# 6. Predications

## 6.1. Naming and describing

### 6.1.0. Overview

We will now begin to study a wider variety of logical forms in which we identify components of sentences that are not also sentences.

6.1.1. A richer grammar

A variety of grammatical categories can be defined using the idea of an individual term, an expression whose function is to name.

6.1.2. Logical predicates

When the subject is removed from a sentence, a grammatical predicate is left behind; a logical predicate is what is left when any number of individual terms are removed.

6.1.3. Extensionality

The truth value of a predication depends only on the reference values of the terms the predicate is applied to, so the meaning of predicate is a function from reference values to truth values.

6.1.4. Identity

We will study the special logical properties of only one predicate, the one expressed by the equals sign.

6.1.5. Analyzing predications

When the analysis of truth-functional structure is complete, we go on to analyze atomic sentences as predications.

6.1.6. Individual terms

While individual terms are not limited to proper names, they do not include all noun phrases, only the ones that function like proper names.

6.1.7. Functors

Individual terms can be formed from other individual terms by operations analogous to predicates.

6.1.8. Examples and problems

These operations enable us to continue the analysis of sentences beyond the analysis of predications by analyzing individual terms themselves.

Glen Helman 15 Aug 2006

### 6.1.1. A richer grammar

While there are more truth-functional connectives that we might study and more questions we might ask about those we have studied, we will now move on from truth-functional logic. The logical forms we will now explore involve ways sentences may be constructed out of expressions that are not yet sentences. Although the kinds of expressions we will identify do not correspond directly to any of the usual parts of speech, our analyses will be comparable in detail to grammatical analyses of short sentences into words.

The simplest case of this sort of analysis is related to, but not identical with, the traditional grammatical analysis into subject and predicate. You might find a grammar text of an old-fashioned sort defining *subject* and *predicate* correlatively as the part of the sentence that is being spoken of and the part that says something about it. Of course, in saying that the subject is being spoken of, there would be no intention to say that the predicate is used to say something about words. So the text might go on to say that a subject contains a word that names the "person, place, thing, or idea" (to quote one of my high school grammar texts) about which something is being said. Thus we have the situation shown in Figure 6.1.1-1.

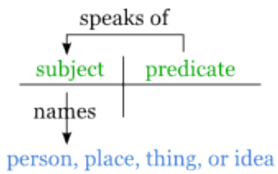


Fig. 6.1.1-1. The traditional picture of grammatical subjects and predicates.

This picture is really not adequate for either grammar or logic, but grammarians and logicians part company in the ways they refine it. Grammarians look for more satisfactory definitions of *subject* and *predicate* that capture, at least roughly, the expressions that have been traditionally labeled in this way. Logicians, on the other hand, accept something like the definitions above and look for expressions that really have the functions they describe, whether or not these expressions would traditionally be labeled subjects and predicates.

"Subjects" and "predicates" in the logical sense provide, along with sentences and connectives, examples of two broad syntactic categories, ***complete expressions*** and ***operations***. Sentences are examples of complete expressions and connectives are examples of operations. Like connectives, operations in general can be thought of as expressions with blanks, expressions that are incomplete in the sense that they are waiting for input. We can classify operations according to the number and kinds of inputs they are waiting for and the kind of output they yield when they receive this input. In the case of connectives, both the input or inputs and the output are sentences.

A "subject" in the logical sense will be a kind of complete expression, an ***individual term***. This is a type of expression whose function is to refer to something; that is, it is an expression which can be described, roughly, as naming a "person, place, thing, or idea." In 6.1.6, we will consider the full range of expressions that count as individual terms but, for now, it will be enough to have in mind some basic examples, proper names (such as Socrates, Indianapolis, Hurricane Isabel, or 3) and simple definite descriptions formed from the definite article the and a common noun (such as the winner, the U.S. president, the park, the book, or the answer).

In the simplest case, a "predicate" in the logical sense—for which we will use the term ***predicate***—is an expression that serves to say something about the object referred to by an individual term. It is an operation whose input is the individual term and whose output is a sentence expressing what is said. So a predicate of this sort amounts to a sentence with a blank waiting to be filled by an individual term. In 6.1.2, we will extend this idea to include predicates that require multiple inputs (i.e., that have several blanks to be filled). Such predicates are certainly not predicates in the grammatical sense; but any predicate in the logical sense will contain the main verb of any sentence it yields as output, so many of the simplest examples of predicates will correspond to verbs or verb phrases.

So the categories of expressions we are working with now include the ones listed below (with simple examples chosen in the simple language used in some popular early elementary school readers from the mid-20th century):

*Complete expressions*

| |
|---|
| sentence<br>Jane ran and Spot barked,<br>Jane ran, Spot barked |
| individual term<br>Jane, Spot |

*Operations*

| *operation* | *input* | *output* |
|---|---|---|
| connective<br>_ and _ | sentence(s) | sentence |
| predicate<br>_ ran,<br>_ barked | individual term(s) | sentence |

Since we now have a number of kinds of expression that might be input or output of an operation, there are many more sorts of operations that we might distinguish according to their input and output, and we will go on to consider some of them. For example, in 6.1.7, we will add a kind of operation which yields individual terms as output (for individual terms as input). And the input and output of operations need not be limited to complete expressions; in later chapters we will add operations that take predicates as input.

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### 6.1.2. Logical predicates

We derived the concept of an individual term from a traditional description of the grammatical subject of a sentence by focusing on the semantic idea of naming. As we will see in 6.1.6, the idea of an individual term is much narrower than the idea of a grammatical subject: not every phrase that could serve as the subject of a sentence counts as an individual term.

We have seen that the opposite is true of our concept of a predicate: it includes grammatical predicates but many other expressions, too. The definition of a predicate in our sense is, like the definition of an individual term, a semantic one: a predicate says something about the about whatever objects are named by the individual terms to which it is applied. The simplest example of this is a grammatical predicate that says something about an object named by an individual term. But consider a sentence that has not only a subject but also a direct object —Ann met Bill for example. This says something about Ann, but it also says something about Bill. From a logical point of view, we could equally well divide the sentence into the subject Ann on the one hand and the predicate met Bill on other or into the subject-plus-verb Ann met and the direct object Bill. And we will be most in the spirit of the idea that predicates are used to say something about individuals if we divide the sentence into the two individual terms Ann and Bill on the one hand and the verb met on the other. The subject and object both are names, and the verb says something about the people they name. That is why we define a ***predicate*** as an operation that forms a sentence when applied to one or more terms. We will speak of the application of this operation as ***predication*** and speak of a sentence that results as ***a predication***.

