it
 Ed tried the door, but it was locked \wedge the window was open, and Ed climbed through it
 (Ed tried the door \wedge the door was locked) \wedge (the window was open \wedge Ed climbed through the window)
 $(T \wedge L) \wedge (O \wedge C)$
 both both T and L and both O and C
 C: Ed climbed through the window; L: the door was locked; O: the window was open; T: Ed tried the door

6.	$A \wedge C$	1
	$B \wedge D$	2
1 Ext	A	
1 Ext	C	(6)
2 Ext	B	(5)
2 Ext	D	(7)
5 QED	B	3
6 QED	C	4
7 QED	D	4
4 Cnj	$C \wedge D$	3
3 Cnj	$B \wedge (C \wedge D)$	

7.	$A \wedge (B \wedge C)$	1
1 Ext	A	(5)
1 Ext	$B \wedge C$	2
2 Ext	B	(6)
2 Ext	C	(8)
5 QED	A	4
6 QED	B	4
4 Cnj	$A \wedge B$	3
8 QED	C	7
	O	A, B, C \neq D
	D	7
7 Cnj	$C \wedge D$	3
3 Cnj	$(A \wedge B) \wedge (C \wedge D)$	
A B C D	$A \wedge (B \wedge C) / (A \wedge B) \wedge (C \wedge D)$	
T T T F	$\textcircled{\text{T}}$ T T $\textcircled{\text{F}}$	

Phi 270 F03 test 1

1. Define entailment by completing the following: $\Gamma \models \phi$ if and only if
(Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds. Remember that Γ is a set, not a sentence, though its members are sentences.)
answer
2. Define absurdity by completing the following: ϕ is absurd if and only if (Your answer need not replicate the wording of the text's definitions, but it should define absurdity in terms of truth values and possible worlds.)
answer
3. Is it possible for there to be a pair of sentences ϕ and ψ where (i) $\phi \simeq \psi$ (i.e., ϕ and ψ are equivalent) and (ii) ϕ and ψ together form an inconsistent set (i.e., the set $\{\phi, \psi\}$ is inconsistent)? If it is possible for both (i) and (ii) to be true of a pair of sentences ϕ and ψ , describe (in terms of the possibilities for truth values) what ϕ and ψ must be like. If it is not possible, explain why in terms of possibilities for truth values. (Hint: this is not a trick question but it may trip you up if you try to answer it intuitively; you'll do better to just think through the consequences of the definitions of equivalence and inconsistency.)
answer
4. Give an example of implicature, presenting a sentence and describing situations in which it (i) is true and not misleading, (ii) is true but misleading, and (iii) is false. Explain your answer.
answer
5. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation (i.e., using **both ... and**). Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

The road was completed and opened to traffic, but it was closed for repairs and has not been re-opened

answer

Use derivations to check whether each of the entailments below holds. If one fails, present a counterexample by providing a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which divides an open gap.

6. $A \wedge B, C \wedge (D \wedge E) \models B \wedge D$

answer

7. $A \wedge B, D \wedge E \models (A \wedge C) \wedge D$

answer

Phi 270 F03 test 1 answers

1. $\Gamma \models \phi$ if and only if there is no possible world in which ϕ is false while all members of Γ are true
2. ϕ is absurd if and only if there is no possible world in which ϕ is true
3. If ϕ and ψ are equivalent they must have the same truth value as each other in every possible world. If they form an inconsistent set, there is no possible world in which both are true. Both of these are possible if there is no possible world in which either ϕ or ψ is true; that is, it is possible if both are absurd.
4. To re-use an example from class, if I say "My class was taught this morning" to someone, my statement (i) would be true and not misleading if someone else taught my class, (ii) would be true but misleading if I taught it, and (iii) would be false if it wasn't taught at all. (i) and (ii) differ regarding the truth of an implicature of the sentence, which arises because saying "My class was taught" instead of "I taught my class" would usually be inappropriate unless I didn't teach my class.
5. **The road was completed and opened to traffic, but it was closed for repairs and has not been re-opened**
The road was completed and opened to traffic \wedge the road was closed for repairs and has not been re-opened
(the road was completed \wedge the road was opened to traffic) \wedge (the road was closed for repairs \wedge the road has not been re-opened)
 $(C \wedge O) \wedge (R \wedge N)$
both both C and O and both R and N
C: the road was completed; N: the road has not been re-opened; O: the road was opened to traffic; R: the road was closed for repairs

