

**Topics for test 4**

The following are the topics to be covered. The proportion of the test covering each will approximate the proportion of the classes so far that have been devoted to that topic. Your homework and the collection of old tests will provide specific examples of the kinds of questions I might ask.

*Analysis.* Be ready to handle any of the key issues discussed in class—for example, the proper analysis of **every**, **no**, and **only** (see §7.2.2), how to incorporate bounds on complementary generalizations (see §7.2.3), ways of handling compound quantifier phrases (such as **only cats and dogs**, see §7.3.2), the distinction between **every** and **any** (see §§7.3.3 and 7.4.2), how to represent multiple quantifier phrases with overlapping scope (see §7.4.1). You should be able restate your analysis using unrestricted quantifiers (see §7.2.1), but you will not need to present it in English notation.

*Synthesis.* You may be given a symbolic form and an interpretation of its non-logical vocabulary and asked to express the sentence in English. Remember that the distinction between **every** and **any** can be important here, too.

*Derivations.* Be able to construct derivations to show that entailments hold and to show that they fail. I may tell you in advance whether an entailment holds or leave it to you to check that using derivations. If a derivation fails, you *may* be asked to present a counterexample, which will involve describing a structure. You will *not* be responsible for the rules introduced in §7.8.1.

**Phi 270 F06 test 4**

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *State your analysis also in a form that expresses any generalizations using unrestricted quantifiers.*

1. **Every door was locked.**  
answer
2. **Only people who had witnessed the event were able to follow the description of it.**

[It is possible for the scope of **only** to change with emphasis; although varying interpretations are less likely with this sentence than with others, you may choose whichever scope seems most plausible to you.]

answer

3. **No key opened every door.**  
[You should understand this sentence to leave open the possibility that some key opened some door.]

answer

Synthesize an English sentence with the following logical form; that is, find a sentence that would have the following analysis:

4.  $(\forall x: Px \wedge Nxa) (Dxm \vee Axm)$   
A: [ **\_ was acted on at \_** ]; D: [ **\_ was discussed at \_** ]; N: [ **\_ was on \_** ];  
P: [ **\_ was a proposal** ]; a: **the agenda**; m: **the meeting**

answer

Use derivations to show that the following arguments are valid. You may use any rules.

5.  $\forall x (Fx \rightarrow (Gx \rightarrow Hx))$   
 $\forall x Gx$

---

$\forall x (Fx \rightarrow Hx)$

answer

6.  $\forall x (Fx \rightarrow \forall y Rxy)$   
 $\forall x Fx$

---

$\forall x \forall y Ryx$

answer

Use a derivation to show that the following argument is not valid and present a counterexample by describing a structure that divides an open gap. (You may describe the structure either by depicting it in a diagram, as answers in the text usually do, or by giving tables.)

7.  $\forall x Rax$   
 $\forall x Rxb$   


---

 $\forall x Rxx$   
answer

**Phi 270 F06 test 4 answers**

- Every door was locked**  
**Every door is such that (it was locked)**  
 $(\forall x: \underline{x} \text{ is a door}) \underline{x} \text{ was locked}$   
 $(\forall x: Dx) Lx$   
 $\forall x (Dx \rightarrow Lx)$   
D: [    is a door]; L: [    was locked]
- only people who had witnessed the event were able to follow the description of it**  
**only people who had witnessed the event are such that (they were able to follow the description of it)**  
 $(\forall x: \neg x \text{ is a person who had witnessed the event}) \neg \underline{x} \text{ was able to follow the description of the event}$   
 $(\forall x: \neg (\underline{x} \text{ is a person} \wedge \underline{x} \text{ had witnessed the event})) \neg Fx(\text{the description of the event})$   
 $(\forall x: \neg (Px \wedge Wxe)) \neg Fx(de)$   
 $\forall x (\neg (Px \wedge Wxe) \rightarrow \neg Fx(de))$   
F: [    was able to follow   ]; P: [    is a person]; W: [    had witnessed   ]; e: **the event**; d: [**the description of**]

Other possible (though less likely) interpretations:  
 $(\forall x: Px \wedge \neg Wxe) \neg Fx(de)$  [**only people who had witnessed...**]  
 $(\forall x: \neg Px \wedge Wxe) \neg Fx(de)$  [**only people who had witnessed ...**]

Not a possibility:  
 $(\forall x: \neg Px \wedge \neg Wxe) \neg Fx(de)$

- No key opened every door**  
**No key is such that (it opened every door)**  
 $(\forall x: \underline{x} \text{ is a key}) \neg x \text{ opened every door}$   
 $(\forall x: Kx) \neg \text{every door is such that (x opened it)}$   
 $(\forall x: Kx) \neg (\forall y: \underline{y} \text{ is a door}) \underline{x} \text{ opened } \underline{y}$   
 $(\forall x: Kx) \neg (\forall y: Dy) Oxy$   
 $\forall x (Kx \rightarrow \neg \forall y (Dy \rightarrow Oxy))$   
D: [    is a door]; K: [    is a key]; O: [    opened   ]

Although there are equivalent analyses, one that differs only in the location of  $\neg$  is likely to be wrong. In particular,  $(\forall x: Kx) (\forall y: Dy) \neg Oxy$  rules out the possibility that some key opened some door.

- $(\forall x: Px \wedge Nxa) (Dxm \vee Axm)$   
 $(\forall x: x \text{ was a proposal} \wedge x \text{ was on the agenda}) (x \text{ was discussed at the meeting} \vee x \text{ was acted on at the meeting})$   
 $(\forall x: x \text{ was a proposal on the agenda}) (x \text{ was discussed or acted on at the meeting})$   
**Every proposal on the agenda is such that (it was discussed or acted on at the meeting)**  
**Every proposal on the agenda was discussed or acted on at the meeting**

- |  |   |
|--|---|
| $\forall x (Fx \rightarrow (Gx \rightarrow Hx))$<br>$\forall x Gx$ | a: 3<br>a: 5  |
| ⓐ  | Fa<br>(4)   |
| 3 UI<br>4 MPP<br>5 UI<br>6 MPP                                     | Fa → (Ga → Ha)<br>Ga → Ha<br>Ga<br>Ha<br>(6)<br>(7) |
| 7 QED  | Ha<br>2   |
| 2 CP   | Fa → Ha<br>1  |
| 1 UG   | $\forall x (Fx \rightarrow Hx)$                     |

Phi 270 F05 test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Restate 1 using an unrestricted quantifier.*

- Everyone knew the tune.** [Remember to restate your answer to this using an unrestricted quantifier.]  
answer
- Sam heard only tunes that he knew.**  
[Remember to restate your answer in 2 using an unrestricted quantifier.]  
answer
- No one liked everything on the menu.**  
answer

Synthesize an English sentence with the following logical form; that is, produce a sentence that would have the following analysis:

- $(\forall x: Px) \rightarrow Fsx$   
P: [ is a person ]; F: [ fit ]; s: the shoe  
answer

Use derivations to show that the following arguments are valid. You may use any rules.

