Phi 270 F99 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for each of the first two.

1. Sam mentioned someone Tina didn't know. [Give this analysis also using an unrestricted quantifier.]

answer

2. Every shoe fit someone. [This sentence is ambiguous. Analyze it in two different ways, and describe a situation in which the sentence is true on one of your interpretations and false on the other.]

answer

3. Sam found at least two pieces.

answer

Analyze the sentence below using each of the two ways of analyzing definite descriptions. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The elephant standing on Sam sighed.

answer

[The following question was on a topic not covered in F06] Put the following sentence into prenex normal form (i.e., into a form which contains no restricted quantifiers and in which no quantifier is in the scope of a connective). Show each step where you move a quantifier past a connective separately.

5. $\neg \forall x\, ((Px \wedge \exists y\, Rxy) \rightarrow \exists z\, Sxz)$

answer

Use derivations to show that the following argument is valid. You may use attachment rules (but not replacement by equivalence).

6.

$\forall x\, \forall y\, (Rxy \rightarrow (Ryx \rightarrow Rxx))$

$\exists x\, \exists y\, (Rxy \wedge Ryx)$

$\exists x\, Fxx$

answer

Use a derivation to show that the following argument is not valid and describe a structure dividing an open gap.

7.

$\exists x\, Fx$

$\exists x\, (Gx \wedge Hx)$

$\exists x\, (Fx \wedge Hx)$

answer

Complete the following to give a definition of entailment by a single sentence (i.e., implication) in terms of truth values and possible worlds:

8. A sentence φ entails a sentence ψ if and only if …

answer

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component.

9.

A B C D	¬ (A ∧ B) → (C ∨ ¬ D)
T F F T	

answer

Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the sentences below all true. (You may use either tables or a diagram.)

10. a = fb, fb = fc, fa = c, Pa, Pb, ¬ Pc, Rab, Rbc, Rc(fb)

answer

Phi 270 F99 test 5 answers

1. Sam mentioned someone Tina didn't know

someone Tina didn't know is such that (Sam mentioned him or her)

(∃x: x is a person Tina didn't know) Sam mentioned x

(∃x: x is a person ∧ ¬ Tina knew x) Sam mentioned x

(∃x: Px ∧ ¬ Ktx) Msx

∃x ((Px ∧ ¬ Ktx) ∧ Msx)

K: [_ knew _]; M: [_ mentioned _]; P: [_ is a person]; s: Sam; t: Tina

2. *first analysis:*

Every shoe fit someone

every shoe is such that (it fit someone)

(∀x: x is a shoe) x fit someone

(∀x: Sx) someone is such that (x fit him or her)

(∀x: Sx) (∃y: y is a person) x fit y

(∀x: Sx) (∃y: Py) Fxy

second analysis:

Every shoe fit someone

someone is such that (every shoe fit him or her)

(∃x: x is a person) every shoe fit x

(∃x: Px) every shoe is such that (it fit x)

(∃x: Px) (∀y: y is a shoe) y fit x

(∃x: Px) (∀y: Sy) Fyx

F: [_ fit _]; P: [_ is a person]; S: [_ is a shoe]
The first is true and the second false if every shoe could be worn but not all by the same person

3. Sam found at least two pieces
at least two pieces are such that (Sam found them)
(∃x: x is a piece) (∃y: y is a piece ∧ ¬ y = x) (Sam found x ∧ Sam found y)

$$(\exists\, x\colon Px)\ (\exists\, y\colon Py \wedge \neg\, y = x)\ (Fsx \wedge Fsy)$$

F: [_ found _]; P: [_ is a piece]; s: Sam

4. *using Russell's analysis:*
The elephant standing on Sam sighed
The elephant standing on Sam is such that (it sighed)
(∃x: x and only x is an elephant standing on Sam) x sighed
(∃x: x is an elephant standing on Sam ∧ (∀y: ¬ y = x) ¬ y is an elephant standing on Sam) Sx
(∃x: (x is an elephant ∧ x is standing on Sam) ∧ (∀y: ¬ y = x) ¬ (y is an elephant ∧ y is standing on Sam)) Sx

$$(\exists x\colon (Ex \wedge Txs) \wedge (\forall y\colon \neg\, y = x)\ \neg\, (Ey \wedge Tys))\ Sx$$

or:

$$(\exists x\colon (Ex \wedge Txs) \wedge (\forall y\colon Ey \wedge Tys)\ x = y)\ Sx$$

using the description operator:
The elephant standing on Sam sighed
S (the elephant standing on Sam)
S (ιx x is an elephant standing on Sam)
S (ιx (x is an elephant ∧ x is standing on Sam))

$$S(\iota x\ (Ex \wedge Txs))$$

E: [_ is an elephant]; S: [_ sighed]; T: [_ is standing on _]; s: Sam

5. [The following question was on a topic not covered in F06]