	$A \wedge B$	1
	$C \wedge (D \wedge E)$	2
1 Ext	A	
1 Ext	B	(5)
2 Ext	C	
2 Ext	$D \wedge E$	3
3 Ext	D	(6)
3 Ext	E	
●		
5 QED	B	4
●		
6 QED	D	4
4 Cnj	$B \wedge D$	

	$A \wedge B$	1
	$D \wedge E$	2
1 Ext	A	(5)
1 Ext	B	
2 Ext	D	(6)
2 Ext	E	
●		
5 QED	A	4
○		
○		
○		
4 Cnj	$A \wedge C$	3
●		
6 QED	D	3
3 Cnj	$(A \wedge C) \wedge D$	

This derivation could be ended after the first dead-end open gap appears at stage 4.

A	B	C	D	E	A ∧ B, D ∧ E / (A ∧ C) ∧ D
T	T	F	T	T	Ⓣ Ⓣ F Ⓣ

Phi 270 F02 test 1

- Define entailment for the special case of three premises by completing the following: $\varphi, \psi, \chi \models \theta$ if and only if ... (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds.)

answer

- Suppose that each of φ and ψ is a tautology (i.e., $\models \varphi$ and also $\models \psi$). What, if anything can you conclude about the equivalence of φ and ψ ? That is, do you have enough information to conclude that they are equivalent (i.e., that $\varphi \simeq \psi$)? to conclude that they aren't equivalent?—or can't you say for sure? Explain your answer by reference to the definitions of tautologousness and equivalence, using the concepts of possible worlds and truth values that appear in those definitions.

answer

- Does a statement always entail everything it implicates? If you answer yes, explain why using the definitions of entailment and implicature? If you answer no, give an example of a case of implicature that isn't a case of entailment.

answer

- Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Jack saw the book and told Jill about it, but she had already read it

answer

Use derivations to check whether each of the entailments below holds. If one fails, present a counterexample—that is, provide a table in which you calculate the truth values of the premises and conclusion on an assignment of truth values which divides an open gap.

5. $(A \wedge B) \wedge (C \wedge D) \models A \wedge D$

answer

6. $A \wedge D \models (A \wedge B) \wedge (C \wedge D)$

answer

- [This question was on a topic not covered in F09] Use replacement principles to put the following sentence into list normal form (i.e., to transform it into a sentence in which no conjunction is the left component of a conjunction and letters appear in alphabetical order without repetition):

$((A \wedge B) \wedge C) \wedge A$

answer

8. Why is single dead-end open gap enough to show that an entailment does not hold? That is, why must all gaps close for a derivation to show that an entailment *does* hold?

answer

Phi 270 F02 test 1 answers

- $\phi, \psi, \chi \models \theta$ if and only if there is no possible world in which ϕ, ψ , and χ are true but θ is false
- ϕ and ψ are equivalent. Because each is a tautology, there is no possible world in which either is false, so there is no possible world in which they have different truth values.
- No. Any true sentence with a false implicature provides an example of an implicature that isn't an entailment because no true sentence can entail a false one, so you need only provide an example of a true sentence with a false implicature.
- Jack saw the book and told Jill about it, but she had already read it
 Jack saw the book and told Jill about it \wedge Jill had already read the book
 (Jack saw the book \wedge Jack told Jill about the book) \wedge Jill had already read the book

$$(S \wedge T) \wedge R$$

both both S and T and R

R: Jill had already read the book; S: Jack saw the book; T: Jack told Jill about the book

5.

	$(A \wedge B) \wedge (C \wedge D)$	1
1 Ext	$A \wedge B$	2
1 Ext	$C \wedge D$	3
2 Ext	A	(5)
2 Ext	B	
3 Ext	C	
3 Ext	D	(6)
5 QED	● A	4
6 QED	● D	4
4 Cnj	$A \wedge D$	

6.

