1. Introduction

1.1. Formal deductive logic

1.1.0. Overview

The topic of this course is the study of reasoning; but we will study only certain aspects of reasoning and study them only from one perspective. The special character of our study is indicated by the label ***formal deductive logic***, and our first task will be to see what this label means. The terms formal and logic specify the way in which we will study reasoning while the term deductive specifies the sort of reasoning we will study. In the course of the subsections listed below, we will look at each of these three terms in a little more detail.

1.1.1. Logic

Logic is concerned with features that make reasoning good in certain respects.

1.1.2. Inference and arguments

The key form of reasoning that we will consider is inference; the premises and conclusion of an inference make up an ***argument***.

1.1.3. Notation for arguments

We will often use some compact ways of referring to arguments and their components generally without identifying particular examples.

1.1.4. Deductive *vs*. non-deductive inference

An inference is ***deductive*** when its conclusion extracts information already present in its premises, and such an inference is risk free.

1.1.5. Deductive bounds on inference

The sentences that constitute risk-free conclusions and those that are absolutely incompatible with the premises form lower and upper bounds on what can be reasonably concluded.

1.1.6. Entailment, exclusion, and inconsistency

Entailment is the relation between the premises and conclusion of a deductive inference, and the terms of exclusion and inconsistency are tied to the idea of absolute incompatibility.

1.1.7. Formal logic

Many cases of entailment can be captured by generalizations concerning certain linguistic forms, and we will use a quasi-mathematical notation to express these forms.

Several features of the page you are looking at will be reflected throughout the text. A special font (this one) is used to mark language that is being displayed rather than used; the text will frequently use this sort of alternative to quotation marks. Another font (***this one***) is used for special terminology that is being introduced; the index to the text lists these terms and provides links to the points where they are explained. In the list of subsections that appears above, headings have a special formatting (like this) that will be used for links. These are links to the subsections themselves, and cross-references in the text with similar formatting will also function as links.

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1.1.1. Logic

Logic is a study of reasoning. However, it does not concern the ways and means by which people actually reason—as psychology does—but rather the sorts of reasoning that count as good. So, while a psychologist is interested as much in cases where people get things wrong as in cases where they get them right, a logician is interested instead in drawing the line between good and bad reasoning without attempting to explain how cases of either sort come about.

Another way of making this distinction between logic and psychology is to say that, in logic, the point of view on reasoning is *internal*: it is a study "from the inside" in a certain sense. As we study reasoning in this way, we will be interested in the norms of reasoning—the rules that reasoners feel bound by, the ideals they strive to reach—rather than the mixed success we observe when we look from outside on their efforts to put norms of reasoning into practice.

This makes logic much like the study of grammar. A linguist studying the grammar of a language will be interested in the sort of things people actually say, but chiefly as evidence of the ways they think words ought to be put together. So, although linguists do not attempt to lay down the rules of grammar for others and see their task as one of description rather than prescription, what they attempt to describe are the (largely unconscious) rules on the basis of which the speakers of a language judge whether utterances are grammatical.

Indeed, one way of understanding logical norms suggests that there is more than an analogy between logic and the study of language. However ineffable language itself may sometimes seem, it is vastly more concrete than thought and it has always served logicians as a tool in their study of reasoning. In the 20th century it acquired an even greater significance because the traditional view of the relation between thought and language (according to which thought is independent of language and language acquires its significance as the expression of thought) came to be reversed, and thought was seen to derive its significance from the possibility of linguistic expression. As a result, the norms of thought have often been seen to derive from the norms of language, specifically from rules governing certain aspects of meaning. This view is not uncontroversial, but we will see in 1.2 that there is a way of describing the norms of reasoning that makes it quite natural to see them as resting on norms of language.

1.1.2. Inference and arguments

The norms studied in logic can concern many different features of reasoning and we will consider several of these. But the most important one and the one that will receive most of our attention is ***inference***, the action of drawing a ***conclusion*** from certain ***premises*** or ***assumptions***.