- $$\frac{\forall x (Fx \wedge Gx)}{\forall x (Gx \wedge Fx)}$$
answer
- $$\frac{\forall x \forall y (Gy \rightarrow Rxy) \quad \forall x (Fx \rightarrow Gx)}{\forall x (Fx \rightarrow \forall y Ryx)}$$
answer

Use a derivation to show that the following argument is not valid and present a counterexample by describing a structure that divides an open gap. (You may describe the structure either by depicting it in a diagram, as answers in the text usually do, or by giving tables.)

- $$\frac{Fa}{\forall x Rxa}$$
answer

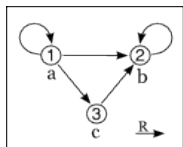
6.

	$\forall x (Fx \rightarrow \forall y Rxy)$	b: 3
	$\forall x Fx$	b: 4
	(a)	
3 UI	Fb $\rightarrow \forall y Rby$	5
4 UI	Fb	(5)
5 MPP	$\forall y Rby$	a: 6
6 UI	Rba	(7)
	●	
7 QED	Rba	2
2 UG	$\forall y Rya$	1
1 UG	$\forall x \forall y Ryx$	

7.

	$\forall x Rax$	a: 2, b: 3, c: 4
	$\forall x Rxb$	a: 5, b: 6, c: 7
	(c)	
2 UI	Raa	
3 UI	Rab	
4 UI	Rac	
5 UI	Rab	
6 UI	Rbb	
7 UI	Rcb	
	$\neg Rcc$	
	○	Raa,Rab,Rac,Rbb,Rcb, $\neg Rcc \Rightarrow \perp$
	⊥	8
8 IP	Rcc	1
1 UG	$\forall x Rxx$	

Counterexample presented by a diagram Counterexample presented by tables



range: 1, 2, 3	a b c	R	1	2	3
	1 2 3	1	T	T	T
		2	F	T	F
		3	F	T	F

Phi 270 F05 test 4 answers

1. **Everyone knew the tune**  
**Everyone is such that (he or she knew the tune)**  
 $(\forall x: \underline{x} \text{ is a person}) \underline{x} \text{ knew the tune}$   
 $(\forall x: Px) Kxt$   
 $\forall x (P \rightarrow Kxt)$

K: [ \_ knew \_ ]; P: [ \_ is a person ]; t: the tune

2. **Sam heard only tunes that he knew**  
**only tunes that Sam knew are such that (Sam heard them)**  
 $(\forall x: \neg x \text{ is a tune that Sam knew}) \neg \underline{\text{Sam heard } x}$   
 $(\forall x: \neg (x \text{ is a tune} \wedge \underline{\text{Sam knew } x})) \neg Hsx$   
 $(\forall x: \neg (Tx \wedge Ksx)) \neg Hsx$

[ \_ heard \_ ]; K: [ \_ knew \_ ]; T: [ \_ is a tune ]; s: Sam

A different but equally plausible interpretation would be to treat tunes as a bounds indicator; this interpretation would be analyzed as  $(\forall x: Tx \wedge \neg Ksx) \neg Hsx$ . This is also the analysis of **Sam heard no tunes he didn't know**.

3. **No one liked everything on the menu**  
**No one is such that (he or she liked everything on the menu)**  
 $(\forall x: \underline{x} \text{ is a person}) \neg x \text{ liked everything on the menu}$   
 $(\forall x: Px) \neg \text{everything on the menu is such that } (x \text{ liked it})$   
 $(\forall x: Px) \neg (\forall y: \underline{y} \text{ is on the menu}) \underline{x} \text{ liked } y$   
 $(\forall x: Px) \neg (\forall y: Oym) Lxy$

L: [ \_ liked \_ ]; O: [ \_ is on \_ ]; P: [ \_ is a person ]; m: the menu

4.  $(\forall x: x \text{ is a person}) \neg \text{the shoe fit } x$   
**No one is such that (the shoe fit him or her)**  
**The shoe fit no one**  
*or*

$(\forall x: x \text{ is a person}) \neg \text{the shoe fit } x$

$(\forall x: x \text{ is a person}) \text{the shoe didn't fit } x$

**Everyone is such that (the shoe didn't fit him or her)**  
**The shoe didn't fit anyone**

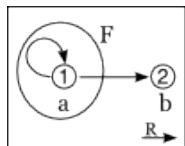
The sentence **The shoe didn't fit everyone** is not the best synthesis since it is likely to be understood as the denial of **The shoe fit everyone**—i.e., as  $\neg (\forall x: Px) Fsx$ .

5.  $\forall x (Fx \wedge Gx)$  a:2
- |       |  |                            |     |
|-------|--|----------------------------|-----|
| 2 UI  |  | Fa $\wedge$ Ga             | 3   |
| 3 Ext |  | Fa                         | (6) |
| 3 Ext |  | Ga                         | (5) |
|       |  | ●                          |     |
| 5 QED |  | Ga                         | 4   |
|       |  | ●                          |     |
| 6 QED |  | Fa                         | 4   |
| 4 Cnj |  | Ga $\wedge$ Fa             | 1   |
| 1 UG  |  | $\forall x (Gx \wedge Fx)$ |     |
6.  $\forall x \forall y (Gy \rightarrow Rxy)$  b:6  
 $\forall x (Fx \rightarrow Gx)$  a:4
- |       |  |  |      |
|-------|--|--|------|
|       |  | Fa   | (5)  |
|       |  | ●  |      |
| 4 UI  |  | Fa $\rightarrow$ Ga                        | 5    |
| 5 MPP |  | Ga   | (8)  |
| 6 UI  |  | $\forall y (Gy \rightarrow Rby)$           | a: 7 |
| 7 UI  |  | Ga $\rightarrow$ Rba                       | 8    |
| 8 MPP |  | Rba  | (9)  |
|       |  | ●  |      |
| 9 QED |  | Rba  | 3    |
| 3 UG  |  | $\forall y Rya$                            | 2    |
| 2 CP  |  | Fa $\rightarrow \forall y Rya$             | 1    |
| 1 UG  |  | $\forall x (Fx \rightarrow \forall y Ryx)$ |      |

7.

	$\forall x (Fx \rightarrow Rax)$	a:1, b:4
	Fa	(2)
1 UI	Fa $\rightarrow$ Raa	2
2 MPP	Raa	
	$\textcircled{b}$	
4 UI	Fb $\rightarrow$ Rab	6
	$\neg$ Rba	
	$\neg$ Fb	
	$\circ$	Fa, Raa, $\neg$ Rba, $\neg$ Fb $\Rightarrow \perp$
	$\perp$	7
7 IP	Fb	6
	Rab	
	$\circ$	Fa, Raa, $\neg$ Rba, Rab $\Rightarrow \perp$
	$\perp$	6
6 RC	$\perp$	5
5 IP	Rba	3
3 UG	$\forall x Rxa$	

Counterexample presented by a diagram



This counterexample divides both gaps; but the specific value for F2 is needed only for the first gap and the specific value for R12 is needed only for the second.