	$A \wedge D$	1
1 Ext	A	(5)
1 Ext	D	(6)
5 QED	● A	3
	○ B	$A, D \not\models B$
3 Cnj	$A \wedge B$	3
	○ C	$A, D \not\models C$
6 QED	● D	4
4 Cnj	$C \wedge D$	2
2 Cnj	$(A \wedge B) \wedge (C \wedge D)$	

This derivation could be ended after the first dead-end open gap appears at stage 3. Any one of the three interpretations below is enough to present as a counterexample (though the third serves as a counterexample only once the second dead-end open gap is reached).

A	B	C	D	$A \wedge D / (A \wedge B) \wedge (C \wedge D)$
T	F	T	T	Ⓣ F Ⓢ T
T	F	F	T	Ⓣ F Ⓢ F
T	T	F	T	Ⓣ T Ⓢ F

7. [This question was on a topic not covered in F09]

$$\begin{aligned}
 & \underline{\underline{((A \wedge B) \wedge C) \wedge A}} \\
 & \approx \\
 & A \wedge \underline{\underline{(A \wedge B) \wedge C}} \\
 & \approx \\
 & \underline{\underline{A \wedge (A \wedge (B \wedge C))}} \\
 & \approx \\
 & \underline{\underline{(A \wedge A) \wedge (B \wedge C)}} \\
 & \approx \\
 & A \wedge \underline{\underline{(B \wedge C)}}
 \end{aligned}$$

8. A dead-end open gap is always divided by an interpretation that also makes the initial resources of the derivation true and its initial goal false. So any dead-end open gap shows that the derivation for which the derivation is constructed can have true premises and a false conclusion.

Phi 270 F00 test 1

- Define (logical) equivalence by completing the following: $\varphi \simeq \psi$ if and only if (Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of truth values and possible worlds.)
answer
- Suppose you know that φ entails ψ (i.e., $\varphi \models \psi$) and that φ is false. What, if anything can you conclude about the truth value of ψ ? Explain your answer by reference to the definitions of entailment and equivalence, making explicit reference to the possibilities of truth and falsity mentioned in these definitions.
answer
- Suppose everything implicated (and thus everything implied) by a sentence φ is actually implied by a sentence ψ (so ψ explicitly says everything that φ suggests) and suppose that ψ has no further implicatures. Could ψ be true but inappropriate (and thus true but misleading)? Explain your answer using the definitions of implicature and implication.
answer
- Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Sam got a red jellybean but wanted a green one; and Tom got a green one even though he wanted a red one

answer

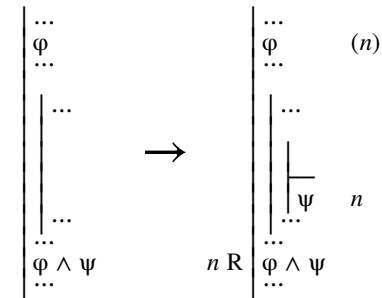
Use derivations (but no replacement rules) to check whether each of the entailments below holds. If one fails, provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which divides an open gap.

- $A \wedge (B \wedge C) \models (A \wedge B) \wedge D$
answer
- $(A \wedge B) \wedge C, D \wedge E \models (D \wedge B) \wedge E$
answer
- [This question was on a topic not covered in F09] Use replacement principles to put the following sentence into list normal form (in which no conjunction is the left component of a conjunction and letters appear in alphabetical order without repetition):

$$(G \wedge B) \wedge (D \wedge G)$$

answer

- Explain why the following derivation rule R would be a legitimate one. That is, explain why it would be legitimate, when φ appears among the active resources of a gap, to develop the gap by replacing a goal $\varphi \wedge \psi$ with the new goal ψ . [When evaluating your answer, I'll be less concerned about your mastery of my technical terminology than in your intuitive understanding of how derivations work. Your answer need not be long; two or three sentences would be enough to give an entirely satisfactory answer.]