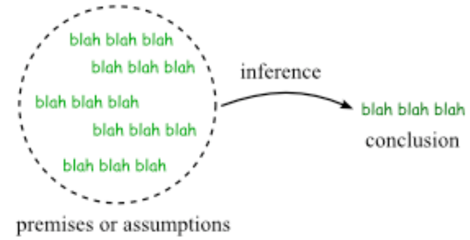

Fig. 1.1.2-1. The action of inference.

Inferences are to be found in science when generalizations are based on data or when a hypothesis is offered to explain some phenomenon. They are also to be found when theorems are proved in mathematics. But the most common case of inference calls less attention to itself. Much of the process of understanding what we hear or read can be seen to involve inference because, when we interpret spoken or written language, our interpretation can usually be formulated as a statement, and we base this statement on statements in the text we interpret.

The terminology we will use to speak of inference deserves some comment. The terms premise and assumption both to refer to the starting points of inference—whether these be observational data, mathematical axioms, or the statements making up something heard or read. The term premise is most appropriate when the claim or claims from which we draw a conclusion are ones that we accept. The term assumption need not carry this suggestion, and we may speak of something being "assumed for the sake of argument." But, in general, we will be far more interested in judging the transition from the starting point of an inference to its conclusion than in judging the soundness of its starting point, so the distinction between premises and assumptions will not have a crucial role for us, and, for the most part, we will use the two terms interchangeably.

If it should seem strange to suppose that you might draw conclusions from claims you do not accept, imagine going over a body of data to check for inconsistencies either within the data or with information from other sources. In this sort of case, you may well draw conclusions from data that you do not accept and, indeed, do this as a way of showing that the data is unacceptable. For example, you might see that the data is unacceptable by seeing that it leads you to draw contradictory conclusions. (Incidentally, although our focus for the moment is on inference, the sort of recognition of unacceptability that inference serves in this kind of example is another aspect of reasoning that we will be studying.)

It is convenient to have a term for a conclusion taken together with the premises or assumptions on which it is based. We will follow tradition and label such a combination of premises and conclusion an ***argument***. A particularly graphic way of writing an argument is to list the premises (in any order) with the conclusion following and separated off by a horizontal line (as shown in Figure 1.1.2-1). The sample argument shown here is a version of a widely used traditional example and has often served as a paradigm of the sort of reasoning studied by deductive logic.

premises	All humans are mortal Socrates is human
conclusion	Socrates is mortal

Fig. 1.1.2-2. The components of an argument.

When we need to represent an argument horizontally, we will use / (***virgule*** or slash) to divide the premises from the conclusion, so the argument above might also be written as All humans are mortal, Socrates is human / Socrates is mortal.

This example serves to emphasize again that the concepts of inference and argument can be applied not only to reasoning from experimental data or mathematical axioms, but to any reasoning where a conclusion is drawn from certain statements. Notice that the information expressed in the conclusion is the result of an interaction between the two premises. In its broadest sense, the traditional term ***syllogism*** (whose etymology might be rendered as 'reckoning together') applies in the first instance to inference that is based on such interaction, and the argument above is a traditional example of a syllogism. The broadest sense of another traditionaly term, ***immediate inference***, covered arguments with a single premise—i.e., arguments in which the conclusion is inferred directly from a premise without the mediation of any further premises.

1.1.3. Notation for arguments

It is useful to have some abstract notation so that we can speak of reasoning generally rather than in specific examples. We will use the lower case Greek letters φ, ψ, and χ to stand for the individual sentences that may appear as the conclusion of an argument or as its premises. And we will use upper case Greek Γ, Σ, and Δ to stand for sets of sentences, such as the set of premises of an argument (or a set of sentences that is rejected as unacceptable). The general form shared by all arguments can then be expressed horizontally as Γ / φ, where Γ is the set of premises and φ is the conclusion.

