

### Phi 270 Fo4 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the special instructions given for **1** and **3**.

- 1. Someone was singing** [Present your analysis also using an unrestricted quantifier.]
- 2. There is a package that isn't addressed to anyone.**
- 3. An airline served each airport.** [This sentence is ambiguous. On one way of interpreting it, it could be true even if no one airline served all airports. Analyze the sentence according to that interpretation of it.]
- 4. At least two people called.**

Analyze the sentence below using each of the two ways of analyzing the definite description **the sleigh Santa drove**. That is, give an analysis that uses Russell's treatment of definite descriptions as quantifier phrases and another analysis that uses the description operator.

- 5. The sleigh Santa drove was red.**

Use derivations to show that the following arguments are valid. You may use any rules.

- 6.  $\exists x (Fx \wedge Gx)$**

$\exists x Gx$

- 7.  $\exists x (Fx \wedge \exists y Rxy)$**

$\exists x \exists y (Fy \wedge Ryx)$

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

- 8. A sentence  $\phi$  is entailed by a set  $\Gamma$  (i.e.,  $\Gamma \Rightarrow \phi$ ) if and only if ...**

Complete the following truth table for the two rows shown. Indicate the value of each component of the sentence on the right by writing the value under the main connective of that component.

- 9. 

A	B	C	D
T	T	F	F
F	F	T	F

 $\neg (A \wedge B) \rightarrow (\neg C \vee D)$**

T T F F

F F T F

Use either tables or a diagram to describe a structure in which the following sentences are true. (That is, do what would be required to present a counterexample when a dead-end gap of a derivation had these sentences as its active resources.)

10.  $a = c, fa = fb, \neg Ga, Gb, G(fc), Ra(fb), Rb(fa)$

answer

### Phi 270 Fo4 test 5 answers

1. **Someone was singing**  
**Someone is such that (he or she was singing)**  
 $(\exists x: x \text{ is a person}) x \text{ was singing}$   
 $(\exists x: Px) Sx$   
 $\exists x (Px \wedge Sx)$   
 P: [ \_ is a person ]; S: [ \_ was singing ]
2. **There is a package that isn't addressed to anyone**  
**Something is a package that isn't addressed to anyone**  
 $\exists x x \text{ is a package that isn't addressed to anyone}$   
 $\exists x (x \text{ is a package} \wedge x \text{ isn't addressed to anyone})$   
 $\exists x (Kx \wedge \neg x \text{ is addressed to someone})$   
 $\exists x (Kx \wedge \neg \text{someone is such that } (x \text{ is addressed to him or her}))$   
 $\exists x (Kx \wedge \neg (\exists y: y \text{ is a person}) x \text{ is addressed to } y)$   
 $\exists x (Kx \wedge \neg (\exists y: Py) Axy)$   
**or:**  $\exists x (Kx \wedge (\forall y: Py) \neg Axy)$   
 A: [ \_ is addressed to \_ ]; K: [ \_ is a package ]; P: [ \_ is a person ]
3. **An airline served each airport**  
**Every airport is such that (an airline served it)**  
 $(\forall x: x \text{ is an airport}) \text{ an airline served } x$   
 $(\forall x: Ax) \text{ some airline is such that (it served } x)$   
 $(\forall x: Ax) (\exists y: y \text{ is an airline}) y \text{ served } x$   
 $(\forall x: Ax) (\exists y: Ly) Syx$   
 P: [ \_ is an airport ]; L: [ \_ is an airline ]; S: [ \_ served \_ ]  
 $(\exists x: Lx) (\forall y: Ay) Sxy$  would be incorrect since it is true only if there is a single airline that serves all airports
4. **At least two people called**  
**At least two people are such that (they called)**  
 $(\exists x: x \text{ is a person}) (\exists y: y \text{ is a person} \wedge \neg y = x) (x \text{ called} \wedge y \text{ called})$   
 $(\exists x: Px) (\exists y: Py \wedge \neg y = x) (Cx \wedge Cy)$   
 C: [ \_ called ]; P: [ \_ is a person ]

5. Using Russell's analysis:

The sleigh Santa drove was red

The sleigh Santa drove is such that (it was red)

$(\exists x: x \text{ is a sleigh Santa drove} \wedge (\forall y: \neg y = x) \neg y \text{ is a sleigh Santa drove}) x \text{ was red}$

$(\exists x: (x \text{ is a sleigh} \wedge \text{Santa drove } x) \wedge (\forall y: \neg y = x) \neg (y \text{ is a sleigh} \wedge \text{Santa drove } y)) x \text{ was red}$

$(\exists x: (Sx \wedge Dsx) \wedge (\forall y: \neg y = x) \neg (Sy \wedge Dsy)) Rx$

Using the description operator:

The sleigh Santa drove was red

R (the thing such that (it is a sleigh Santa drove))

R (lx x is a sleigh Santa drove)

R (lx (x is a sleigh  $\wedge$  Santa drove x))

R(lx (Sx  $\wedge$  Dsx))

D: [ \_ drove \_ ]; R: [ \_ was red ]; S: [ \_ is a sleigh ]; s: Santa

6.

<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>\exists x (Fx \wedge Gx)</math></td> <td style="padding: 5px;">1</td> <td style="padding: 5px;"><b>or</b></td> <td style="border-right: 1px solid black; padding: 5px;"><math>\exists x (Fx \wedge Gx)</math></td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Ⓐ</td> <td></td> <td></td> <td style="border-right: 1px solid black; padding: 5px;">Ⓐ</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Fa <math>\wedge</math> Ga</td> <td style="padding: 5px;">2</td> <td></td> <td style="border-right: 1px solid black; padding: 5px;">Fa <math>\wedge</math> Ga</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">2 Ext</td> <td></td> <td></td> <td style="border-right: 1px solid black; padding: 5px;">2 Ext</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; 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8. A sentence  $\phi$  is entailed by a set  $\Gamma$  if and only if there is no possible world in which  $\phi$  is false while all members of  $\Gamma$  are true

**or**

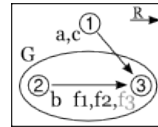
A sentence  $\phi$  is entailed by a set  $\Gamma$  if and only  $\phi$  is true in every possible world in which all members of  $\Gamma$  are true

9. 

A	B	C	D	$\neg (A \wedge B) \rightarrow (\neg C \vee D)$
T	T	F	F	F T <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">T</span> T T
F	F	T	F	T F <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">F</span> F F

10. range: 1, 2, 3

a	b	c	$\tau$	$f\tau$	$\tau$	$G\tau$	R	1	2	3
1	2	1	1	3	1	F	1	FFT	2	FFT
2	3	3	2	T	2	T	2	FFF	3	FFF
3	3	3	3	T	3	T	3	FFF		



(The diagram provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

<i>alias sets</i>	<i>IDs</i>	<i>values</i>	<i>resources</i>	<i>values</i>
a	1	a: 1	$\neg Ga$	G1: F
c		c: 1	Gb	G2: T
b	2	b: 2	G(fc)	G3: T
fa	3	f1: 3	Ra(fb)	R13: T
fb		f2: 3	Rb(fa)	R23: T
fc		f1: 3		