answer

Phi 270 F00 test 1 answers

- $\varphi \simeq \psi$ if and only if there is no possible world in which φ and ψ have different truth values.
- You can conclude nothing. Although the definition of entailment requires that ψ be true φ is true, it places no constraints on its value when φ is false; that is, it doesn't rule out either the possibility that φ and ψ are both false or the possibility that ψ is true even though φ isn't.
- No. Since ψ implicates nothing beyond what φ does and implies all of φ 's implicatures, anything required for ψ to be appropriate is required for it to be true.
- Sam got a red jellybean but wanted a green one; and Tom got a green one even though he wanted a red one
 Sam got a red jellybean but wanted a green one \wedge Tom got a green jellybean even though he wanted a red one
 (Sam got a red jellybean \wedge Sam wanted a green jellybean) \wedge (Tom got a green jellybean \wedge Tom wanted a red jellybean)

$$(R \wedge G) \wedge (D \wedge N)$$

both both R and G and both D and N

D: Tom got a green jellybean; G: Sam wanted a green jellybean; N: Tom wanted a red jellybean; R: Sam got a red jellybean

5.	$A \wedge (B \wedge C)$	1
1 Ext	A	(5)
1 Ext	$B \wedge C$	2
2 Ext	B	(6)
2 Ext	C	
5 QED	A	4
6 QED	B	4
4 Cnj	$A \wedge B$	3
	\circ	$A, B, C, \neq D$
	D	3
3 Cnj	$(A \wedge B) \wedge D$	
A B C D	$A \wedge (B \wedge C) / (A \wedge B) \wedge D$	
T T T F	\oplus T T \oplus	

6.	$(A \wedge B) \wedge C$	1
	$D \wedge E$	2
1 Ext	$A \wedge B$	3
1 Ext	C	
2 Ext	D	(6)
2 Ext	E	(8)
3 Ext	A	
3 Ext	B	(7)
6 QED	D	5
7 QED	B	5
5 Cnj	$D \wedge B$	4
	\bullet	
8 QED	E	4
4 Cnj	$(D \wedge B) \wedge E$	

7. [This question was on a topic not covered in F09]

$$\begin{aligned}
 & \underline{(G \wedge B) \wedge (D \wedge G)} \\
 & \quad \simeq \\
 & \underline{(B \wedge G) \wedge (D \wedge G)} \\
 & \quad \simeq \\
 & B \wedge \underline{(G \wedge (D \wedge G))} \\
 & \quad \simeq \\
 & B \wedge \underline{((D \wedge G) \wedge G)} \\
 & \quad \simeq \\
 & B \wedge (D \wedge \underline{(G \wedge G)}) \\
 & \quad \simeq \\
 & B \wedge (D \wedge G)
 \end{aligned}$$

8. A gap with $\phi \wedge \psi$ as a goal represents the question whether $\phi \wedge \psi$ is entailed by the gap's active resources. In this case, the question is whether $\phi \wedge \psi$ is entailed by a group of assumptions including ϕ . That will be so if and only if each of ϕ and ψ is entailed and ϕ is bound to be entailed by a group of assumptions that includes it, so the only question is whether ψ is entailed by these assumptions. And that's the question represented by gap that results when this rule is applied.

Phi 270 F99 test 1

1. Define tautologousness by completing the following: $\models \phi$ if and only if (Your answer need not replicate the wording of the text's definitions, but it should define tautologousness in terms of truth values and possible worlds.)

answer

2. Suppose you know that $\phi \models \chi$ and that $\psi \models \chi$ (i.e., χ is implied, or entailed, by each one of ϕ and ψ). Can you conclude that $\phi \simeq \psi$ (i.e., ϕ and ψ are equivalent)? Explain why or why not by reference to the definitions of entailment and equivalence, making explicit reference to the possibilities of truth and falsity mentioned in these definitions.

answer

3. Give your own example of a true sentence with a false implicature, using the definition of implicature to explain why it is an example. [The originality of the example counts for something here but your explanation is the more important aspect of the answer.]

answer

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Sam finished the job even though he was tired and it wasn't urgent.

answer

Use the basic system of derivations (i.e., no replacement rules) to check whether each of the entailments below holds. If one fails, provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which divides an open gap.