Counterexample presented by tables

range: 1, 2

a	b	$\tau$	F $\tau$	R	1	2
1	2	1	T	1	T	T
2	1	2	F	2	F	F

### Phi 270 F04 test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Restate 2 using an unrestricted quantifier.*

1. **Sam checked every lock**

answer

2. **No one who was in the office answered the call**

[Remember to restate your answer in 2 using an unrestricted quantifier.]

answer

3. **Ralph got the joke if anyone did**

answer

4. **Only bestsellers were on every list**

answer

Use derivations to show that the following arguments are valid. You may use any rules.

5. 
$$\frac{\forall x Fx \quad \forall x \neg Gx}{\forall x (Fx \wedge \neg Gx)}$$

answer

6. 
$$\frac{\forall x (Rxa \rightarrow \forall y Txy)}{\forall x \forall y (Rya \rightarrow Tyx)}$$

answer

Use a derivation to show that the following argument is not valid and present a counterexample by describing a structure that divides an open gap. (You may describe the structure either by depicting it in a diagram, as answers in the text usually do, or by giving tables.)

7. 
$$\frac{\forall x Rax}{\forall x (Rxa \rightarrow Rxx)}$$

answer

### Phi 270 F04 test 4 answers

1. **Sam checked every lock**

**Every lock is such that (Sam checked it)**

( $\forall x$ : x is a lock) Sam checked x

( $\forall x$ : Lx) Csx

C: [ \_ checked \_ ]; L: [ \_ is a lock ]; s: Sam

2. No one who was in the office answered the call  
 No one who was in the office is such that (he or she answered the call)

$(\forall x: x \text{ is a person who was in the office}) \rightarrow \underline{x} \text{ answered the call}$

$(\forall x: \underline{x} \text{ is a person} \wedge \underline{x} \text{ was in the office}) \rightarrow Axc$

$(\forall x: Px \wedge Nxo) \rightarrow Axc$

$\forall x ((Px \wedge Nxo) \rightarrow \neg Axc)$

A: [ \_ answered \_ ]; P: [ \_ is a person ]; N: [ \_ was in \_ ]; c: the call; o: the office

3. Ralph got the joke if anyone did  
 Everyone is such that (Ralph got the joke if he or she did)

$(\forall x: x \text{ is a person}) \text{ Ralph got the joke if } x \text{ did}$

$(\forall x: Px) (\text{Ralph got the joke} \leftarrow \underline{x} \text{ got the joke})$

$(\forall x: Px) (Grj \leftarrow Gxj)$

$(\forall x: Px) (Gxj \rightarrow Grj)$

P: [ \_ is a person ]; G: [ \_ got \_ ]; j: the joke

4. Only bestsellers were on every list  
 Only bestsellers are such that (they were on every list)

$(\forall x: \neg x \text{ is a bestseller}) \rightarrow x \text{ was on every list}$

$(\forall x: \neg Bx) \rightarrow \text{every list is such that } (x \text{ was on it})$

$(\forall x: \neg Bx) \rightarrow (\forall y: y \text{ is a list}) x \text{ was on } y$

$(\forall x: \neg Bx) \rightarrow (\forall y: Ly) Nxy$

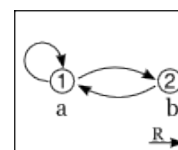
B: [ \_ is a bestseller ]; L: [ \_ is a list ]; N: [ \_ was on \_ ]

5.	$\forall x Fx$ $\forall x \neg Gx$	a: 3 a: 5
3 UI	$\textcircled{a}$ $Fa$ <span style="display: inline-block; width: 10px; height: 10px; background-color: black; border-radius: 50%;"></span>	(4)
4 QED	$Fa$	2
5 UI	$\neg Ga$ <span style="display: inline-block; width: 10px; height: 10px; background-color: black; border-radius: 50%;"></span>	(6)
6 QED	$\neg Ga$	2
2 Cnj	$Fa \wedge \neg Ga$	1
1 UG	$\forall x (Fx \wedge \neg Gx)$	

6.	$\forall x (Rxa \rightarrow \forall y Txy)$ c:4 $\textcircled{b}$ $\textcircled{c}$ $Rca$ $Rca \rightarrow \forall y Tcy$ $\forall y Tcy$ $Tcb$ <span style="display: inline-block; width: 10px; height: 10px; background-color: black; border-radius: 50%;"></span>	c:4 (5) 5 b: 6 (7)
4 UI	$Tcb$	3
5 MPP	$Rca \rightarrow Tcb$	2
6 UI	$\forall y (Rya \rightarrow Tyb)$	1
7 QED	$\forall x \forall y (Rya \rightarrow Tyx)$	

7.	$\forall x Rax$ a:4, b:5 $\textcircled{b}$ $Rba$ $\neg Rbb$ $Raa$ $Rab$ $\circ$ $\perp$	a:4, b:5    Rba, $\neg Rbb$ , Raa, Rab $\not\Rightarrow \perp$ 3
4 UI	$Rbb$	2
5 UI	$Rba \rightarrow Rbb$	1
3 IP	$\forall x (Rxa \rightarrow Rxx)$	

Counterexample presented by a diagram Counterexample presented by tables



range: 1, 2	a b	R	1	2
	1 2	1	T	T
	2	T	T	F

**Phi 270 F03 test 4**

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Restate 2 using an unrestricted quantifier.*

- No one called the new number**  
answer
- Sam asked everyone he could think of** [Remember to restate this one using an unrestricted quantifier.]  
answer
- If any door was opened, the alarm sounded**  
answer
- Only people who'd read everything the author had written were asked to review the book**  
answer

Use derivations to show that the following arguments are valid. You may use any rules.

$$\frac{\forall x (Fx \wedge Gx)}{\forall x Gx}$$

answer

$$\frac{\forall x (Fx \rightarrow Gx) \quad \forall x \forall y (Gy \rightarrow Rxy)}{\forall x \forall y (Fy \rightarrow Rxy)}$$

answer

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides an open gap.

$$\frac{\forall x (Fx \rightarrow Rxa)}{Fa \rightarrow \forall x Rxx}$$

answer

**Phi 270 F03 test 4 answers**

- No one called the new number**  
**No one is such that (he or she called the new number)**  
( $\forall x: x$  is a person)  $\neg$  x called the new number)  
( $\forall x: Px$ )  $\neg$  Cxn  
C: [ \_ called \_ ]; P: [ \_ is a person ]; n: the new number

- Sam asked everyone he could think of everyone Sam could think of is such that (Sam asked him or her)**  
( $\forall x: x$  is a person Sam could think of) Sam asked x  
( $\forall x: x$  is a person  $\wedge$  Sam could think of x) Asx  
( $\forall x: Px \wedge Tsx$ ) Asx  
 $\forall x ((Px \wedge Tsx) \rightarrow Asx)$