We can present predicates in this sense graphically by considering sentences containing any number of blanks. For example, the predication Jane called Spot might be depicted as follows:

Individual terms: Jane Spot
Predicate: ____ called ____

The number of different terms to which a predicate may be applied is its number of ***places***, so the predicate [ _ called _ ] has 2 places while predicates, like [ _ ran] and [ _ barked], that are also predicates in the grammatical sense will have one place. We will discuss our notation for predicates more in 6.2.1, but we will often (as has been done here) indicate a predicate by surrounding with brackets the English sentence-with-blanks that expresses it.

In the example above, the two-place predicate is a transitive verb and the second individual term functions as a direct object in the resulting sentence. The individual terms that serve as input to predicates also often appear as indirect objects or as the objects of prepositional phrases that modify a verb—as in the following examples:

Individual terms: Jane Spot the ball
Predicate: ____ threw ____ ______

Individual terms: the ball the window the fishbowl
Predicate: ______ went through _________ into __________

Other examples of many-place predicates are provided by sentences containing comparative constructions or relative terms. Even conjoined subjects can indicate a many-place predicate when and is used to indicate the terms of a relation rather than to state a conjunction:

Individual terms: Jane Sally
Predicate: ____ is older than ____

Individual terms: 2 5
Predicate: _ < _

Individual terms: Jane Sally
Predicate: ____ is a sister of ____

Individual terms: Jane Sally
Predicate: ____ and ____ are sisters

Although you will rarely run into predicates with more than three or four places, it is not hard make up examples of predicates with arbitrarily large numbers of places. For example, imagine the predicate you would get by analyzing a sentence that begins Sam travelled from New York to Los Angeles via Newark, Easton, Bethlehem, …. and goes on to state the full itinerary of a trans-continental bus trip.

The places of a many-place predicate come in a particular order. For example, the sentences Jane is older than Sally and Sally is older than Jane are certainly not equivalent, so it matters which of Jane and Sally is in the first place and which in the second when we identify them as the inputs of the predicate [ _ is older than _ ]. Even when the result of reordering individual terms is equivalent to the original sentence, we will count the places as having a definite order and treat any reordering of the terms filling them as a different sentence. So Dick is the same age as Jane and Jane is the same age as Dick will count as different sentences even though [ _ is the same age as _ ] is symmetric in the sense that

$$\sigma \text{ is the same age as } \tau \Leftrightarrow \tau \text{ is the same age as } \sigma$$

for any terms σ and τ.

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### 6.1.3. Extensionality

The only restriction on an analysis of a sentence into a predicate and individual terms is that the contribution of an individual term to the truth value of a sentence must lie only in its reference value. That is, all that matters is what a term names if it names something; and, if it names nothing if it does not and has the nil reference value mentioned in 1.3.5, that is all that matters. This means that the predicates we will consider are like truth-functional connectives in being ***extensional operations***: the extension of their output depends only on the extensions of their inputs.

In the specific case of predicates, this requirement is sometimes spoken of as a requirement of ***referential transparency***. When evaluating the truth-value of a sentence we sometimes look through individual terms and pay attention only to their reference values while in other cases we pay attention to the terms themselves or the ways in which they refer to their values because differences of this sort make a difference for the truth value of the sentence. For example, in deciding the truth of The U. S. president is over 40, all that matters about the individual term the U. S. president is who it refers to. On the other hand, the sentence For the past two centuries, the U. S. president has been over 35 is true while the sentence For the past two centuries, George Bush has been over 35 is false—even when the terms the U. S. president and George Bush refer to the same person. So, in this second case, we must pay attention to differences between terms that have the same reference value. When this is so the occurrences of these terms are said to be ***referentially opaque***; that is, we cannot look through them to their reference values. The restriction on the analysis of sentences into predicates and individual terms is then that we can count an occurrence of an individual term as filling the place of a predicate only when that occurrence is referentially transparent. Occurrences that are referential opaque cannot be separated from the predicate and must remain part of it.

Hints of idea of a predicate as an incomplete expression can be found in the Middle Ages but it was first presented explicitly a little over a century ago by Gottlob Frege. Frege applied the idea of an incomplete expression not only to predicates but also to mathematical expressions for functions. Indeed, Frege spoke of predicates as signs for a kind of function, a function whose value is not a number but rather a truth value. That is, just as a function like + takes numbers as input and issues a number as output, a predicate is a sign for a function that takes the possible references of individual terms as input and issues a truth value as output. It does this by saying something true or false about its input.

We will speak of the truth-valued function associated with a 1-place predicate as a ***property*** and speak of the function associated with a predicate of two or more places as a ***relation***. Thus a predicate is a sign for a property or relation in the way a truth-functional connective is a sign for a truth function.

Just as a truth-functional connective can be given a truth table, the extensionality of predicates means that a table can capture the way the truth values of the their output sentences depend on the reference values of their input. For example, consider the predicate __ divides __ (evenly). Just as there can be addition or multiplication tables displaying the output of arithmetic functions for a limited range of input, we can give a table indicating the output of the relation expressed by this predicate. For the first half dozen positive integers, we would have the table shown below. Here the input for the first place of the predicate is shown by the row labels at the left and the input for the second place by the column labels at the top. The first row of the table then shows that 1 divides all six integers evenly, the second row shows that 2 divides only 2, 4, and 6 evenly, and the final column shows that each of 1, 2, 3, and 6 divides 6 evenly.

| divides | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | T | T | T | T | T | T |
| 2 | F | T | F | T | F | T |
| 3 | F | F | T | F | F | T |
| 4 | F | F | F | T | F | F |
| 5 | F | F | F | F | T | F |
| 6 | F | F | F | F | F | T |

Of course, this table does not give a complete account of the meaning of the predicate; and, for many predicates, no finite table could. But such tables like this will still be of interest to us because we will consider cases where there are a limited number of reference values and, in such cases, tables can give full accounts of predicates.

As was noted in 1.3.5, we assume that sentences have truth values even when they contain terms that do not refer to anything. This means that we must assume that predicates yield a truth value as output even the nil value is part of their input; that is, we assume that predicates are ***total***. The truth value that is issued as output when the input includes the nil value is usually not settled by the ordinary meaning of an English predicate. It is analogous to the supplements to contexts of use suggested in 1.3.7 as a way of handling cases of vagueness. As in that case, we try to avoid making anything depend on the particular output in cases of undefined input but instead look at relations among sentences that hold no matter how such output is stipulated.

### 6.1.4. Identity

Although all the connectives that figured in our analyses of logical form received special notation and had logical properties we studied, only one predicate will count as ***logical vocabulary*** in this sense. Other predicates and all unanalyzed individual terms will be, like unanalyzed component sentences, part of the ***non-logical vocabulary*** which is assigned a meaning only by an interpretation.

The predicate that is part of our logical vocabulary will be referred to as ***identity***. It is illustrated in the following sentences:

George Bush is the U.S. president

The winner was Funny Cide

n = 3

The morning star and the evening star are the same thing.

We will refer to such sentences as ***equations***; they constitute a particular kind of predication.