Although we speak of the premises of an argument as forming a set, in practice what appears above a vertical line or to the left of the sign / will often be a list of sentences, and a symbol like Γ can often best be thought of as standing for such a list. The reason for speaking of sets at all is that we will have no interest in the order of the premises or the number of times a premise appears in the list. We ignore just such features of a list when we move from the list to the set whose members it lists—as we do when we use the notation $\{a_1, a_2, ..., a_n\}$ for a set with members $a_1, a_2, ..., a_n$. This means that we regard two arguments that share a conclusion as the same when their premises constitute the same set. There are other features of sets, however, which are of little use to us. In particular, we have no need to distinguish between a sentence φ and the set {φ} that has φ as its only member, and we will not attempt to preserve the distinction between the two in our notation for arguments.

If we regard the capital Greek letters as standing for lists of sentences, it makes sense to write Γ, φ / ψ to speak of an argument whose premises consist of the members of Γ together with φ. The set of premises of this argument is the ***union*** Γ ∪ {φ} of the sets Γ and {φ}—i.e., it is the set whose members are their members taken together. Since this idea does not exclude the possibility that φ is already a member of Γ, it provides convenient way to refer to any argument whose premises include the sentence φ. We will use an analogous convention in the vertcial notation for arguments. So, if Γ is the set {φ, ψ, χ} (i.e., the set whose members are φ, ψ, and χ) and Σ is the set {ψ, χ}, then all of the following refer to the same argument:

horizontal:	Γ / θ	φ, ψ, χ / θ	ψ, φ, χ, φ / θ	Σ, φ / θ	Γ, φ / θ	φ, Γ / θ	
vertical:	Γ —— θ	φ ψ χ —— θ	ψ φ χ φ —— θ	Σ φ —— θ	Γ φ —— θ	φ Σ —— θ	Γ = {φ, ψ, χ} Σ = {ψ, χ}

Fig. 1.1.3-1. Alternative expressions for the same argument (where Γ is the set whose members are φ, ψ, and χ and Σ is the set whose members are ψ and χ).

The equivalence of the expressions after the first can be traced to the equivalence among the following ways of referring to the set whose members are φ, ψ, and χ:

$$\{\varphi, \psi, \chi\} = \{\psi, \varphi, \chi, \varphi\} = \{\psi, \chi\} \cup \{\varphi\} = \{\varphi, \psi, \chi\} \cup \{\varphi\} = \{\varphi\} \cup \{\psi, \chi\}$$

1.1.4. Deductive *vs.* non-deductive reasoning

Although all good reasoning is of interest to logic, we will focus on reasoning—and, more specifically, on inference—that is good in a special way. To see what this way is, let us begin with a rough distinction between two kinds of reasoning the scientist will typically employ when attempting to account for a body of experimental data.

An example of the first kind is the extraction of information from the data. For instance, the scientist may notice that no one who has had disease A has also had disease B. Even though this conclusion is more than a simple restatement of the data and could well be an important observation, it is closely related to what is already given by the data. It may require perceptiveness to see it, but what is seen does not go beyond the information the data provides. This sort of close tie between a conclusion and the premises on which it is based is characteristic of ***deductive reasoning***. This sort of reasoning appears also in mathematical proof and in some of the inferences we draw in the course of interpreting oral or written language. It is found whenever we draw conclusions that do not go beyond the content of the premises on which they are based and thus introduce no new risk. It is this kind of reasoning that we will study. The traditional name for its study is ***deductive logic***.

Science is not limited to the extraction information from data. There usually is some attempt to go beyond data either to make a generalization that applies to other cases or to offer an explanation of the case at hand. A conclusion of either sort brings us closer to the goals of science than does the mere extraction of information, so an inference that generalizes or explains the data is given more attention. But generalizations and explanations call attention to themselves also because they are risky, and this riskiness distinguishes them from the extraction of information.