A: [ \_ asked \_ ]; P: [ \_ is a person ]; T: [ \_ could think of \_ ]; s: Sam

- If any door was opened, the alarm sounded every door is such that (if it was opened, the alarm sounded)**  
( $\forall x: x$  is a door) if x was opened, the alarm sounded  
( $\forall x: Dx$ ) (x was opened  $\rightarrow$  the alarm sounded)  
( $\forall x: Dx$ ) (Ox  $\rightarrow$  Sa)

D: [ \_ is a door ]; O: [ \_ was opened ]; S: [ \_ sounded ]; a: the alarm

- Only people who'd read everything the author had written were asked to review the book**  
**Only people who'd read everything the author had written are such that (they were asked to review the book)**  
( $\forall x: \neg$  x is a person who'd read everything the author had written)  $\neg$  x was asked to review the book  
( $\forall x: \neg$  (x is a person  $\wedge$  x had read everything the author had written))  $\neg$  Axb  
( $\forall x: \neg$  (x is a person  $\wedge$  everything the author had written is such that (x had read it)))  $\neg$  Axb  
( $\forall x: \neg$  (Px  $\wedge$  ( $\forall y: y$  is a thing the author had written) x had read y))  $\neg$  Axb  
( $\forall x: \neg$  (Px  $\wedge$  ( $\forall y: \text{the author had written } y$ ) Rxy))  $\neg$  Axb  
( $\forall x: \neg$  (Px  $\wedge$  ( $\forall y: \text{Way}$ ) Rxy))  $\neg$  Axb

A: [ \_ was asked to review \_ ]; P: [ \_ is a person ]; R: [ \_ had read \_ ];  
R: [ \_ had written \_ ]; a: the author; b: the book

$$\begin{array}{l|l}
 \forall x (Fx \wedge Gx) & \text{a: 2} \\
 \hline
 2 \text{ UI} & \textcircled{a} Fa \wedge Ga & 3 \\
 3 \text{ Ext} & Fa & \\
 3 \text{ Ext} & Ga & (4) \\
 & \bullet & \\
 4 \text{ QED} & Ga & 1 \\
 \hline
 1 \text{ UG} & \forall x Gx & 
 \end{array}$$

6.

	$\forall x (Fx \rightarrow Gx)$	b:4
	$\forall x \forall y (Gy \rightarrow Rxy)$	a:6
	(a)	
	(b)	
	Fb	(5)
4 UI	$Fb \rightarrow Gb$	5
5 MPP	Gb	(8)
6 UI	$\forall y (Gy \rightarrow Ray)$	b:7
7 UI	$Gb \rightarrow Rab$	8
8 MPP	Rab	(9)
	●	
9 QED	Rab	3
3 CP	$Fb \rightarrow Rab$	2
2 UG	$\forall y (Fy \rightarrow Ray)$	1
1 UG	$\forall x \forall y (Fy \rightarrow Rxy)$	

7.

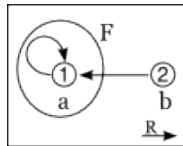
	$\forall x (Fx \rightarrow Rxa)$	a:2, b:5
	Fa	(3)
2 UI	$Fa \rightarrow Raa$	3
3 MPP	Raa	
	(b)	
5 UI	$Fb \rightarrow Rba$	7
	$\neg Rbb$	
	$\neg Fb$	
	○	Fa, Raa, $\neg Rbb, \neg Fb \not\Rightarrow \perp$
	⊥	8
8 IP	Fb	7
	Rba	
	○	Fa, Raa, $\neg Rbb, Rba \not\Rightarrow \perp$
	⊥	7
7 RC	⊥	6
6 IP	Rbb	4
4 UG	$\forall x Rxx$	1
1 CP	$Fa \rightarrow \forall x Rxx$	

Counterexample presented by tables

range: 1, 2	a b	$\tau$   F $\tau$	R   1 2
	1 2	1   T	1   T F
		2   F	2   T F

(This interpretation divides both gaps; the value of F2 is needed only for the 1st and the value of R21 only for the 2nd.)

Counterexample presented by a diagram



### Phi 270 F02 test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

- Only bears performed.  
answer
- If everyone cheered, the elephant bowed. [In this case, restate your answer using an unrestricted quantifier.]  
answer
- No one laughed at any performers except clowns.  
answer

Synthesize an English sentence with the following logical form:

- $(\forall x: Px \wedge Cxt) Ctx$   
C: [ \_ called \_ ]; P: [ \_ is a person ]; t: Tom  
answer

Use derivations to establish the validity of the following arguments. You may use attachment rules.

- $$\frac{\forall x Fx \quad \forall x \neg (Fx \wedge Gx)}{\forall x \neg Gx}$$
answer
- $$\frac{\forall x \forall y (Fy \rightarrow Rxy)}{\forall x (Fx \rightarrow \forall y Ryx)}$$
answer

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

- $$\frac{\forall x Rax \quad \forall x (Rbx \rightarrow \neg Rxa)}{\forall x \neg Rbx}$$
answer



Phi 270 F02 test 4 answers

1. Only bears performed

$(\forall x: \neg x \text{ is a bear}) \neg x \text{ performed}$

$(\forall x: \neg Bx) \neg Px$

B: [ \_ is a bear ]; P: [ \_ performed ]

2. If everyone cheered, the elephant bowed

everyone cheered  $\rightarrow$  the elephant bowed

$(\forall x: x \text{ is a person}) x \text{ cheered} \rightarrow \text{the elephant bowed}$

$(\forall x: Px) Cx \rightarrow Be$

$\forall x (Px \rightarrow Cx) \rightarrow Be$

B: x bowed; C: x cheered; P: x is a person; e: the elephant

Incorrect:

$(\forall x: Px) (Cx \rightarrow Be)$  or:  $\forall x (Px \rightarrow (Cx \rightarrow Be))$

these say: If anyone cheered, the elephant bowed

3. No one laughed at any performers except clowns

all performers except clowns are such that (no one laughed at them)

$(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown})$  no one laughed at x

$(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown}) (\forall y: y \text{ is a person}) \neg y$  laughed at x

$(\forall x: Fx \wedge \neg Cx) (\forall y: Py) \neg Lyx$

C: [ \_ is a clown ]; F: [ \_ is a performer ]; P: [ \_ is a person ]; L: [ \_

laughed at \_ ]

Incorrect:

$(\forall y: Py) \neg (\forall x: Fx \wedge \neg Cx) Lyx$

says: No one laughed at all performers who weren't clowns

4.  $(\forall x: x \text{ is a person} \wedge x \text{ called Tom})$  Tom called x

$(\forall x: x \text{ is a person who called Tom})$  Tom called x

everyone who called Tom is such that (Tom called him or her)

Tom called everyone who called him

5.

$\forall x Fx$  a:2  
 $\forall x \neg (Fx \wedge Gx)$  a:3

		$\textcircled{a}$		
2 UI		Fa	(4)	
3 UI		$\neg (Fa \wedge Ga)$	4	
4 MPT		$\neg Ga$	(5)	
		●		
5 QED		$\neg Ga$	1	
1 UG		$\forall x \neg Gx$		

6.