5. $(A \wedge B) \wedge C \models C \wedge A$

answer

6. $A \wedge D, E \wedge A \models (A \wedge B) \wedge C$

answer

7. [This question was on a topic not covered in F09] Use replacement principles to put the following sentence into list normal form (in which no conjunction is the left component of a conjunction and letters appear in alphabetical order without repetition):

$$A \wedge ((B \wedge A) \wedge C)$$

answer

1. $\models \phi$ if and only if there is no possible world in which ϕ is false.
2. No. The information about entailment tells you that χ is true in every possible world in which ϕ is true and also in every one where ψ is true. But there could still be possible worlds where one of ϕ and ψ is true while the other is false, and it is such worlds that would have to be ruled out for us to be sure that ϕ and ψ are equivalent.
3. [An example of the kind of thing you might say:] **Al got into a house** would be true when said of someone who entered his own house through an unlocked door. But the sentence would ordinarily implicate that Al entered a house that wasn't his own and faced some difficulty in doing so because such factors would be the most likely reason for it to be appropriate to use **a house** rather than **his house** and **got into** rather than **went into**.
4. **Sam finished the job even though he was tired and it wasn't urgent**
 $\text{Sam finished the job} \wedge \text{Sam was tired and the job wasn't urgent}$
 $\text{Sam finished the job} \wedge (\text{Sam was tired} \wedge \text{the job wasn't urgent})$

$$F \wedge (T \wedge N)$$

both F and both T and N

F: **Sam finished the job**; N: **the job wasn't urgent**; T: **Sam was tired**

	$(A \wedge B) \wedge C$	1
1 Ext	$A \wedge B$	2
1 Ext	C	(4)
2 Ext	A	(5)
2 Ext	B	
4 QED	● C	3
5 QED	● A	3
3 Cnj	C \wedge A	

6.

	$A \wedge D$	1
	$E \wedge A$	2
1 Ext	A	(5)
1 Ext	D	
2 Ext	E	
2 Ext	A	
5 QED	●	
	A	4
	○	$A, D, E \not\models B$
	B	4
4 Cnj	$A \wedge B$	3
	○	$A, D, E \not\models C$
	C	3
3 Cnj	$(A \wedge B) \wedge C$	

Only one of the following counterexamples need be presented. The first two divide the first open gap and the last two divide the second one.

A	B	C	D	E	$A \wedge D, E \wedge A / (A \wedge B) \wedge C$
T	F	T	T	T	Ⓟ Ⓟ F Ⓟ
T	F	F	T	T	Ⓟ Ⓟ F Ⓟ
T	T	F	T	T	Ⓟ Ⓟ T Ⓟ

7. [This question was on a topic not covered in F09]

$$\begin{aligned}
 & A \wedge ((B \wedge A) \wedge C) \\
 & \quad \simeq \\
 & A \wedge ((A \wedge B) \wedge C) \\
 & \quad \simeq \\
 & \underline{A \wedge (A \wedge (B \wedge C))} \\
 & \quad \simeq \\
 & \underline{(A \wedge A) \wedge (B \wedge C)} \\
 & \quad \simeq \\
 & A \wedge (B \wedge C)
 \end{aligned}$$

Phi 270 F98 test 1

1. Define entailment by completing the following: $\Gamma \models \phi$ if and only if (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds. Remember that Γ is a set, not a sentence, so it does not itself have a truth-value.)

answer

2. Suppose you know that $\models \phi$ (i.e., ϕ is a tautology) and that $\phi \models \psi$. Can you conclude that $\phi \simeq \psi$? Explain why or why not by considering possibilities of truth and falsity.

answer

3. Explain how a sentence can implicate something it doesn't imply. (While you don't need to state the definitions of implication and implicature, you will need to employ the ideas used in them.)

answer

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Although Carol called Dave, she didn't reach him; but Ed stopped by and helped her finish the job.

answer

5. Use the basic system of derivations (i.e., no replacement rules) to check whether the entailment below holds. If it fails, provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which makes the premises true and conclusion false.

$$A \wedge (B \wedge C), B \wedge D \models (C \wedge D) \wedge E$$

answer

6. [This question was on a topic not covered in F09] Use replacement principles to put the following sentence into list normal form (in which no conjunction is the left component of a conjunction and letters appear in alphabetical order without repetition):

$$(B \wedge C) \wedge (A \wedge B)$$

answer

Phi 270 F98 test 1 answers

- $\Gamma \models \phi$ if and only if there is no possible world in which ϕ is false while every member of Γ is true.
- Yes. In every possible world, ϕ is true (because it is a tautology) so ψ is

true too (since $\phi \models \psi$), so there is no possible world in which ϕ and ψ have different truth values.