There is no very good term—other than ***non-deductive***—for the sort of reasoning involved in inferences where we generalize or offer explanations. The term ***inductive inference*** has been used for some kinds of non-deductive reasoning. But it has often been limited to cases of generalization, and the conclusions of many non-deductive inferences are not naturally stated as generalizations. Although scientific explanations typically employ general laws, they usually employ other sorts of information, too; and other cases of inference to the best explanation of some data—for example, the sort of inferences a detective draws from the evidence at a crime scene—will often focus on conclusions about particular people, things, or events.

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1.1.5. Deductive bounds on reasoning

Let's look at the relations between deductive and non-deductive reasoning a little more closely with the aim to sketch the role of deductive logic. First notice that there is a close tie between the riskiness of an inference and the question of whether is merely extracts information or does something more. The information extracted from data may be no more reliable than the data it is extracted from, but it certainly will be no less reliable. On the other hand, even the generalization or explanatory hypothesis that is most strongly supported by a body of data must go beyond the data if it is to generalize or explain it. And, if it goes beyond what the data says, there is a possibility it is wrong even when the data is completely accurate.

The extraction of information can be a first step towards generalization or inference to an explanation. And we have seen that extracting information does not merely prepare us to go further: it maps out the territory that we can reach without making the leap to a generalization or explanatory hypothesis. That is, deductive logic serves to distinguish safe from risky inferences. And this sets a lower bound for inference by marking the range of conclusions that come for free.

But deductive logic sets bounds for inference in another respect as well. Another aspect of reasoning is the recognition of tension or incompatibility within collections of sentences; and this, too, has a deductive side when the incompatibility is a direct conflict among the informational content of the sentences and there is no chance that the sentences could be all be accurate. This sets a sort of upper bound for inference by marking the range of conclusions that could not be supported by any amount of further research, such as generalizations to which the data provides counterexamples.

These two bounds are depicted in the following diagram.

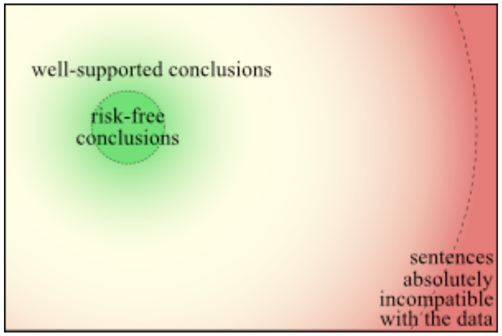

Fig. 1.1.5-1. Deductive bounds on inference.

Sentences in the small circle are the conclusions that are the result of deductive reasoning. They merely extract information and are risk-free and always well-supported. Beyond this circle is a somewhat larger circle with fuzzy boundaries that also includes generalizations and inferences to explanations that are well supported by the data but go beyond it and are at least somewhat risky. Beyond the circle at the right are sentences deductively incompatible with the data. These are claims that cannot be accurate if the data is. The sentences near this circle are not absolutely ruled out by the data but are in real conflict with it.

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1.1.6. Entailment, exclusion, and inconsistency

When the conclusion of an argument merely states information extracted from the premises and is therefore risk free, we will say that it is ***entailed by*** the premises. Using this vocabulary, cases of extraction of information may characterized by a relation of ***entailment*** between the initial data and the information extracted from it. If we speak in terms of arguments, entailment is a relation that may or may not hold between given premises and a conclusion, and we can speak of an argument as having the property of ***validity*** if its premises do entail its conclusion. We will say also that the conclusion of an argument with this property is a ***valid conclusion*** from its premises. Figure 1.1.6-1 summarizes these ways of stating the relation of entailment between a set of premises or assumptions Γ and a conclusion φ.

the assumptions Γ ***entail*** the conclusion φ
the conclusion φ ***is entailed by*** the assumptions Γ
the conclusion φ ***is a valid conclusion from*** the assumptions Γ
the argument Γ / φ ***is valid***

Fig. 1.1.6-1. Several ways of stating a relation of entailment.