In our symbolic notation, we will follow the third example and use the sign = to mark identity. As English notation, we will use the word is. We will represent unanalyzed individual terms by lower case letters, so we can analyze the sentences above as follows:

George Bush is the U.S. president
George Bush = the U.S. president

g = p
g is p

g: George Bush; p: the U.S. president

The winner was Funny Cide
the winner = Funny Cide

w = f
w is f

f: Funny Cide; w: the winner

n = 3

n = t
n is t

n: n; t: 3

The morning star and the evening star are the same thing
the morning star = the evening star

m = e
m is e

m: the morning star; e: the evening star

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## 6.1.5. Analyzing predications

In our symbolic notation for predications other than equations, the predicate will come first followed by the individual terms that are its input. So we might begin an analysis of Bill introduced himself to Ann as follows:

| | Bill introduced himself to Ann |
|---|---|
| Identify (referentially transparent) occurrences of individual terms within the sentence, making sure they are all independent by replacing pronouns by their antecedents | Bill introduced Bill to Ann |
| Separate the terms from the rest of the sentence | Bill Bill Ann<br>___ introduced ___ to ___ |
| Preserve the order of the terms, and form a predicate from the remainder of the sentence | Bill Bill Ann<br>[ _ introduced _ to _ ] ___ ___ ___ |
| Write the terms in the places of the predicate | [ _ introduced _ to _ ] Bill Bill Ann |

Underlining will often be used, as it is here, to mark the places of predicates when they are filled by English expressions. In examples and answers to exercises, we will move directly from the second of these steps to the last, so the process can be thought of as one of removing terms, placing them (in order and with any repetitions) after the sentence they are removed from, and enclosing sentence-with-blanks in brackets.

In general, an application of an *n*-place predicate θ to a series of *n* individual terms $\tau_1, \ldots, \tau_n$ takes the form

$$\theta\tau_1 \ldots \tau_n$$

and our English notation is this:

$$\theta \text{ fits (series) } \tau_1, \ldots, \text{ an' } \tau_n$$

The use of the verb fit here is somewhat artificial. It provides a short verb that enables $\theta\tau_1\ldots\tau_n$ to be read as a sentence, and it is not too hard to understand it as saying that θ is true of $\tau_1, \ldots, \tau_n$. Another artificial aspect of this notation is the unemphasized form an' of and, which is designed to distinguish the use of and here to join the terms of a relation from its use as a truth-functional connective. The role of the term series, which will rarely be needed, is discussed in 6.1.7. We will use the general notation $\theta\tau_1\ldots\tau_n$ when we wish to speak of all predications, so we will take it to apply to equations, too, even though the predicate = is written between the two terms to which it is applied.

In our fully symbolic analyses, unanalyzed non-logical predicates will be abbreviated by capital letters. This is consistent with our use of capital letters for unanalyzed sentences since predicates have sentences as their output. When we add non-logical operations that yield individual terms as output, they will be abbreviated by lower case letters just as unanalyzed individual terms are.

As was done in the display above, we will use the Greek letters θ, π, μ, and ρ to refer to stand for any predicates, so they may stand for single letters and = and also for predicates, which we will consider in the next subsections, whose internal structure has been analyzed. For the time being, all terms will be single letters in our symbolic notation; but in the next section we will consider compound terms, so we will use the Greek letters τ, σ, and υ to stand for any terms, simple or compound.

If we continue the analysis of Bill introduced himself to Ann into fully symbolic form, we would get the following:

Bill introduced himself to Ann
Bill introduced Bill to Ann
[ _ introduced _ to _ ] Bill Bill Ann

Tbba
T fits b, b, an' a

T: [ _ introduced _ to _ ]; a: Ann; b: Bill

The bracketed English sentence-with-blanks does not appear in the final analysis but it does appear in the key.

When sentences contain truth-functional structure, that structure should be analyzed first; an analysis into predicates and individual terms should begin only when no further analysis by connectives is possible. Here is an example:

If either Ann or Bill was at the meeting, then Carol has seen the report and will call you about it
Either Ann or Bill was at the meeting → Carol has seen the report and will call you about it
(Ann was at the meeting ∨ Bill was at the meeting) → (Carol has seen the report ∧ Carol will call you about the report)
([ _ was at _ ] Ann the meeting ∨ [ _ was at _ ] Bill the meeting) → ([ _ has seen _ ] Carol the report ∧ [ _ will call _ about _ ] Carol you the report)

$$(\text{Aam} \lor \text{Abm}) \to (\text{Scr} \land \text{Lcor})$$

if either A fits a an' m or A fits b an' m then both S fits c an' r and L fits c, o, an' r

A: [ _ was at _ ]; L: [ _ will call _ about _ ]; S: [ _ has seen _ ]; a: Ann; b: Bill; c: Carol; m: the meeting; o: you; r: the report

When analyzing atomic sentences into predicates and terms be sure to watch for repetitions of predicates from one atomic sentence to another like that of [ _ was at _ ] in this example. Such repetitions are an important part of the logical structure of the sentence.

Since the notation for identity is different from that used for non-logical predicates, you need to watch for atomic sentences that count as equations. These will usually, but not always, be marked by some form of the verb to be but, of course, forms of to be have other uses, too. Consider the following example:

If Tom was told of the nomination, then if he was the winner he wasn't surprised

Tom was told of the nomination → if Tom was the winner he wasn't surprised

Tom was told of the nomination → (Tom was the winner → Tom wasn't surprised)

Tom was told of the nomination → (Tom was the winner → ¬ Tom was surprised)

[ _ was told of _ ] Tom the nomination → (Tom = the winner → ¬ [ _ was surprised] Tom)

$$Ltn \to (t = r \to \neg St)$$

if L fits t an' n then if t is r then not S fits t

L: [ _ was told of _ ]; S: [ _ was surprised]; t: Tom; n: the nomination

It is fairly safe to assume that a form of to be joining to individual terms indicates an equation, but it is wise to always think about what is being said: an equation is a sentence that says its component individual terms have the same reference value. Notice also that identity does not appear in the key to the analysis. That is because it is part of the logical vocabulary; that is, it is like the connectives, which also do not appear in keys.



### 6.1.6. Individual terms

The chief examples of individual terms are proper names, for the central function of a proper name is to refer to the bearer of the name. But a proper name is not the only sort of expression that refers to an individual; the phrase the first U. S. president serves as well as the name George Washington. In general, descriptive phrases coupled with the definite article the at least purport to refer of individuals. These phrases are the ***definite descriptions*** discussed briefly in 1.3.5, and we have been counting them as individual terms. Still other examples of individual terms can be found in nouns and noun phrases modified by possessives—for example, Mt. Vernon's most famous owner. Indeed, expressions of this sort can generally be paraphrased by definite descriptions (such as the most famous owner of Mt. Vernon). A final group of examples are demonstrative pronouns this and that and other pronouns whose references are determined by the context of use—such as I, you, and certain uses of third person pronouns. On the other hand, while ***anaphoric pronouns***, pronouns that have other noun phrases as their antecedents, count grammatically as individual terms, they do not have independent reference values and will be treated differently in our analyses. We will look at their role more closely in 6.2.3; but, for now, it is safe to think of them as place-holders for individual terms.

