Overview

Basic system

Exploitation and planning rules		
sentence	*as a resource*	*as a goal*
atomic sentence	*none*	IP
negation $\neg \varphi$	CR (if φ not atomic & goal is ⊥)	RAA
conjunction $\varphi \wedge \psi$	Ext	Cnj
disjunction $\varphi \vee \psi$	PC	PE
conditional $\varphi \rightarrow \psi$	RC (if goal is ⊥)	CP
universal $\forall x\ \theta x$	UI	UG
existential $\exists x\ \theta x$	PCh	NcP

Rules for closing gaps			
when to close			*rule*
co-aliases	*resources*	*goal*	
	φ	φ	QED
	φ and $\neg \varphi$	$\bot$	Nc
	any	$\top$	ENV
	$\bot$	*any*	EFQ
τ—υ	*any*	$\tau = \upsilon$	EC
τ—υ	$\neg \tau = \upsilon$	$\bot$	DC
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$	$P\upsilon_1 \ldots \upsilon_n$	QED=
τ_1—υ_1, …, τ_n—υ_n	$P\tau_1 \ldots \tau_n$ $\neg P\upsilon_1 \ldots \upsilon_n$	$\bot$	Nc=

Detachment rules (optional)		
required resources		*rule*
main	*auxiliary*	
$\neg(\varphi \wedge \psi)$	φ or ψ	MPT
$\varphi \vee \psi$	$\neg^{\pm} \varphi$ or $\neg^{\pm} \psi$	MTP
$\varphi \rightarrow \psi$	φ	MPP
	$\neg^{\pm} \psi$	MTT

In addition, if the conditions for applying a rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "="; QED= and Nc= are examples of this.

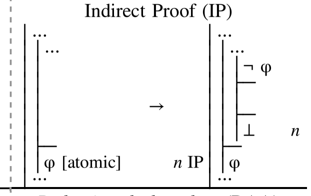
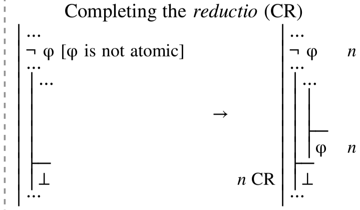
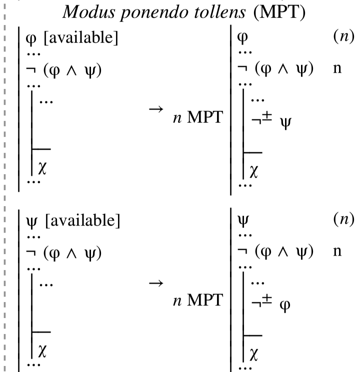
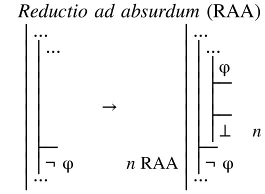
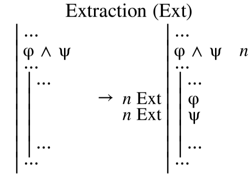
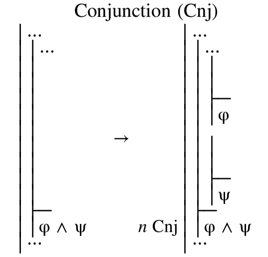
Additional rules