We will use the sign ⇒ (***rightwards double arrow***) as shorthand for the verb entails, so we add to the English expressions in Figure 1.1.6-1 the symbolic expression Γ ⇒ φ as a way of saying that the assumptions Γ entail the conclusion φ. Using this sign, we can express the validity of argument in Figure 1.1.2-2 by writing

All humans are mortal, Socrates is human ⇒ Socrates is mortal

and we will sometimes use the sign ⇐ (***leftwards double arrow***) as shorthand for is entailed by. Either way, the entailed conclusion appears next to the head of the arrow, and the assumptions that entail it are next to the tail.

Entailment represents the positive side of deductive reasoning. The negative side is represented by the idea of a statement φ that cannot be accurate when a set Γ of statements are all accurate. In this sort of case, we will say that φ is ***excluded by*** Γ, and we will say that cases of this sort are characterized by the relation of ***exclusion***. We will see later that it is possible to adapt the notation for entailment to express exclusion, so we will not introduce special notation for this relation.

Entailment and exclusion are natural opposites, but the nature of the opposition means that the very different roles of premises and conclusion in entailment are not found when we say that a set Γ excludes a sentence φ. When we say that Γ ⇒ φ, we are saying that there is no chance that φ will fail to be accurate when the members of Γ are all accurate. When we say that Γ excludes φ, we are saying that there is no chance that φ will succeed in being accurate along with the members of Γ. In the latter case, we are really saying that a set consisting of sentence consisting of the members of Γ together with φ cannot be wholely accurate, so it is natural to trace the relation of exclusion to a property of ***inconsistency*** that characterize such sets: we will say that a set of sentences is ***inconsistent*** when its members cannot be jointly accurate. Then to say that φ is excluded by Γ is to say that φ is ***inconsistent with*** Γ in the sense that adding φ to Γ would produce an inconsistent set. The symmetry in the roles of terms in a relation of exclusion is reflected in ordinary ways of expressing this side of deductive reasoning. Difference between saying That hypothesis is inconsistent with our data and Our data is inconsistent with that hypothesis is merely stylistic.

One aspect of the notation we will use for arguments and entailment deserves a final comment. The signs / and $\Rightarrow$ differ not only in their content but also in their grammatical role. A symbolic expression of the form Γ / φ is a noun phrase since it abbreviate the English expression the argument formed of premises Γ and conclusion φ, so it is comparable in this respect to an expression like $x + y$ (which abbreviates the English the sum of x and y). On the other hand, an expression of the form $\Gamma \Rightarrow \varphi$ is a sentence, and it is thus analogous to an expression like $x < y$. In short, $\Rightarrow$ functions as a verb, but the sign / functions as a noun. In Γ / φ, the symbols Γ and φ appear not as subject and object of a verb but as nouns used to specify the reference of a term, much as the names Linden and Crawfordsville do in the term the distance between Linden and Crawfordsville. And the relation between the claims

$$\Gamma \Rightarrow \varphi$$
$$\Gamma / \varphi \text{ is valid}$$

is analogous to the relation between the claims

Linden is close to Crawfordsville
The distance between Linden and Crawfordsville is short

1.1.7. Formal logic

The subject we will study has traditional been given a variety of names. "Deductive logic" is one. Another is ***formal logic***, and this term reflects an important aspect of the way we will study deductive reasoning. Even among the inferences that are deductive, we will consider only ones that do not depend on the *subject matter* of the data. This means that these inferences will not depend on the concepts employed to describe particular subjects, and it also means that they will not depend the mathematical structures (systems of numbers, shapes, etc.) that might be employed in such descriptions. This can be expressed by saying that we will limit ourselves to inferences that depend only on the *form* of the claims involved.