There is no traditional grammatical category or part of speech that includes individual terms but no other expressions. In particular, the class of nouns and noun phrases is too broad because it includes simple common nouns, such as president, as well as ***quantifier phrases***—such as no president, every president, or a president. And neither common nouns nor quantifier phrases make the kind of reference that is required for an individual term.

The following table collects the examples we have just seen on both sides of the line between individual terms and other noun phrases:

| *Individual terms* | *Noun phrases that are not individual terms* |
| --- | --- |
| proper names (e.g., George Washington) | common nouns (e.g., president) |
| definite descriptions (e.g., the first U. S. president) | quantifier phrases (e.g., no president, every president, a president) |
| noun phrases with possessive modifiers (e.g., Mt. Vernon's most famous owner) | |
| non-anaphoric pronouns (e.g., this, you) | |
| anaphoric pronouns (e.g., he, she, it) | |

In a moment, we will look further at the reasons for drawing the line in this way; but one way of seeing the difference between individual terms and other nouns and noun phrases is to note that, while a proper name or a definite description provides a direct answer to the question Which person, place, thing, or idea are you referring to?, a common noun or quantifier phrase either provides no answer at all or, as in the case of a president, constitutes only an incomplete or evasive one.

Perhaps the most that can be done in general by way of defining the idea of an ***individual term*** is to give the following rough semantic description: an individual term is

*an expression that refers, or purports to refer, to a single object in a definite way*

At any rate, this formula can be elaborated to explain the reasons for rejecting the noun phrases at the right of the table above.

The formula above is intended as a somewhat more precise statement of the idea that an individual term "names a person, place, thing or idea." It uses object in place of the list person, place, thing, or idea partly for compactness and partly because that list is incomplete. Indeed it would be hard to ever list all the kinds of things that might be referred to by individual terms. If the term ***object*** and other terms like ***entity***, ***individual***, and ***thing*** are used in a broad abstract sense, they can apply to anything that an individual term might refer to. In particular, in this sort of usage, these terms apply to people. The main force of the formula above then lies in the ideas of *referring to a single thing* and *referring in a definite way*.

The requirement that reference be to a single thing rules out most of noun phrases on the right of the table above. First of all, if a common noun can be said to refer at all, it refers not to a single thing but to a class, such as the class of all presidents. Now this class can be thought of as a single thing and can be referred to by the definite description just used—i.e., the class of all presidents—but the common noun president "refers" to this class in a different way. Common nouns are sometimes labeled ***general terms*** and distinguished from ***singular terms***, an alternative label for individual terms. The function of a general term is to indicate a general kind (e.g., dogs) from which individual things may be picked out rather than to pick out a single thing of that kind (e.g., Spot), as an individual term does. Thus the individual term the first U. S. president picks out an individual within the class indicated by the common noun president; and the class of all presidents picks out an individual within the class indicated by the common noun class. That is, a general term indicates a range of objects from which a particular object might be chosen while an individual term picks out a particular object. Although there is much that might be said about the role of general terms in deductive reasoning, we will never identify them as separate components in our analyses of logical form, and the word term without qualification will be used as an abbreviated alternative to individual term.

The remaining noun phrases at the right of the table are like individual terms in making use of a common noun's indication of a class of objects. However, they do not do this to pick out a single member of the class but instead to help make claims about the class as a whole. The claims to which they contribute say something about the number of members of a class that have or lack a certain property, and that is the reason for describing them as "quantifier" phrases. It's probably clear that the phrases every president and no president, even though they are grammatically singular, do not serve the function of picking out a single object. But that may be less clear in the case of a president.

Sentences containing quantifier phrases like a president and some president share with those containing definite descriptions, such as the president, the feature that they can be true because of a fact about a single object. For example, The first U. S. president wore false teeth and A president wore false teeth can be said to both be true because of a fact about Washington. The difference between the two sorts of expression can be seen by considering what might make such sentences false. If Washington had not worn false teeth, The first U. S. president wore false teeth would be false but A president wore false teeth might still be true. That's because the second could be true because of facts about many different presidents (in many different countries), so its truth is not tied to facts about any one of them. If the expression a president is thought of as referring at all, its reference is an indefinite one. That is one reason for adding the qualification definite to the formula for individual terms given above, but this qualification also serves as a reminder that the presence of a definite article marks an individual term while an indefinite article indicates a quantifier phrase.

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### 6.1.7. Functors

Truth-functional connectives express truth-valued functions of truth values, and predicates express truth-valued functions of reference values. A third sort of function not only takes reference values as input but also issues them as output. We will refer to this sort of function as a ***reference function*** or, in contexts where we do not need a more general concept, simply as a ***function***. We will refer to expressions that are signs for these functions as ***functors*** and refer to the operation of applying a functor as ***function application***. We can speak of the result of a function application as a ***compound term***.

Functors are incomplete expressions that stand to individual terms as connectives stand to sentences, so we can extend the table of operations in 6.1.1 as follows:

| *operation* | *input* | *output* |
|---|---|---|
| connective | sentence(s) | sentence |
| predicate | individual term(s) | sentence |
| functor | individual term(s) | individual term |

We will add further incomplete expressions to this list in later chapters when we consider operations that take predicates as input.

Signs for mathematical functions provide examples of functors. The expression 7 + 5 can be analyzed as

Individual terms: 7 5
Functor: _ + _

But functors are not limited to mathematical vocabulary. Any individual term that contains one or more individual terms can be seen as the result of applying a functor to those component terms. Thus the oldest child of Ann and Bill can be analyzed as

Individual terms: Ann Bill
Functor: the oldest child of ___ and ___

And the more complex individual term the book that Ann's father mentioned has the following analysis:

Individual term: Ann
Functors: ___'s father
the book that ________ mentioned

Possessives and prepositional phrases often give rise to functors but all that is needed to have a functor is an individual term that contains an individual term.

Our notation for functors will be analogous to that for predicates. Functors can be represented in semi-symbolic notation by individual-terms-with-blanks surrounded by brackets. Using this notation, the first two examples above could be given the analyses:

[ _ + _ ] 7 5
[the oldest child of _ and _ ] Ann Bill

In the case of the third example, we will use parentheses to show grouping

[the book that _ mentioned] ([ _'s father] Ann)

In fact, there is no danger of ambiguity here; but the structure is clearer with parentheses, and, in the full symbolic notation, compound terms should be enclosed in parentheses when they fill a place of a functor or predicate.