$\forall x \forall y (Fy \rightarrow Rxy)$  b:4

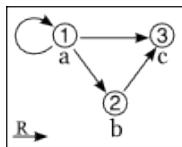
		$\textcircled{a}$		
		Fa	(6)	
		$\textcircled{b}$		
4 UI		$\forall y (Fy \rightarrow Rby)$	a:5	
5 UI		Fa $\rightarrow$ Rba	6	
6 MPP		Rba	(7)	
		●		
7 QED		Rba	3	
3 UG		$\forall y Rya$	2	
2 CP		Fa $\rightarrow \forall y Rya$	1	
1 UG		$\forall x (Fx \rightarrow \forall y Ryx)$		

7.

	$\forall x Rax$	a:3,b:4,c:5
	$\forall x (Rbx \rightarrow \neg Rxa)$	c:6,a:8,b:10
	$\textcircled{c}$	
	Rbc	(7)
	Raa	(9)
3 UI	Rab	
4 UI	Rac	
5 UI	Rbc $\rightarrow \neg Rca$	7
6 UI	$\neg Rca$	
7 MPP	Rba $\rightarrow \neg Raa$	9
8 UI	$\neg Rba$	
9 MTT	Rbb $\rightarrow \neg Rba$	11
10 UI		
	$\neg Rbb$	
	$\textcircled{O}$	Rbc,Raa,Rab,Rac, $\neg Rca$ , $\neg Rba$ , $\neg Rbb \not\Rightarrow \perp$
	$\perp$	12
12 IP	Rbb	11
	$\neg Rba$	
	$\textcircled{O}$	Rbc,Raa,Rab,Rac, $\neg Rca$ , $\neg Rba \not\Rightarrow \perp$
	$\perp$	11
11 RC	$\perp$	2
2 RAA	$\neg Rbc$	1
1 UG	$\forall x \neg Rbx$	

Counterexample presented by tables      Counterexample presented by a diagram

range: 1, 2, 3	a b c	R	1	2	3
	1 2 3	1	T	T	T
		2	F	F	T
		3	F	F	F



Grayed values are not required to divide either gap;  
the value for R22 is not required to divide the 2nd gap

### Phi 270 F00 test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

- Only necessary projects were funded.** [Different interpretations of the scope of *only* are possible here; any of them will do.]  
answer
- Tom can solve the puzzle if anyone can.** [In this case, restate your answer using an unrestricted quantifier.]  
answer
- No one received every vote**  
answer

Use derivations to establish the validity of the following arguments. You may use attachment rules. English interpretations are suggested but remember that they play no role in derivations, and don't hesitate to ignore them if they don't help you think about the derivations.

- $$\frac{\forall x (Dx \rightarrow Mx) \quad \forall x (\neg Ax \rightarrow \neg Mx)}{\forall x (Dx \rightarrow Ax)}$$

answer

A: [ \_ is an animal ]; D: [ \_ is dog ]; M: [ \_ is a mammal ]

- $$\frac{\forall x \forall y ((Py \wedge Byx) \rightarrow Dyx)}{\forall x (Px \rightarrow \forall y (Bxy \rightarrow Dxy))}$$

answer

**Everyone who has built anything is proud of it / Everyone is proud of everything he or she has built**

- Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

$$\frac{\forall x (Rxx \rightarrow \neg Fx) \quad \forall x Rxc}{\forall x \forall y (Fy \rightarrow \neg Rxy)}$$

answer

Phi 270 F00 test 4 answers

1. Only necessary projects were funded

$(\forall x: \neg x \text{ was a necessary project}) \rightarrow x \text{ was funded}$

$(\forall x: \neg (x \text{ was a project} \wedge x \text{ was necessary})) \rightarrow x \text{ was funded}$

$(\forall x: \neg (Px \wedge Nx)) \rightarrow Fx$

or:  $(\forall x: Px \wedge \neg Nx) \rightarrow Fx$ —i.e., No unnecessary projects were funded;

or:  $(\forall x: Nx \wedge \neg Px) \rightarrow Fx$ —i.e., Among the necessities only projects were funded

F: [ \_ was funded]; N: [ \_ was necessary]; P: [ \_ was a project]

2. Tom can solve the puzzle if anyone can

$(\forall x: x \text{ is a person}) \text{ Tom can solve the puzzle if } x \text{ can}$

$(\forall x: Px) (\text{Tom can solve the puzzle} \leftarrow x \text{ can solve the puzzle})$

$(\forall x: Px) (S \text{ Tom the puzzle} \leftarrow S x \text{ the puzzle})$

$(\forall x: Px) (Stp \leftarrow Sxp) [or: (\forall x: Px) (Sxp \rightarrow Stp)]$

$\forall x (Px \rightarrow (Stp \leftarrow Sxp)) [or: \forall x (Px \rightarrow (Sxp \rightarrow Stp))]$

P: [ \_ is a person]; S: [ \_ can solve \_ ]; p: the puzzle; t: Tom

3. No one received every vote

$(\forall x: x \text{ is a person}) \rightarrow x \text{ received every vote}$

$(\forall x: Px) \rightarrow x \text{ received every vote}$

$(\forall x: Px) \rightarrow (\forall y: y \text{ is a vote}) x \text{ received } y$

$(\forall x: Px) \rightarrow (\forall y: Vy) Rxy$

P: [ \_ is a person]; R: [ \_ received \_ ]; V: [ \_ is a vote]

Incorrect answers:

$(\forall x: Px) (\forall y: Vy) \rightarrow Rxy$  says No one received any vote

$\neg (\forall x: Px) (\forall y: Vy) Rxy$  says Not everyone received every vote

$(\forall y: Vy) \rightarrow (\forall x: Px) Rxy$  says No vote is such that everyone received it

4.

	$\forall x (Dx \rightarrow Mx)$	a:3
	$\forall x (\neg Ax \rightarrow \neg Mx)$	a:5
	Ⓐ	
	Da	(4)
3 UI	Da → Ma	4
4 MPP	Ma	(6)
5 UI	¬Aa → ¬Ma	6
6 MTT	Aa	(7)
	●	
7 QED	Aa	2
2 CP	Da → Aa	1
1 UG	$\forall x (Dx \rightarrow Ax)$	

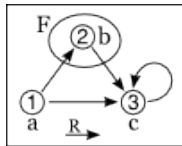
5.

	$\forall x \forall y ((Py \wedge Byx) \rightarrow Dyx)$	b:5
	Ⓐ	
	Pa	(9)
	Ⓑ	
	Bab	(10)
5 UI	$\forall y ((Py \wedge Byb) \rightarrow Dyb)$	a:6
6 UI	$(Pa \wedge Bab) \rightarrow Dab$	8
	¬Dab	(8)
8 MTT	¬(Pa ∧ Bab)	9
9 MPT	¬Bab	(10)
	⊥	7
10 Nc	Dab	4
7 IP	Bab → Dab	3
4 CP	$\forall y (Bay \rightarrow Day)$	2
3 UG	$Pa \rightarrow \forall y (Bay \rightarrow Day)$	1
2 CP	$\forall x (Px \rightarrow \forall y (Bxy \rightarrow Dxy))$	
1 UG		

[This can be done without the *reductio* argument begun at stage 7 by using Adj to derive Pa ∧ Bab in order to exploit (Pa ∧ Bab) → Dab for a]

6.