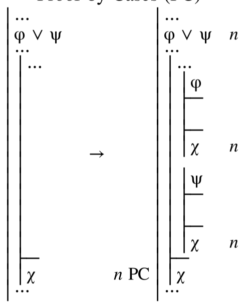
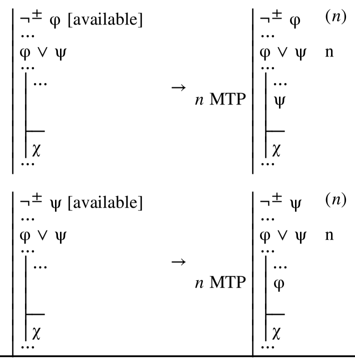
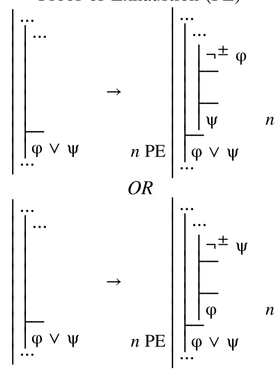
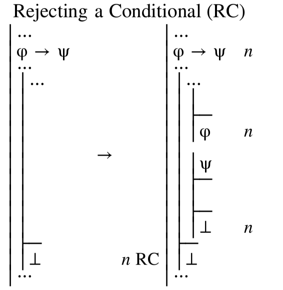
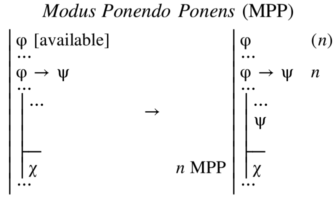
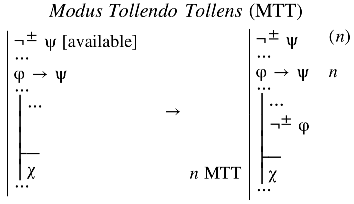
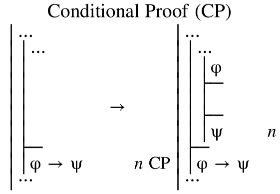
Attachment rules	
added resource	*rule*
$\varphi \wedge \psi$	Adj
$\neg(\varphi \wedge \psi)$	Wk
$\varphi \vee \psi$	
$\varphi \rightarrow \psi$	
$\tau = \upsilon$	CE
$\theta\upsilon_1 \ldots \upsilon_n$	Cng
$\exists x\ \theta x$	EG

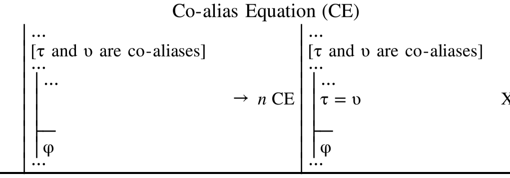
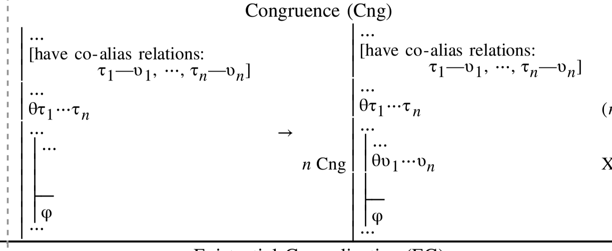
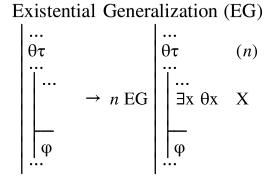
Rule for lemmas	
prerequisite	*rule*
the goal is ⊥	LFR

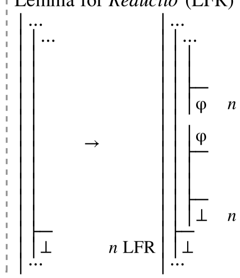
Derivation rules

Basic system

Rules for developing gaps		
logical form	*as resource*	*as goal*
atomic sentence	*no rule*	Indirect Proof (IP) 
negation $\neg \varphi$	Completing the *reductio* (CR)  *Modus ponendo tollens* (MPT) 	*Reductio ad absurdum* (RAA) 
conjunction $\varphi \wedge \psi$	Extraction (Ext) 	Conjunction (Cnj) 

Rules for developing gaps		
logical form	*as resource*	*as goal*
disjunction $\varphi \lor \psi$	Proof by Cases (PC)  *Modus Tollendo Ponens* (MTP) 	Proof of Exhaustion (PE) 
conditional $\varphi \to \psi$	Rejecting a Conditional (RC)  *Modus Ponendo Ponens* (MPP)  *Modus Tollendo Tollens* (MTT) 	Conditional Proof (CP) 

Attachment rules		
what is required	*added resource*	*rule*
τ and υ are co-aliases	$\tau = \upsilon$	Co-alias Equation (CE) 
have co-alias relations τ_1—υ_1, …, τ_n—υ_n and $\theta\tau_1\cdots\tau_n$ is available	$\theta\upsilon_1\cdots\upsilon_n$	Congruence (Cng) 
$\theta\tau$ is available	$\exists x\ \theta x$	Existential Generalization (EG) 

Rule for lemmas	
prerequisite	*rule*
the goal is $\bot$	Lemma for *Reductio* (LFR) 

Additional rules (not guaranteed to be progressive)