In that notation, unanalyzed functors will be represented by lower case letters and will be written before the individual terms filling their places. The general form of a compound term is this

$$\zeta\tau_1 \ldots \tau_n$$

and our English notation will be

$$\zeta \text{ of (series) } \tau_1, \ldots, \text{ an' } \tau_n$$

or

$$\zeta \text{ applied to (series) } \tau_1, \ldots, \text{ an' } \tau_n$$

both of which are in keeping with the usual way of reading a function application, but one or the other will work better in certain contexts. When we need a general variable for functors we will use ζ or ξ.

Using this symbolic and English notation, we can express the final analyses of the examples above as follows:

| *symbolic notation* | *English notation* | *key* |
|---|---|---|
| psf | p of s an' f | p: [ _ + _ ]; f: 5; s: 7 |
| oab | o of a an' b | o: [the oldest child of _ and _ ]; a: Ann; b: Bill |
| b(fa) | b of f of a | b: [ the book that _ mentioned]; f: [ _'s father]; a: Ann |

The symbolic notation for functors that is used here is fairly common in work on logic but is different from the most common mathematical notation for function applications. Here are some examples for comparison

| *common mathematical notation* | *symbolic notation used here* | *English notation* |
|---|---|---|
| f(a) | fa | f of a |
| f(a, b) | fab | f of a an' b |
| f(g(a)) | f(ga) | f of g of a |
| f(a, g(b)) | fa(gb) | f of a an' g of b |
| f(g(a), b) | f(ga)b | f of series g of a an' b |
| f(g(a, b)) | f(gab) | f of g of series a an' b |

The notation we will use is designed to minimize parentheses and commas. The general rule for interpreting it is this: (i) after a predicate —i.e., after a capital letter—each unparenthesized letter and each parenthetical unit occupies one place of the predicate and (ii) within a parenthetical unit the first letter is a functor and each following unparenthesized letter and each parenthetical unit occupies one place of this functor.

The last two examples above show the role of the optional term series in avoiding ambiguity. Because the letters used to represent functors and non-logical predicates do not have a fixed number of places associated with them, when a single an' follows two occurrences of of, it can be unclear where the series of terms marked by an' actually began. There are other ways of handling this ambiguity. Parentheses suffice in written notation and parentheses, like other punctuation, can be reflected in speech. For example, it is natural to mark the difference between f of (g of a) an' b and f of (g of a an' b), respectively, by varying the speed with which they are spoken in ways that might be indicated by "f of g-of-a an' b" and "f of g of a-an'-b".

In the presence of functors, the potential for undefined terms increases considerably. Even if the cat on the mat has a non-nil reference value, the cat on the refrigerator may not—to say nothing of the cat on the house of Ann's father's best friend or the cat on 6. That is, functors accept a large variety of inputs and can be expected to issue output with undefined reference for some of them. This problem can be reduced (though not eliminated) by limiting functors to input of certain sorts. That is usually done by assigning individual terms to various ***types*** and allowing only individual terms of certain types to serve as inputs to a given functor. For example, the functor [ _ + _ ] might be restricted to numerical input. We will not follow this approach (which complicates the description of logical forms considerably), but it can serve to capture a number of features, both syntactic and semantic, of a natural language like English.

Glen Helman 15 Aug 2006

### 6.1.8. Examples and problems

We will begin with a couple of extended but straightforward examples.

If Dan is the winner and Portugal is the place he would most like to visit, he will visit there before long

Dan is the winner and Portugal is the place he would most like to visit → Dan will visit Portugal before long

(Dan is the winner ∧ Portugal is the place Dan would most like to visit) → Dan will visit Portugal before long

(Dan is the winner ∧ Portugal is the place Dan would most like to visit) → Dan will visit Portugal before long

(Dan = the winner ∧ Portugal = the place Dan would most like to visit) → [ _ will visit _ before long] Dan Portugal

(d = n ∧ p = [the place _ would most like to visit] Dan) → Vdp

(d = n ∧ p = ld) → Vdp

if both d is n and p is l of d then V fits d an' p

V: [ _ will visit _ before long]; l: [the place _ would most like to visit]; d: Dan; n: the winner; p: Portugal

Al won't sign the contract Barb's lawyer made out without speaking to his lawyer

¬ Al will sign the contract Barb's lawyer made out without speaking to his lawyer

¬ (Al will sign the contract Barb's lawyer made out ∧ ¬ Al will speak to his lawyer)

¬ (Al will sign the contract Barb's lawyer made out ∧ ¬ Al will speak to Al's lawyer)

¬ ([ _ will sign _ ] Al the contract Barb's lawyer made out ∧ ¬ [ _ will speak to _ ] Al Al's lawyer)

¬ (S a (the contract Barb's lawyer made out) ∧ ¬ P a (Al's lawyer))

¬ (S a ([the contract _ made out] Barb's lawyer) ∧ ¬ P a ([ _'s lawyer] Al))

¬ (S a (c ([ _'s lawyer] Barb)) ∧ ¬ Pa(la))

¬ (Sa(c(lb)) ∧ ¬ Pa(la))

not both S fits a an' c of l of b and not P fits a an' l of a

P: [ _ will speak to _ ]; S: _ will sign _ ; c: [the contract _ made out]; l: [ _'s lawyer]; a: Al; b: Barb

When analyzing either a predication or an individual term, make sure that you remove all the largest individual terms it contains. That is, if you identify a component individual term, make sure that it is not part of a compound term that is itself a component of the sentence or term you are analyzing. To analyze Al will speak to his lawyer as [ _ will speak to _'s lawyer] Al Al would be to ignore an important aspect of its structure. Of course, when applying this maxim, it is important to distinguish individual terms from other noun phrases. For example, although Dan is the winner of the contest can be analyzed initially as Dan = the winner of the contest, the grammatically similar sentence Dan is a winner of the contest should be analyzed as [ _ is a winner of _ ] Dan the contest because a winner of the contest is not an individual term.

Also, when you locate a definite description, make sure that you have identified the whole of it. What you are most likely to miss are modifiers, usually prepositional phrases or relative clauses, that follow the main common noun of the definite description. For example, although the place might be an individual term in its own right in other cases, in the example above is it only part of the term the place Dan would most like to visit. Similarly, the contract is only the beginning of the individual term the contract Barb's lawyer made out. In both of the these cases, the rest of the definite description is a relative clause with a suppressed relative pronoun; that is, they might have been stated more fully as the place that Dan would most like to visit and the contract that Barb's lawyer made out, respectively. It might help here to think of prepositional phrases and relative clauses as modifying a common noun before the definite article is attached. That is, the phrases above have the form the (place Dan would most like to visit) and the (contract Barb's lawyer made out), so any component of these sentences containing the initial the must also contain the whole of the following parenthesized expressions.