	$\forall x (Rxx \rightarrow \neg Fx)$	b:4, c:9, a:11
	$\forall x Rxc$	a:6, b:7, c:8
	(a)	
	(b)	
	Fb	(5)
4 UI	$Rbb \rightarrow \neg Fb$	5
5 MTT	$\neg Rbb$	
6 UI	Rac	
7 UI	Rbc	
8 UI	Rcc	(10)
9 UI	$Rcc \rightarrow \neg Fc$	10
10 MPP	$\neg Fc$	
11 UI	$Raa \rightarrow \neg Fa$	13
	Rab	
	$\neg Raa$	
	○	$Fb, \neg Rbb, Rac, Rbc, Rcc, \neg Fc, Rab, \neg Raa \not\Rightarrow \perp$
	⊥	14
14 IP	Raa	13
	$\neg Fa$	
	○	$Fb, \neg Rbb, Rac, Rbc, Rcc, \neg Fc, Rab, \neg Fa \not\Rightarrow \perp$
	⊥	13
13 RC	⊥	12
12 RAA	$\neg Rab$	3
3 CP	$Fb \rightarrow \neg Rab$	2
2 UG	$\forall y (Fy \rightarrow \neg Ray)$	1
1 UG	$\forall x \forall y (Fy \rightarrow \neg Rxy)$	



divides both open gaps

Phi 270 F99 test 4

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

1. Sam invited every vertebrate to the party, but only people accepted his invitation

answer

2. Tom didn't send anything to the printer

answer

3. No game that every child liked was complete

answer

Synthesize an English sentence whose analysis would yield the following form.

4.  $(\forall x: Px) (\forall y: Ry \wedge Txy) Sy$

P: [ \_ is a person ]; R: [ \_ is a room ]; S: [ \_ was reserved ]; T: [ \_ thought of \_ ]

answer

Use derivations to establish the validity of the following arguments. You may use attachment rules.

5.  $\forall x (Fx \rightarrow Gx)$

$\forall x Fx \rightarrow \forall x Gx$

answer

6.  $\forall x \forall y (Fyx \rightarrow \neg Py)$

$\forall x (Px \rightarrow \forall y \neg Fxy)$

answer

7. Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

$\forall x \forall y (Fy \rightarrow \neg Rxy)$

$\forall x Rxx$

$\forall x \forall y \neg Rxy$

answer

Phi 270 F99 test 4 answers

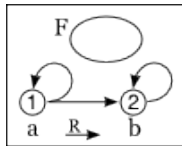
- Sam invited every vertebrate to the party, but only people accepted his invitation  
 Sam invited every vertebrate to the party  $\wedge$  only people accepted Sam's invitation  
 every vertebrate is such that (Sam invited it to the party)  $\wedge$  only people are such that (they accepted Sam's invitation)  
 $(\forall x: \underline{x}$  is a vertebrate) Sam invited  $\underline{x}$  to the party  $\wedge$   $(\forall x: \neg \underline{x}$  is a person)  $\neg \underline{x}$  accepted Sam's invitation  
 $(\forall x: \forall x) \text{Isxp} \wedge (\forall x: \neg Px) \neg Ax(\text{Sam's invitation})$   
 $(\forall x: \forall x) \text{Isxp} \wedge (\forall x: \neg Px) \neg Ax(\text{is})$   
 A: [ accepted ]; I: [ invited to ]; P: [ is a person]; V: [ is a vertebrate]; i: [ 's invitation]; p: the party; s: Sam
- Tom didn't send anything to the printer  
 everything is such that (Tom didn't send it to the printer)  
 $\forall x$  Tom didn't send x to the printer  
 $\forall x \neg$  Tom sent  $\underline{x}$  to the printer  
 $\forall x \neg \text{Stxp}$   
 S: [ sent to ]; p: the printer; t: Tom
- No game that every child liked was complete  
 No game that every child liked is such that (it was complete)  
 $(\forall x: x$  was a game that every child liked)  $\neg x$  was complete  
 $(\forall x: x$  was a game  $\wedge$  every child liked x)  $\neg Cx$   
 $(\forall x: x$  was a game  $\wedge$  every child is such that (he or she liked x))  $\neg Cx$   
 $(\forall x: Gx \wedge (\forall y: y$  was a child) y liked x)  $\neg Cx$   
 $(\forall x: Gx \wedge (\forall y: Dy) Lyx) \neg Cx$   
 C: [ was complete]; D: [ was a child]; G: [ was a game]; L: [ liked ]
- $(\forall x: x$  is a person)  $(\forall y: y$  is a room  $\wedge$  x thought of y) y was reserved  
 $(\forall x: x$  is a person)  $(\forall y: y$  is a room x thought of) y was reserved  
 $(\forall x: x$  is a person) every room x thought of was such that (it was reserved)  
 $(\forall x: x$  is a person) every room x thought of was reserved  
 everyone is such that (every room he or she thought of was reserved)  
 every room anyone thought of was reserved

- |       |   |     |
|-------|---|-----|
|       | $\forall x (Fx \rightarrow Gx)$         | a:3 |
|       | $\forall x Fx$                          | a:4 |
| 3 UI  | $Fa \rightarrow Ga$                     | 5   |
| 4 UI  | $Fa$                                    | (5) |
| 5 MPP | $Ga$                                    | (6) |
|       | $\perp$                                 |     |
| 6 QED | $Ga$                                    | 2   |
| 2 UG  | $\forall x Gx$                          | 1   |
| 1 CP  | $\forall x Fx \rightarrow \forall x Gx$ |     |
- |       |   |     |
|-------|---|-----|
|       | $\forall x \forall y (Fyx \rightarrow \neg Py)$ | b:5 |
|       | $Pa$  | (8) |
|       | $Fab$   | (7) |
| 5 UI  | $\forall y (Fyb \rightarrow \neg Py)$           | a:6 |
| 6 UI  | $Fab \rightarrow \neg Pa$                       | 7   |
| 7 MPP | $\neg Pa$                                       | (8) |
|       | $\perp$   |     |
| 8 Nc  | $\neg Fab$                                      | 4   |
| 4 RAA | $\neg Fab$                                      | 3   |
| 3 UG  | $\forall y \neg Fay$                            | 2   |
| 2 CP  | $Pa \rightarrow \forall y \neg Fay$             | 1   |
| 1 UG  | $\forall x (Px \rightarrow \forall y \neg Fxy)$ |     |

7.