3. Since a true sentence can still be inappropriate for other reasons, something can be required for a sentence to be appropriate (and thus be implicated by it) without being required for its truth (and thus without being implied by it).
4. *Although Carol called Dave, she didn't reach him; but Ed stopped by and helped her finish the job*
Although Carol called Dave, she didn't reach him \wedge Ed stopped by and helped Carol finish the job
(Carol called Dave \wedge Carol didn't reach Dave) \wedge (Ed stopped by \wedge Ed helped Carol finish the job)

$$(C \wedge D) \wedge (E \wedge F)$$

both both C and D and both E and F

C: Carol called Dave; D: Carol didn't reach Dave; E: Ed stopped by;

F: Ed helped Carol finish the job

5.

	$A \wedge (B \wedge C)$	1
	$B \wedge D$	2
1 Ext	A	
1 Ext	$B \wedge C$	3
2 Ext	B	
2 Ext	D	(7)
3 Ext	B	
3 Ext	C	(6)
6 QED	● C	5
7 QED	● D	5
5 Cnj	C \wedge D	4
	○	A, B, C, D \neq E
	E	4
4 Cnj	(C \wedge D) \wedge E	
A B C D E	A \wedge (B \wedge C), B \wedge D / (C \wedge D) \wedge E	
T T T T F	⊕ T ⊕ T ⊕	

6. [This question was on a topic not covered in F09]

$$(B \wedge C) \wedge (A \wedge B)$$

\approx

$$(A \wedge B) \wedge (B \wedge C)$$

\approx

$$A \wedge (B \wedge (B \wedge C))$$

\approx

$$A \wedge ((B \wedge B) \wedge C)$$

\approx

$$A \wedge (B \wedge C)$$

Phi 270 F97 test 1

1. Define (a special case of) entailment by completing the following: $\phi, \psi \models \chi$ if and only if (Your answer need not replicate the wording of the text's definitions, but it should define entailment in terms of truth values and possible worlds.)
answer
2. Suppose you know that $\phi \simeq \psi$ and that $\chi \simeq \psi$. Can you conclude that $\phi \simeq \chi$? Explain why or why not by considering possibilities of truth and falsity.
answer
3. Provide an example of a true sentence that has a false implicature. (Be sure to state both the sentence and its implicature and to explain why one is true and the other false and also why one implicates the other.)
answer
4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Ann and Bill helped to plan the campaign, but Carol directed it and reported its results

5. Use the basic system of derivations (i.e., no replacement rules) to establish the following:

$$A \wedge B, C \wedge D \models (A \wedge C) \wedge B$$

6. Use the basic system of derivations (i.e., no replacement rules) to show that the entailment below fails; provide a table in which you calculate the truth values of the premises and conclusion on an extensional interpretation (i.e., an assignment of truth values) which makes the premises true and conclusion false:

$$(A \wedge B) \wedge C \models A \wedge (D \wedge C)$$

answer

Phi 270 F97 test 1 answers

1. $\phi, \psi \models \chi$ if and only if there is no possible world in which χ is false while ϕ and ψ are both true.
2. Yes. In any possible world, each of ϕ and χ must have the same truth value as ψ , so they must have the same truth value as each other.
3. **I pushed the button but the motor shut off** is true even if the button is

an emergency shutoff button for the motor, but it implicates (at least) **The button did not shut off the motor**, which is false in this case. The implicature arises because the word *but* is not appropriate in cases where the clause it introduces describes an expected result of the truth of the other clause.

4. **Ann and Bill helped to plan the campaign, but Carol directed it and reported its results**
Ann and Bill helped to plan the campaign \wedge **Carol directed the campaign and reported its results**
(Ann helped to plan the campaign \wedge **Bill helped to plan the campaign)** \wedge **(Carol directed the campaign** \wedge **Carol reported the campaign's results)**

$$(A \wedge B) \wedge (D \wedge R)$$

both both **A and B** and both **D and R**

A: Ann helped to plan the campaign; B: Bill helped to plan the campaign; D: Carol directed the campaign; R: Carol reported the campaign's results

5.