There are some cases where a prepositional phrase or relative clause following a common noun should not be counted as part of a definite description. Some prepositional phrases can modify both nouns and verbs, and a prepositional phrase following a noun within a grammatical predicate might be understood to modify either it or the main verb. The sentence The dog chased the cat on the mat is ambiguous in this way since the mat might be understood to be either the location of the chase or the location of the cat, who might have been chased elsewhere. This sort of ambiguity can be clarified by converting the prepositional phrase into a relative clause, which can only modify a noun; if this transformation—e.g.,

The dog chased the cat that was on the mat

—preserves meaning, then the prepositional phrase is part of the definite description. On the other hand, since anaphoric pronouns cannot accept modifiers, replacing a possible noun phrase by a pronoun will show the result of taking a prepositional phrase to modify the verb. This can be done by moving the noun phrase to the front of the sentence, joining it to the remaining sentence-with-a-blank by the phrase is such that, and filling the blank with an appropriate pronoun (he, she, or it). In this example, that would give us

The cat is such that the dog chased it on the mat

So, the prepositional phrase on the mat should be taken to modify cat or chased depending on whether the first or second of the displayed sentences best captures the meaning of the original. Of course, when a potentially ambiguous sentence is taken out of context, it may not be clear which of two alternatives does best capture the original meaning; in such a case, either analysis is legitimate.

Not all relative clauses contribute to determining reference. Those that do are ***restrictive*** clauses, and it is these that should be included in definite descriptions. Other relative clauses are ***non-restrictive***. Non-restrictive clauses cannot use the word that and, when punctuated, are marked off by commas. Restrictive clauses are not marked off by commas in standard English punctuation and may use that (but are not limited to this relative pronoun), and they can in some cases be expressed without a relative pronoun. It is easiest to tell what sort of relative clause you are faced with when more than one of these differences is exhibited. For example, the relative clause The cat that the dog had chased was asleep or The cat the dog had chased was asleep is clearly restrictive while the one in The cat, who the dog had chased, was asleep is clearly non-restrictive. This means that the relative clause in the first is part of the definite description the cat that the dog had chased. The relative clause in the second would instead be analyzed as a separate conjunct to give the dog had chased the cat ∧ the cat was asleep as the initial step of the analysis.

Another indication of the difference between the two sorts or relative clause is that a non-restrictive clause can modify a proper name—as in Puff, who the dog had chased, was asleep. And, since neither prepositional phrases nor restrictive relative clauses can modify a proper name, putting a proper name in a blank that was left when you removed an apparent individual term can show whether you really removed the whole of the term. For example, Puff on the mat was asleep and Puff that the dog had chased was asleep are both ungrammatical.

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### 6.1.s. Summary

1 We move beyond truth-functional logic by recognizing complete expressions other than sentences and operations other than connectives. Our additions are motivated by a traditional description of grammatical subjects and predicates. The new complete expressions are individual terms, whose function is to name. Given this idea, we can define a predicate as an operation that forms a sentence from one or more individual terms.

2 A predicate corresponds to an English sentence with blanks that might be filled by terms. These blanks are the predicate's places and the operation of filling them is predication.

3 We will maintain something analogous to truth-functionality by requiring that predicates be extensional. This means that all places of a predicate must be referentially transparent (rather than referentially opaque): when judging the truth value of a sentence formed by the predicate, we must be able see through the terms filling these places to what those terms refer to. Thus, just as a connective expresses a truth function, a predicate expresses a function that takes reference values as input and issues truth values as output. Such a function may be called a property if it has one place and a relation if it has 2 or more. In symbolic notation, it takes the form $\sigma = \tau$ and, in English notation, it takes the form $\sigma$ is $\tau$.

4 While recognizing quite a variety of non-logical vocabulary in our analyses, we recognize only one new item of logical vocabulary, the predicate identity. This is a 2-place predicate that forms an equation, which is true when its component terms have the same reference value.

5 In our symbolic notation, we use lower case letters to stand for unanalyzed individual terms, the equal sign for identity, and capital letters to stand for non-logical predicates. Non-logical predicates, both capital letters and predicate abstracts are written in front of the terms they apply to (with a predicate abstract enclosed in brackets), and = is written between the terms to which it applies. In English notation, predications other than equations are written as $\theta$ fits (series) $\tau_1$, ..., an' $\tau_n$.

6 In addition to proper names, the individual terms include definite descriptions and various non-anaphoric pronouns. They do not include certain other noun phrases, quantifier phrases in particular.

We will speak of the “person, place, thing, or idea” referred to by an individual term by using such words as object, entity, individual, and thing, understanding these to apply to anything that might be named. Common nouns are also not individual terms. Indeed, they may be labeled general terms to distinguish their function of indicating a class of objects from the function of individual terms, also called singular terms, which is to refer to a single individual in a definite way. The word term will often be used as shorthand for individual term.

7 A functor is an operation that takes one or more individual terms as input and yields an individual term as output. Just like other operations, it expresses a reference function, which yields reference values when applied to reference values. Although a reference function is a particular sort of function we will use that term primarily for reference functions. The operation of combining a functor with input is application, and the individual term that is the output is a compound term. For any functor, there will almost always be some terms for which the application of the functor yields an undefined term. Although this problem can be reduced by limiting the input of functors to objects of certain types, we will not include this complication in our account of logical forms.

8 It can be difficult to recognize the individual terms that fill the places of a predicate or a functor. It is important in include in a definite description all the modifiers that are part of it. Some of these may be prepositional phrases or relative clauses which follow the common noun. In some cases, a prepositional phrase in this position might either be part of a definite or modify a verb; but such an ambiguity cannot arise with relative clauses so a prepositional phrase can be made into a relative clause in order to test what it modifies. Relative clauses must therefore be part of the definite description when they are restrictive; on the other hand, non-restrictive clauses (the sort set off by commas) are analyzed using conjunction.



### 6.1.x. Exercise questions

**1.** Analyze each of the following sentences in as much detail as possible.

- **a.** Ann introduced Bill to Carol.
- **b.** Ann gave the book to either Bill or Carol.
- **c.** Ann gave the book to Bill and he gave it to Carol.
- **d.** Tom had the package sent to Sue, but it was returned to him.
- **e.** Georgia will see Ed if she gets to Denver before Saturday.
- **f.** If the murderer is either the butler or the nephew, then I’m Sherlock Holmes.
- **g.** Neither Ann nor Bill saw Tom speak to either Mike or Nancy.
- **h.** Tom will agree if each of Ann, Bill, and Carol asks him.
- **i.** Reagan’s vice president was the 41st president.
- **j.** Tom found a fly in his soup and he called the waiter.
- **k.** Tom found the book everyone had talked to him about and he bought a copy of it.
- **l.** Wabash College is located in Crawfordsville, which is the seat of Montgomery County.
- **m.** Sue and Tom set the date of their wedding but didn’t decide on its location.