The distinction between form and content is a relative one. For example, the use of numerical methods to extract information can be said to depend on content by comparison with the sort of inferences we will study. However, it can count as formal by comparison with other ways of extracting information since all that matters for much of the numerical analysis of data is the numbers that appear in a body of measurements, not the nature of the quantities measured.

Our study is formal in a similar sense but to a greater degree. What matters for formal logic is the appearance of certain words or grammatical constructions that can be employed in statements concerning any subject matter. Examples of such logical words are and, not, or, if, is (in the sense of is identical to), every, and some. While this list does not include all the logical words we will consider, it does provide a fair indication of the forms of statements we will study. Indeed, these seven words could serve as titles for chapters 2-8, respectively. The way in which a statement is put together using words like these (and using logically significant grammatical constructions not directly marked by words) is its ***logical form***, and formal logic is a study of reasoning that focuses on the logical forms of statements.

So the subject we will study will be not only deductive logic but formal logic. That means that the norms of deductive reasoning that we will study will be general rules applying to all statements with certain logical forms. It happens that we can give an exhaustive account of such rules in the case of the logical forms that we will consider, so the content of the course can be defined by these forms. ***Truth-functional logic***, which will occupy us through chapter 5, is concerned with logical forms that can be expressed using the words and, not, or, and if while ***first-order logic (with identity)*** is concerned with the full list above, adding to truth-functional logic forms that can be expressed by the words is, every, and some.

Another traditional label for the subject we will study is the term ***symbolic logic*** that appears in the course title. Most of what this term indicates about the content of our study is captured already by the term ***formal logic*** because most of the symbols we use will serve to represent logical forms. Certain of the logical forms that appear in the study of truth-functional logic are analogous to patterns appearing in the symbolic statements of algebraic laws. Analogies of this sort were recognized by G. W. Leibniz (1646-1716) and by others after him, but they were first pursued extensively by George Boole (1815-1864), who adopted a notation for logic that was modeled after algebraic notation. The style of symbolic notation that is now standard among logicians owes something to Boole (though the individual symbols are different) and something also to the notation used by Gottlob Frege (1848-1925), who noted analogies between first-order logic and the mathematical theory of functions. This interest in analogies with mathematical theories distinguished logic as studied by Boole and Frege from its more traditional study, and the term symbolic has often been used to capture this distinction. The phrase ***mathematical logic*** would be equally appropriate, and it has often been used as a label for the subject we will study; but this label is also used a little more narrowly for an application of logic to mathematical theories, making them objects of mathematical study in their own right and producing a kind of research that is also known as ***metamathematics*** (which means, roughly, 'the mathematics of mathematics').

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1.1.s. Summary

The following summarizes this section subsection by subsection. Much of the special terminology introduced in the section appears in the summaries with links by to its initial explanation.

1 Logic studies reasoning not to explain actual processes of reasoning but instead to describe the norms of good reasoning.

2 The central focus of our study of logic will be inference. We will refer to the starting points of inference as assumptions or premises and its end as a conclusion. These two aspects of a stretch of reasoning can be referred to jointly as an argument. We will separate them by a horizontal line when they are listed vertically and by the sign / when they are listed horizontally.

3 We use the lower case Greek φ, ψ, and χ to stand for individual sentences and upper case Greek Γ, Σ, and Δ to stand for sets of sentences. Our notation for arguments will not distinguish sets from lists of their members; and, in considering the norms of inference, we will not distinguish between lists of sentences that determine the same set.

4 Inference that merely extracts information from premises or assumptions and thus brings no risk of new error is deductive. Inference that goes beyond the content of the premises to, for example, generalize or explain is then non-deductive. Deductive inference may be distinguished as risk free in the sense that it adds no further chance of error to the data. The study of the norms of deductive inference is deductive logic, and it is topic of this course.

5 Since deductive inferences are risk free, they provide a lower bound on the inferences that are good. Deductive reasoning also sets an upper bound on good inference by rejecting certain conclusions as absolutely incompatible with given premises.