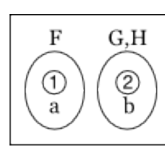
$$\neg\, \forall x\, ((Px \wedge \exists y\, Rxy) \rightarrow \exists z\, Sxz)$$
$$\exists x\, \neg\, ((Px \wedge \exists y\, Rxy) \rightarrow \exists z\, Sxz)$$
$$\exists x\, \neg\, (\exists y\, (Px \wedge Rxy) \rightarrow \exists z\, Sxz)$$
$$\exists x\, \neg\, \forall y\, ((Px \wedge Rxy) \rightarrow \exists z\, Sxz)$$
$$\exists x\, \exists y\, \neg\, ((Px \wedge Rxy) \rightarrow \exists z\, Sxz)$$
$$\exists x\, \exists y\, \neg\, \exists z\, ((Px \wedge Rxy) \rightarrow Sxz)$$
$$\exists x\, \exists y\, \forall z\, \neg\, ((Px \wedge Rxy) \rightarrow Sxz)$$

6.

	∀x ∀y (Rxy → (Ryx → Rxx))	a:4
	∃x ∃y (Rxy ∧ Ryx)	1
	ⓐ	
	∃y (Ray ∧ Rya)	2
	ⓑ	
	Rab ∧ Rba	3
3 Ext	Rab	(6)
3 Ext	Rba	(7)
4 UI	∀y (Ray → (Rya → Raa))	b:5
5 UI	Rab → (Rba → Raa)	6
6 MPP	Rba → Raa	7
7 MPP	Raa	(8)
8 EG	∃x Rxx	X, (9)
	●	
9 QED	∃x Rxx	2
2 PCh	∃x Rxx	1
1 PCh	∃x Rxx	

7.

	∃x Fx	1
	∃x (Gx ∧ Hx)	2
	ⓐ	
	Fa	(7)
	ⓑ	
	Gb ∧ Hb	3
3 Ext	Gb	
3 Ext	Hb	(8)
	∀x ¬ (Fx ∧ Hx)	a:5, b:6
5 UI	¬ (Fa ∧ Ha)	7
6 UI	¬ (Fb ∧ Hb)	8
7 MPT	¬ Ha	
8 MPT	¬ Fb	
	○	Fa,Gb,Hb,¬Ha,¬Fb ⇏ ⊥
	⊥	4
4 NcP	∃x (Fx ∧ Hx)	2
2 PCh	∃x (Fx ∧ Hx)	1
1 PCh	∃x (Fx ∧ Hx)	



8. A sentence φ entails a sentence ψ if and only if there is no possible world in which φ is true but ψ is false (*or:* if and only if ψ is true in every possible world in which φ is true)

9.

A	B	C	D	¬ (A ∧ B) → (C ∨ ¬ D)
T	F	F	T	T F Ⓕ F F

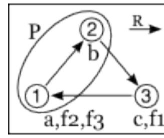
10. range: 1, 2, 3

a	b	c
1	2	3

τ	fτ
1	3
2	1
3	1

τ	Pτ
1	T
2	T
3	F

R	1	2	3
1	F	T	F
2	F	F	T
3	T	F	T



The diagram above provides a complete answer, as do the tables to its left. The tables below illustrate a way of finding this structure.

alias sets	*IDs*	*values*
a	1	a: 1
fb		f2: 1
fc		f3: 1
b	2	b: 2
c	3	c: 3
fa		f1: 3

resources	*values*
Pa	P1: **T**
Pb	P2: **T**
¬ Pc	P3: **F**
Rab	R12: **T**
Rbc	R23: **T**
Rc(fb)	R31: **T**