**2.** Synthesize idiomatic English sentences that express the propositions associated with the logical forms below by the intensional interpretations that follow them.

- **a.** $Wci \land Scl$
  S: [ _ is south of _ ]; W: [ _ is west of _ ]; c: Crawfordsville; i: Indianapolis; l: Lafayette
- **b.** $Mab \rightarrow Mba$
  M: [ _ has met _ ]; a: Ann; b: Bill
- **c.** $Iacb \land Iadb$
  I: [ _ introduced _ to _ ]; a: Alice; b: Boris; c: Clarice; d: Doris

**d.** Wab ∧ Kabab
K: [ _ asked _ to write _ about _ ]; W: [ _ wrote to _ ]; a: Alice; b: Boris

**e.** g = c → (f = s ∧ p = t)
c: the city; f: football; g: Green Bay; p: the Packers; s: the sport; t: the team

**f.** (Sab ∧ ¬ Sa(fc)) → ¬ b = fc
S: [ _ has spoken to _ ]; f: _'s father; a: Ann; b: Bill; c: Carol

**g.** (B(fa)(mb) ∨ S(ma)(fb)) → Cab
B: [ _ is a brother of _ ]; C: _ and _ are cross-cousins; S: [ _ is a sister of _ ]; f: _'s father; m: [ _'s mother]; a: Ann; b: Bill

**h.** Pab(m(sb)(sc)) ∧ Pac(m(sb)(sc))
P: [ _ persuaded _ to accept _ ]; m: the best compromise between _ and _ ; s: [ _ 's proposal]; a: Ann; b: Bill; c: Carol



## 6.1.xa. Exercise answers

**1. a.** Ann introduced Bill to Carol
[ _ introduced _ to _ ] Ann Bill Carol
Iabc
I fits a, b, an' c
I: [ _ introduced _ to _ ]; a: Ann; b: Bill; c: Carol

**b.** Ann gave the book to either Bill or Carol
Ann gave the book to Bill ∨ Ann gave the book to Carol
[ _ gave _ to _ ] Ann the book Bill ∨ [ _ gave _ to _ ] Ann the book Carol
Gakb ∨ Gakc
either G fits a, k, an' b or G fits a, k, an' c
G: [ _ gave _ to _ ]; a: Ann; b: Bill; c: Carol; k: the book

**c.** Ann gave the book to Bill and he gave it to Carol
Ann gave the book to Bill ∧ Bill gave the book to Carol
[ _ gave _ to _ ] Ann the book Bill ∧ [ _ gave _ to _ ] Bill the book Carol
Gakb ∧ Gbkc
both G fits a, k, an' b and G fits b, k, an' c
G: [ _ gave _ to _ ]; a: Ann; b: Bill; c: Carol; k: the book

**d.** Tom had the package sent to Sue, but it was returned to him
Tom had the package sent to Sue ∧ the package was returned to Tom
[ _ had _ sent to _ ] Tom the package Sue ∧ [ _ was returned to _ ] the package Tom
Htps ∧ Rpt
both H fits t, p, an' s and R fits p an' t
H: [ _ had _ sent to _ ]; R: [ _ was returned to _ ]; p: the package; s: Sue; t: Tom

**e.** Georgia will see Ed if she gets to Denver before Saturday
Georgia will see Ed ← Georgia will get to Denver before Saturday
[ _ will see _ ] Georgia Ed ← [ _ will get to _ before _ ] Georgia Denver Saturday
Sge ← Ggds
Ggds → Sge
if G fits g, d, an' s then S fits g an' e
G: [ _ will get to _ before _ ]; S: [ _ will see _ ]; d: Denver; e: Ed; g: Georgia; s: Saturday

**f.** If the murderer is either the butler or the nephew, then I'm Sherlock Holmes

the murderer is either the butler or the nephew → I'm Sherlock Holmes

(the murderer is the butler ∨ the murderer is the nephew) → I = Sherlock Holmes

(the murderer = the butler ∨ the murderer = the nephew) → i = s

$$(m = b \lor m = n) \rightarrow i = s$$

if either m is b or m is n then i is s

b: the butler; i: I; m: the murderer; n: the nephew; s: Sherlock Holmes

**g.** Neither Ann nor Bill saw Tom speak to either Mike or Nancy

¬ (Ann saw Tom speak to either Mike or Nancy ∨ Bill saw Tom speak to either Mike or Nancy)

¬ ((Ann saw Tom speak to Mike ∨ Ann saw Tom speak to Nancy) ∨ (Bill saw Tom speak to Mike ∨ Bill saw Tom speak to Nancy))

¬ (([ _ saw _ speak to _ ] Ann Tom Mike ∨ [ _ saw _ speak to _ ] Ann Tom Nancy) ∨ ([ _ saw _ speak to _ ] Bill Tom Mike ∨ [ _ saw _ speak to _ ] Bill Tom Nancy))

$$\neg ((Satm \lor Satn) \lor (Sbtm \lor Sbtn))$$

not either either S fits a, t, an' m or S fits a,t, an' n or either S fits b,t, an' m or S fits b,t, an' n

S: [ _ saw _ speak to _ ]; a: Ann; b: Bill; m: Mike; n: Nancy; t: Tom

**h.** Tom will agree if each of Ann, Bill, and Carol asks him

Tom will agree ← each of Ann, Bill, and Carol will ask Tom

Tom will agree ← ((Ann will ask Tom ∧ Bill will ask Tom) ∧ Carol will ask Tom)

[ _ will agree] Tom ← (([ _ will ask _ ] Ann Tom ∧ [ _ will ask _ ] Bill Tom) ∧ [ _ will ask _ ] Carol Tom)

$$Gt \leftarrow ((Aat \land Abt) \land Act)$$

$$((Aat \land Abt) \land Act) \rightarrow Gt$$

if both both A fits a an' t and A fits b an' t and A fits c an' t then G fits t

A: [ _ will ask _ ]; G: [ _ will agree]; a: Ann; b: Bill; c: Carol; t: Tom

The function of each here is to indicate a group of two-place predication rather than a single four-place predicate [ _, _, and _ will ask _ ], which is what would be required in order to express instead the idea of Ann, Bill, and Carol making the request as a group.