	$\forall x \forall y (Fy \rightarrow \neg Rxy)$	a:4,b:5
	$\forall x Rxx$	a:6,b:7
	(a) $\neg Fa$	
	(b) $Rab$	(11)
4 UI	$\forall y (Fy \rightarrow \neg Ray)$	a:8, b:9
5 UI	$\forall y (Fy \rightarrow \neg Rby)$	a:12, b:13
6 UI	$Raa$	(10)
7 UI	$Rbb$	(14)
8 UI	$Fa \rightarrow \neg Raa$	10
9 UI	$Fb \rightarrow \neg Rab$	11
10 MTT	$\neg Fa$	
11 MTT	$\neg Fb$	
12 UI	$Fa \rightarrow \neg Rba$	15
13 UI	$Fb \rightarrow \neg Rbb$	14
14 MTT	$\neg Fb$	
	$\neg Fa$	
	$\circ$	$\neg Fa, \neg Fb, Rab, Raa, Rbb \not\Rightarrow \perp$
	$\perp$	16
16 IP	$Fa$	15
	$\neg Rba$	
	$\circ$	$\neg Fa, \neg Fb, Rab, Raa, Rbb, \neg Rba \not\Rightarrow \perp$
	$\perp$	15
15 RC	$\perp$	3
3 RAA	$\neg Rab$	2
2 UI	$\forall y \neg Ray$	1
1 UI	$\forall x \forall y \neg Rxy$	

The structure below divides both gaps



### Phi 270 F98 test 4

(questions 1-2 are from quiz 4 and 3-8 are from quiz 5 out of 6 quizzes—these two quizzes addressed the part of the course your test is designed to cover)

1. Identify individual terms and quantifier phrases in the following sentence and indicate links between pronouns and their antecedents. (You can do this by marking up an English sentence; you are *not* being asked to provide a symbolic analysis.)

Sam ordered a book, but instead of it he received a book he didn't want.

answer

2. Analyze the following generalization in as much detail as possible. Provide a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

No one saw the book that was lying on the table.

answer

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

3. No one except numismatists understood the joke

answer

4. The movie delighted all boys and girls

answer

5. If anyone relayed the message to everyone, then no one understood every part of it

answer

Use derivations to establish the validity of the following arguments. You may use attachment rules.

6.  $\forall x (Fx \vee Gx)$

$$\frac{\forall x \neg Gx}{\forall x Fx}$$

$$\forall x Fx$$

answer

7.  $\forall x (Fx \rightarrow \forall y (Pxy \rightarrow Rxy))$

$$\frac{\forall y \forall x ((Fx \wedge Pxy) \rightarrow Rxy)}{\forall x \forall y (Pxy \rightarrow Rxy)}$$

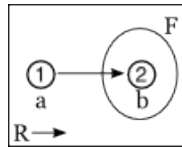
answer



8.

	$\forall x (Fx \rightarrow \neg Rxx)$	b:5, a:7
	(a) (b)	
	Fb	(6)
	Rab	
5 UI	Fb $\rightarrow$ $\neg$ Rbb	6
6 MPP	$\neg$ Rbb	
7 UI	Fa $\rightarrow$ $\neg$ Raa	8
	$\neg$ Fa	
	$\circ$	Fb, Rab, $\neg$ Rbb, $\neg$ Fa $\not\Rightarrow \perp$
	$\perp$	9
9 IP	Fa	8
	$\neg$ Raa	
	$\circ$	Fb, Rab, $\neg$ Rbb, $\neg$ Raa $\not\Rightarrow \perp$
	$\perp$	8
8 RC	$\perp$	4
4 RAA	$\neg$ Rab	3
3 CP	Fb $\rightarrow$ $\neg$ Rab	2
2 UG	$\forall y (Fy \rightarrow \neg Ray)$	1
1 UG	$\forall x \forall y (Fy \rightarrow \neg Rxy)$	

This structure divides both gaps:



**Phi 270 F97 test 4**

(questions 1-3 are from quiz 4 and 4-9 are from quiz 5 out of 6 quizzes—these two quizzes addressed the part of the course your test is designed to cover)

1. Identify individual terms and quantifier phrases in the following sentence and indicate links between pronouns and their antecedents. (You can do this by marking up an English sentence; you are *not* being asked to provide a symbolic analysis.)

Everyone who Carol lent the book to spoke to her at length about it.

answer

Analyze the following generalizations in as much detail as possible. Provide a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer *and restate the result using an unrestricted quantifier.*

2. Bob called no one.  
answer
3. Among contestants, only professionals were finalists.  
answer

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer.

4. Bob doesn't own any map showing Dafter.  
answer
5. Nothing anyone said bothered Dave.  
answer

Use derivations to establish the validity of the following arguments. You may use attachment rules.

6.  $\forall x (Fx \wedge Gx)$   
 $\forall x Fx$   
answer
7.  $\forall x (Rxa \rightarrow \forall y Rxy)$   
 $\forall x (\forall y Rxy \rightarrow Rxb)$   
answer



8. Use a derivation to show that the following argument is not valid and describe a structure dividing one of the derivation's open gaps. (You will *not* need the rules UG+ and ST of 7.8 that were designed to avoid unending derivations.)

$$\frac{\forall x (Fx \rightarrow Rax)}{\forall x (Fx \rightarrow Rxa)}$$

$$\forall x (Fx \rightarrow Rxa)$$

answer

You will receive credit for *one* of the following (but you may attempt both):

- 9a. Synthesize an English sentence whose analysis would yield the following form.

$$(\forall x: Dx) (Okx \rightarrow (\forall y: Dy) Oky)$$

D: [ is a door ]; O: [ opens ]; k: the key

answer

- 9b. Use derivations to establish the validity of the following argument. You may use attachment rules.

$$\forall x \forall y (Rxy \rightarrow \neg Fy)$$

$$\frac{\forall x (Fx \rightarrow Rxx)}{\forall x \neg Fx}$$

$$\forall x \neg Fx$$

answer

### Phi 270 F97 test 4 answers

1. Everyone who Carol lent the book to spoke to her at length about it
- Q            T            T

2. Bob called no one  
no one is such that (Bob called him or her)

$$(\forall x: \underline{x} \text{ is an person}) \neg \underline{\text{Bob}} \text{ called } \underline{x}$$

$$(\forall x: Px) \neg Cbx$$

$$\forall x (Px \rightarrow \neg Cbx)$$

C: [ called ]; P: [ is person ]; b: Bob

3. Among contestants, only professionals were finalists  
Among contestants, only professionals are such that (they were finalists)

$$(\forall x: x \text{ was a contestant} \wedge \neg x \text{ was a professional}) \neg x \text{ was a finalist}$$

$$(\forall x: Cx \wedge \neg Px) \neg Fx$$

$$\forall x ((Cx \wedge \neg Px) \rightarrow \neg Fx)$$

C: [ was a contestant ]; F: [ was a finalist ]; P: [ was a professional ]