	$A \wedge B$	1				
	$C \wedge D$	2				
1 Ext	A	(5)				
1 Ext	B	(7)				
2 Ext	C	(6)				
2 Ext	D					
5 QED	<table style="border-collapse: collapse; margin-left: 5px;"> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">●</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">A</td></tr> </table>		●		A	4
	●					
	A					
6 QED	<table style="border-collapse: collapse; margin-left: 5px;"> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">●</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">C</td></tr> </table>		●		C	4
	●					
	C					
4 Cnj	<table style="border-collapse: collapse; margin-left: 5px;"> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">A \wedge C</td></tr> </table>		A \wedge C	3		
	A \wedge C					
7 QED	<table style="border-collapse: collapse; margin-left: 5px;"> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">●</td></tr> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">B</td></tr> </table>		●		B	3
	●					
	B					
3 Cnj	<table style="border-collapse: collapse; margin-left: 5px;"> <tr><td style="border-left: 1px solid black; padding-left: 2px;"> </td><td style="text-align: center;">(A \wedge C) \wedge B</td></tr> </table>		(A \wedge C) \wedge B			
	(A \wedge C) \wedge B					

6.

	$(A \wedge B) \wedge C$	1
1 Ext	$A \wedge B$	2
1 Ext	C	(6)
2 Ext	A	(5)
2 Ext	B	
	●	
5 QED	A	3
	○	A, B, C \neq D
	D	4
	●	
6 QED	C	4
4 Cnj	D \wedge C	3
3 Cnj	A \wedge (D \wedge C)	
A B C D	$(A \wedge B) \wedge C / A \wedge (D \wedge C)$	
T T T F	T ⊕ ⊕ F	

Phi 270 F96 test 1

1. Define equivalence by completing the following: $\phi \simeq \psi$ if and only if
(Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of truth values and possible worlds.)

answer

2. Suppose you know that $\phi, \psi \models \chi$ and that $\theta \models \psi$. Can you conclude that $\phi, \theta \models \chi$? Explain why or why not by considering possibilities of truth and falsity.

answer

3. Give an example of a case of implicature that is not a case of implication, referring to definitions of implication and implicature to explain why your example is a case of one but not the other.

answer

4. Analyze the sentence below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

Although Mikita and Stapleton assisted him, Hull scored the goal

answer

5. Use the basic system of derivations (i.e., no replacement rules) to establish the following:

$$A, C \wedge D \models C \wedge (A \wedge D)$$

answer

6. Use the basic system of derivations (i.e., no replacement rules) to show that the entailment below fails (as a generalization); give an extensional interpretation (i.e., an assignment of truth values) which makes its premises true and conclusion false:

$$(D \wedge C) \wedge B \models A \wedge C$$

answer

Phi 270 F96 test 1 answers

1. $\phi \simeq \psi$ if and only if there is no possible world in which ϕ and ψ differ in their truth values.
2. Yes. In any possible world in which ϕ and θ are true, ψ will be true (since $\theta \models \psi$), so χ will be true (since $\phi, \psi \models \chi$). So there can be no world in which ϕ and θ are true while χ is false.
3. For example: A car is in my driveway implicates but does not imply A car other than mine is in my driveway; for, while it would in general be inappropriate to say A car is in my driveway (rather than say, for

example, *My car is in the driveway*) if the car is mine, it would not be false when the driveway contained only my car.

4. Although Mikita and Stapleton assisted him, Hull scored the goal
 Mikita and Stapleton assisted Hull \wedge Hull scored the goal
 (Mikita assisted Hull \wedge Stapleton assisted Hull) \wedge Hull scored the

$$\text{goal} \\ (M \wedge S) \wedge H$$

both both M and S and H

H: Hull scored the goal; M: Mikita assisted Hull ; S: Stapleton assisted Hull

5.

	A	(5)
	$C \wedge D$	1
1	C	(3)
1	D	(6)
3 QED	C	2
5 QED	A	4
6 QED	D	4
4 Cnj	$A \wedge D$	2
2 Cnj	$C \wedge (A \wedge D)$	

6.

	$(D \wedge C) \wedge B$	1
1 Ext	$D \wedge C$	2
1 Ext	B	
2 Ext	D	
2 Ext	C	(4)
	O	B, C, D \neq A
	A	3
4 QED	C	3
3 Cnj	$A \wedge C$	
A B C D	$(D \wedge C) \wedge B / A \wedge C$	
F T T T	T ① ②	