**i.** Reagan's vice president was the 41st president.

Reagan's vice president = the 41st president

[ _'s vice president] Reagan = [the _th president] 41

$$vr = pf$$

v of r is p of f

p: [the _th president]; v: _ 's vice president; f: 41; r: Reagan

**j.** Tom found a fly in his soup and he called the waiter

Tom found a fly in his soup ∧ Tom called the waiter

Tom found a fly in Tom's soup ∧ Tom called the waiter

[ _ found a fly in _ ] Tom Tom's soup ∧ [ _ called _ ] Tom the waiter

Ft(Tom's soup) ∧ Ctr

Ft([ _'s soup] Tom) ∧ Ctr

$$Ft(st) \land Ctr$$

both F fits t an' s of t and C fits t an' r

C: [ _ called _ ]; F: _ found a fly in _ ]; s: [ _'s soup]; r: the waiter; t: Tom

**k.** Tom found the book everyone had talked to him about and he bought a copy of it

Tom found the book everyone had talked to him about ∧ Tom bought a copy of the book everyone had talked to him about

Tom found the book everyone had talked to Tom about ∧ Tom bought a copy of the book everyone had talked to Tom about

[ _ found _ ] Tom the book everyone had talked to Tom about ∧ [ _ bought a copy of _ ] Tom the book everyone had talked to Tom about

Ft(the book everyone had talked to Tom about) ∧ Bt(the book everyone had talked to Tom about)

Ft([the book everyone had talked to _ about] Tom) ∧ Bt([the book everyone had talked to _ about] Tom)

$$Ft(bt) \land Bt(bt)$$

both F fits t an' b of t and B fits t an' b of t

B: [ _ bought a copy of _ ]; F: _ found _ ; b: [the book everyone had talked to _ about]; t: Tom

**l.** Wabash College is located in Crawfordsville, which is the seat of Montgomery County

Wabash College is located in Crawfordsville ∧ Crawfordsville is the seat of Montgomery County

[ _ is located in _ ] Wabash College Crawfordsville ∧ Crawfordsville = the seat of Montgomery County

Lbc ∧ c = [the seat of _ ] Montgomery County

Lbc ∧ c = sm

both L fits b an' c and c is s of m

L: [ _ is located in _ ]; s: the seat of _ ; b: Wabash; c: Crawfordsville; m: Montgomery County

**m.** Sue and Tom set the date of their wedding but didn't decide on its location

Sue and Tom set the date of their wedding
∧ Sue and Tom didn't decide on the location of their wedding

Sue and Tom set the date of Sue and Tom's wedding
∧ ¬ Sue and Tom decided on the location of Sue and Tom's wedding

[ _ and _ set _ ] Sue Tom the date of Sue and Tom's wedding
∧ ¬ [ _ and _ decided on _ ] Sue Tom the location of Sue and Tom's wedding

Sst(the date of Sue and Tom's wedding)
∧ ¬ Dst(the location of Sue and Tom's wedding)

Sst([the date of _ ] Sue and Tom's wedding)
∧ ¬ Dst([the location of _ ] Sue and Tom's wedding)

Sst(d(Sue and Tom's wedding)) ∧ ¬ Dst(l(Sue and Tom's wedding))

Sst(d([ _ and _'s wedding] Sue Tom))
∧ ¬ Dst(l([ _ and _'s wedding] Sue Tom))

Sst(d(wst)) ∧ ¬ Dst(l(wst))

both S fits s, t, an' d of (w of s an' t) and not D fits s, t, an' l of (w of s an' t)

D: [ _ and _ decided on _ ]; S: _ and _ set _ ; d: [the date of _ ]; l: the location of _ ; w: [ _ and _'s wedding]; s: Sue; t: Tom

**2. a.** [ _ is west of _ ] Crawfordsville Indianapolis
∧ [ _ is south of _ ] Crawfordsville Lafayette

Crawfordsville is west of Indianapolis ∧ Crawfordsville is south of Lafayette

Crawfordsville is west of Indianapolis and south of Lafayette

**b.** [ _ has met _ ] Ann Bill → [ _ has met _ ] Bill Ann

Ann has met Bill → Bill has met Ann

If Ann has met Bill then he has met her

**c.** [ _ introduced _ to _ ] Alice Clarice Boris
∧ [ _ introduced _ to _ ] Alice Doris Boris

Alice introduced Clarice to Boris ∧ Alice introduced Doris to Boris

Alice introduced Clarice and Doris to Boris

**d.** [ _ wrote to _ ] Alice Boris
∧ [ _ asked _ to write _ about _ ] Alice Boris Alice Boris

Alice wrote to Boris ∧ Alice asked Boris to write Alice about Boris

Alice wrote to Boris ∧ Alice asked Boris to write her about himself

Alice wrote to Boris and asked him to write her about himself

**e.** g = c → (f = s ∧ p = t)

Green Bay = the city → (football = the sport ∧ the Packers = the team)

Green Bay is the city → (football is the sport ∧ the Packers are the team)

Green Bay is the city → football is the sport and the Packers are the team

If Green Bay is the city, then football is the sport and the Packers are the team

**f.** ([ _ has spoken to _ ] Ann Bill ∧ ¬ [ _ has spoken to _ ] Ann ([ _'s father] Carol)) → ¬ Bill = [ _'s father] Carol

(Ann has spoken to Bill ∧ ¬ [ _ has spoken to _ ] Ann Carol's father)
→ ¬ Bill = Carol's father

(Ann has spoken to Bill ∧ ¬ Ann has spoken to Carol's father) → ¬ Bill is Carol's father

(Ann has spoken to Bill ∧ Ann hasn't spoken to Carol's father) → Bill isn't Carol's father

Ann has spoken to Bill but not to Carol's father → Bill isn't Carol's father

If Ann has spoken to Bill but not to Carol's father, then Bill isn't Carol's father

**g.** (B([ _'s father] Ann)([ _'s mother] Bill) ∨ S([ _'s mother] Ann)([ _'s father] Bill)) → [ _ and _ are cross-cousins] Ann Bill

([ _ is a brother of _ ] Ann's father Bill's mother ∨ [ _ is a sister of _ ] Ann's mother Bill's father) → Ann and Bill are cross-cousins

(Ann's father is a brother of Bill's mother ∨ Ann's mother is a sister of Bill's father) → Ann and Bill are cross-cousins

Ann's father is a brother of Bill's mother or Ann's mother is a sister of Bill's father → Ann and Bill are cross-cousins

If Ann's father is a brother of Bill's mother or Ann's mother is a sister of Bill's father, then Ann and Bill are cross-cousins

**h.** Pab(m([ _'s proposal] Bill)([ _'s proposal] Carol))
∧ Pac(m([ _'s proposal] Bill)([ _'s proposal] Carol))
Pab([the best compromise between _ and _ ] Bill's proposal Carol's proposal)
∧ Pac([the best compromise between _ and _ ] Bill's proposal Carol's proposal)
[ _ persuaded _ to accept _ ] Ann Bill the best compromise between Bill's proposal and Carol's proposal
∧ [ _ persuaded _ to accept _ ] Ann Carol the best compromise between Bill's proposal and Carol's proposal
Ann persuaded Bill to accept the best compromise between his and Carol's proposals
∧ Ann persuaded Carol to accept the best compromise between Bill's proposal and hers
Ann persuaded each of Bill and Carol to accept the best compromise between their proposals