4. Bob doesn't own any map showing Dafter  
every map showing Dafter is such that (Bob doesn't own it)

$$(\forall x: x \text{ is a map showing Dafter}) \neg \underline{\text{Bob}} \text{ owns } \underline{x}$$

$$(\forall x: \underline{x} \text{ is a map} \wedge \underline{x} \text{ shows Dafter}) \neg \text{Obx}$$

$$(\forall x: Mx \wedge Sxd) \neg \text{Obx}$$

M: [ is a map ]; O: [ owns ]; S: [ shows ]; b: Bob; d: Dafter

5. Nothing anyone said bothered Dave  
everyone is such that (nothing he or she said bothered Dave)

$$(\forall x: x \text{ is a person}) \text{nothing } x \text{ said bothered Dave}$$

$$(\forall x: Px) \text{nothing } x \text{ said is such that (it bothered Dave)}$$

$$(\forall x: Px) (\forall y: y \text{ is a thing } x \text{ said}) \neg \underline{y} \text{ bothered } \underline{\text{Dave}}$$

$$(\forall x: Px) (\forall y: x \text{ said } y) \neg \text{Byd}$$

$$(\forall x: Px) (\forall y: Sxy) \neg \text{Byd}$$

B: [ bothered ]; P: [ is a person ]; S: [ said ]; d: Dave

- 6.
- |       |   |         |     |
|-------|---|---------|-----|
| 2 UI  | ⓐ | Fa ∧ Ga | 3   |
| 3 Ext |   | Fa      |     |
| 3 Ext |   | Ga      | (4) |
| 4 QED |   | Fa      | 1   |
| 1 UG  |   | ∀x Fx   |     |

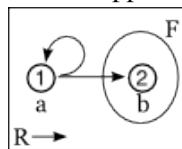
- 7.
- |       |   |                   |     |
|-------|---|-------------------|-----|
| 3 UI  | ⓐ | ∀y Rcy            | a:3 |
| 4 QED |   | Rcb               | (4) |
| 2 CP  |   | ∀y Rcy → Rcb      | 1   |
| 1 UG  |   | ∀x (∀y Rxy → Rxb) |     |
- |       |   |                   |     |
|-------|---|-------------------|-----|
| 3 UI  | ⓐ | Rca               | (5) |
| 4 UI  |   | Rca → ∀y Ryc      | 5   |
| 5 MPP |   | ∀y Ryc            | b:6 |
| 6 UI  |   | Rbc               | (7) |
| 7 QED |   | Rbc               | 2   |
| 2 CP  |   | ∀y Rcy → Rbc      | 1   |
| 1 UG  |   | ∀x (∀y Rxy → Rbx) |     |

[The first premise is never used in the derivation for this question (shown at the left). The fact that it was not needed was a slip on my part in making up the question; at the right is the sort of example I probably had in mind.]

8.

	$\forall x (Fx \rightarrow Rax)$	b:3, a:5
	⊖	
	Fb	(4)
3 UI	Fb $\rightarrow$ Rab	4
4 MPP	Rab	
5 UI	Fa $\rightarrow$ Raa	7
	$\neg$ Rba	
	$\neg$ Fa	
	○	Fb, Rab, $\neg$ Rba, $\neg$ Fa $\nRightarrow \perp$
	⊥	8
8 IP	Fa	7
	Raa	
	○	Fb, Rab, $\neg$ Rba, Raa $\nRightarrow \perp$
	⊥	7
7 RC	⊥	6
6 IP	Rba	2
2 CP	Fb $\rightarrow$ Rba	1
1 UG	$\forall x (Fx \rightarrow Rxa)$	

The structure below divides both gaps; it will divide only the first gap if the arrow from 1 to itself is dropped.



9b.

	$\forall x \forall y (Rxy \rightarrow \neg Fy)$	a:2
	$\forall x (Fx \rightarrow Rxx)$	a:4
	⊖	
2 UI	$\forall y (Ray \rightarrow \neg Fy)$	a:6
	Fa	(5), (8)
4 UI	Fa $\rightarrow$ Raa	5
5 MPP	Raa	(7)
6 UI	Raa $\rightarrow \neg$ Fa	7
7 MPP	$\neg$ Fa	(8)
	●	
8 Nc	⊥	3
3 RAA	$\neg$ Fa	1
1 UG	$\forall x \neg Fx$	

- 9a.  $(\forall x: x \text{ is a door})$  (the key opens  $x \rightarrow (\forall y: y \text{ is a door})$  the key opens  $y$ )  
 $(\forall x: x \text{ is a door})$  (the key opens  $x \rightarrow$  every door is such that (the key opens it))  
 $(\forall x: x \text{ is a door})$  (the key opens  $x \rightarrow$  the key opens every door )  
 $(\forall x: x \text{ is a door})$  if the key opens  $x$ , then it opens every door  
every door is such that (if the key opens it, then it opens every door)  
If the key opens any door, then it opens every door



5. Everyone who worked on any part of the project was honored  
 Every part of the project is such that (everyone who worked on it was honored)

( $\forall x: x$  is a part of the project) everyone who worked on  $x$  was honored

( $\forall x: Rxj$ ) ( $\forall y: y$  is a person who worked on  $x$ )  $y$  was honored

( $\forall x: Rxj$ ) ( $\forall y: y$  is a person  $\wedge y$  worked on  $x$ )  $H_y$

( $\forall x: Rxj$ ) ( $\forall y: Py \wedge Wyx$ )  $H_y$

H: [ \_ was honored]; P: [ \_ is a person]; R: [ \_ is a part of \_ ]; W: [ \_ worked on \_ ]; j: the project

6. ( $\forall x: x$  is a person)  $\neg \forall y x$  ate  $y$

( $\forall x: x$  is a person)  $\neg x$  ate everything

No one is such that (he or she ate everything)

No one ate everything

7.  $\forall x Fx$  a:2  
 $\forall x Gx$  a:3

2 UI  $Fa$  (5)  
 3 UI  $Ga$  (6)

5 QED  $\bullet$   
 $Fa$  4

6 QED  $\bullet$   
 $Ga$  4

4 Cnj  $Fa \wedge Ga$  1

1 UG  $\forall x (Fx \wedge Gx)$  1

8.  $\forall x (Fx \rightarrow Rxa)$  c:4  
 $\forall x (Rxa \rightarrow \forall y Ryx)$  c:6

4 UI  $Fc$  (5)  
 5 MPP  $Fc \rightarrow Rca$  5  
 6 UI  $Rca$  (7)  
 7 MPP  $Rca \rightarrow \forall y Ryc$  7  
 8 UI  $\forall y Ryc$  b:8  
 $Rbc$  (9)

9 QED  $\bullet$   
 $Rbc$  3

3 CP  $Fc \rightarrow Rbc$  2

2 UG  $\forall y (Fy \rightarrow Rby)$  1

1 UG  $\forall x \forall y (Fy \rightarrow Rxy)$

9.  $\forall x Rxx$  a:1,b:2,c:5

1 UI  $Raa$   
 2 UI  $Rbb$

5 UI  $Rab$   
 $\odot$   $Rcc$

$\neg Rca$   
 $\circ$   
 $\perp$  6

6 IP  $Rca$  4

4 UG  $\forall x Rxa$  3

3 CP  $Rab \rightarrow \forall x Rxa$

$Raa, Rab, Rbb, Rcc, \neg Rca \not\Rightarrow \perp$