Attachment rules		
what is required	*added resource*	*rule*
φ and ψ are both available	φ ∧ ψ	Adjunction (Adj) … φ [available] … ψ [available] … | … | χ … → *n* Adj … φ (n) … ψ (n) … | … φ ∧ ψ X | χ …
¬± φ or ¬± ψ is available	¬ (φ ∧ ψ)	Weakening (Wk) … ¬± φ [available] … | … | χ … → *n* Wk … ¬± φ (*n*) … | … ¬ (φ ∧ ψ) X | χ … … ¬± ψ [available] … | … | χ … → *n* Wk … ¬± ψ (*n*) … | … ¬ (φ ∧ ψ) X | χ …
φ or ψ is available	φ ∨ ψ	… φ [available] … | … | χ … → *n* Wk … φ (*n*) … | … φ ∨ ψ X | χ … … ψ [available] … | … | χ … → *n* Wk … ψ (*n*) … | … φ ∨ ψ X | χ …
¬± φ or ψ is available	φ → ψ	¬± φ [available] … | … | χ … → *n* Wk ¬± φ (*n*) … | … φ → ψ X | χ … ψ [available] … | … | χ … → *n* Wk ψ (*n*) … | … φ → ψ X | χ …

Rules for developing gaps		
logical form	*as resource*	*as goal*
universal ∀x θx	Universal Instantiation (UI) … ∀x θx … | … | φ … → *n* UI … ∀x θx τ:*n* … | … θτ | φ …	Universal Generalization (UG) … | … | ∀x θx … → *n* UG … | … ⓐ | θa *n* | ∀x θx …
existential ∃x θx	Proof by Choice (PCh) … ∃x θx … | … | φ … → *n* PCh … ∃x θx *n* … | … ⓐ | θa | φ *n* | φ …	Non-constructive Proof (NcP) … | … | ∃x θx … → *n* NcP … | … | ∀x ¬± θx | ⊥ *n* | ∃x θx …

The parameter a used in UG and PCh should be new to the derivation; that is, it should appear only to the right of the scope line it labels

Rules for closing gaps (truth-functional logic)		
when to close		*rule*
resources	*goal*	
		Quod Erat Demonstrandum (QED)
φ	φ	… φ [available] … … φ … → *n* QED … φ (*n*) … … ● φ …
		Non-contradiction (Nc)
φ and ¬ φ	⊥	… ¬ φ [available] … φ [available] … … ⊥ … → *n* Nc … ¬ φ (*n*) … φ (*n*) … … ● ⊥ …
		Ex Nihilo Verum (ENV)
any	⊤	… … ⊤ … → *n* ENV … … ● ⊤ …
		Ex Falso Quodlibet (EFQ)
⊥	*any*	… ⊥ … … φ … → *n* EFQ … ⊥ (*n*) … … ● φ …

Rules for closing gaps (equations)

In addition to the following rules for closing gaps, if the conditions for applying any rule are met except for differences between co-aliases, then the rule can be applied and is notated by adding "=" to its label; QED= and Nc= below are examples of this in the case of rules for closing gaps.

when to close			*rule*
co-aliases	*resources*	*goal*	Equated Co-aliases (EC)
τ—υ	*any*	τ = υ	… [τ and υ are co-aliases] … … τ = υ … → *n* EC … [τ and υ are co-aliases] … … ● τ = υ …
			Distinguished Co-aliases (DC)
τ—υ	¬ τ = υ	⊥	… [τ and υ are co-aliases] … ¬ τ = υ … … ⊥ … → *n* DC … [τ and υ are co-aliases] … ¬ τ = υ (*n*) … … ● ⊥ …
			QED given equations (QED=)
τ_1—υ_1, ⋯, τ_n—υ_n	P$\tau_1 \cdots \tau_n$	P$\upsilon_1 \cdots \upsilon_n$	… [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1 \cdots \tau_n$ … … P$\upsilon_1 \cdots \upsilon_n$ … → *n* QED= … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1 \cdots \tau_n$ (*n*) … … ● P$\upsilon_1 \cdots \upsilon_n$ …
			Non-contradiction given equations (Nc=)
τ_1—υ_1, ⋯, τ_n—υ_n	P$\tau_1 \cdots \tau_n$ ¬ P$\upsilon_1 \cdots \upsilon_n$	⊥	… [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1 \cdots \tau_n$ … ¬ P$\upsilon_1 \cdots \upsilon_n$ … … ⊥ … → *n* Nc= … [have co-alias relations: τ_1—υ_1, ⋯, τ_n—υ_n] … P$\tau_1 \cdots \tau_n$ (*n*) … ¬ P$\upsilon_1 \cdots \upsilon_n$ (*n*) … … ● ⊥ …