6 The relation between premises and a conclusion that can be deductively inferred from them is entailment. When the premises and conclusion of an argument are related in this way, the argument is said to be valid. Our symbolic notation for this relation are the signs ⇒ and ⇐, where $\Gamma \Rightarrow \varphi$ says that the premises Γ entail the conclusion φ and $\varphi \Leftarrow \Gamma$ says that φ is entailed by Γ. A set of sentences is inconsistent when its members are mutually incompatible, and a sentence φ is excluded by a set Γ when φ and the members of Γ are mutually incompatible.

7 We will be interested in the deductive inferences whose validity is a result of the logical form of their premises and conclusions; so our study will be an example of formal logic. The norms of deductive reasoning based on logical form are analogous to some laws of mathematics. The recognition of these analogies (especially by Boole and Frege) has influenced the development of notation for formal deductive logic over the last two centuries, and logic studied from this perspective is often referred to as symbolic logic.

1.1.x. Exercise questions

1. Some of the following references to arguments refer to the same argument in different ways (remember that changing the order of premises or the number of times a given premise is referred to does not change the argument being referred to). If Γ stands for the sentences φ, χ, θ, what are the different arguments referred to below? Identify each by listing the sentences making up its premises and conclusion, and tell which of the following refer that argument:

a.	φ, ψ, χ / θ	**f.**	φ, θ, ψ, θ / χ
b.	θ, φ, ψ / χ	**g.**	Γ, φ / ψ
c.	χ, φ, ψ / θ	**h.**	Γ / θ
d.	Γ / ψ	**i.**	χ, θ, φ / ψ
e.	Γ, ζ / ψ	**h.**	Γ, ψ / χ

2. The basis for testing a scientific hypothesis can often be presented as an argument whose conclusion is a prediction about the result of the test and whose premises consist of the hypothesis being tested together with certain assumptions about the test (e.g., about the operation of any apparatus being used to perform the test).

$$\begin{array}{rcl} \text{hypothesis to be tested:} & \textit{hypothesis} & \\ \text{assumptions about the test:} & \left\{\begin{array}{c}\textit{assumption}\\ \vdots \\ \textit{assumption}\end{array}\right. & \text{premises} \\ \hline \text{prediction of the test result:} & \textit{prediction} & \text{conclusion} \end{array}$$

Suppose that the prediction is entailed by the hypothesis together with the assumptions about the test (i.e., suppose that the argument shown above is valid) and answer the following questions:

a. Can you conclude that the hypothesis is true on the basis of a successful test (i.e., one for which the prediction is true)? Why or why not?

b. Can you conclude that the hypothesis is false on the basis of an unsuccessful test (i.e., one for which the prediction is false)? Why or why not?

1.1.xa. Exercise answers

1.

	arguments	*references to them*
(1)	φ, χ, ψ / θ	**a**, **c**
(2)	θ, φ, ψ / χ	**b**, **f**
(3)	θ, φ, χ / ψ	**d**, **g**, **i**
(4)	ζ, θ, φ, χ / ψ	**e**
(5)	θ, φ, χ / θ	**h**
(6)	θ, φ, χ, ψ / χ	**j**

2. **a.** Nothing definite can be concluded. The successful test tells you that some true information has been extracted from the hypothesis and auxiliary assumptions. But that can be so even if the hypothesis is not true since a body of information that is not true as a whole can still contain true information. For example, even if the prediction of the result of one test holds true, predictions about other tests may not.

b. You can conclude that the hypothesis is false *provided that the auxiliary assumptions are all true*. The unsuccessful test tells you that a false prediction has been extracted from the hypothesis together with auxiliary assumptions about the test, but this can happen even if the information provided by the hypothesis itself is entirely accurate. The prediction may have failed, for example, because of incorrect assumptions about the way some apparatus would work.