Phi 270 F02 test 5

Analyze the following sentences in as much detail as possible, providing a key to the non-logical vocabulary (upper and lower case letters) appearing in your answer. Notice the additional instructions given for the first.

1. Al received a card that made him laugh [Give this analysis also using an unrestricted quantifier.]
   answer

2. There is a toy that every child wanted
   answer

3. Santa left at least two packages
   answer

Analyze the sentence below using each of the two ways of analyzing the definite description the battery. That is, analyze it using Russell's analysis of definite descriptions as quantifier phrases and then analyze it again using the description operator.

4. The battery is dead
   answer

Use derivations to show that the following argument is valid. You may use any rules.

5. ∃x (Fx ∧ Gx)
   ∃x (Gx ∧ Fx)
   answer

Use a derivation to show that the following argument is not valid and use either tables or a diagram to describe a structure dividing an open gap.

6. ∃x ∃y Rxy
   ∃x Rax
   answer

Complete the following to give a definition of entailment in terms of truth values and possible worlds:

7. A set Γ entails a sentence φ (i.e., Γ ⇒ φ) if and only if ...
   answer

Complete the following truth table by calculating the truth value of the sentence on the given assignment. Show the value of each component by writing it under the main connective of that component, and circle the truth value of the sentence as a whole.

8. A B C D │ (A → B) ∧ ¬ (C ∨ ¬ D)
   T F F T │
   answer

Give at least two restatements of the following sentence as an expansion on a term appearing in it (i.e., as an abstract applied to such a term):

9. Raba

⬚answer⬚

Phi 270 F02 test 5 answers

1. Al received a card that made him laugh
some card that made Al laugh is such that (Al received it)
(∃x: x is a card that made Al laugh) Al received x
(∃x: x is a card ∧ x made Al laugh) Rax

$$(∃x: Cx ∧ Lxa)\ Rax$$
$$∃x\,((Cx ∧ Lxa) ∧ Rax)$$

C: [_ is a card]; L: [_ made _ laugh]; R: [_ received _]; a: Al

2. There is a toy that every child wanted
Something is a toy that every child wanted
Something is such that (it is a toy that every child wanted)
∃x x is a toy that every child wanted
∃x (x is a toy ∧ every child wanted x)
∃x (Tx ∧ every child is such that (he or she wanted x))
∃x (Tx ∧ (∀y: y is a child) y wanted x)

$$∃x\,(Tx ∧ (∀y: Cy)\ Wyx)$$

C: [_ is a child]; T: [_ is a toy]; W: [_ wanted _]

3. Santa left at least two packages
at least two packages are such that (Santa left them)
(∃x: x is a package) (∃y: y is a package ∧ ¬ y = x) (Santa left
  x ∧ Santa left y)

$$(∃x: Px) (∃y: Py ∧ ¬ y = x) (Lsx ∧ Lsy)$$

L: [_ left _]; P: [_ is a package]; s: Santa

4. *using Russell's analysis:*
The battery is dead
The battery is such that (it is dead)
(∃x: x and only x is a battery) x is dead
(∃x: x is a battery ∧ (∀y: ¬ y = x) ¬ y is a battery) x is dead

$$(∃x: Bx ∧ (∀y: ¬ y = x) ¬ By)\ Dx$$
or:
$$(∃x: Bx ∧ (∀y: By)\ x = y)\ Dx$$

B: [_ is a battery]; D: [_ is dead]

using the description operator:

The battery is dead
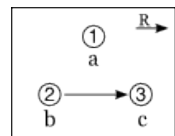
D the battery

D(ɩx x is a battery)

$\qquad$ D(ɩx Bx)

5.

		∃x (Fx ∧ Gx)	1
	ⓐ		
		Fa ∧ Ga	2
2 Ext		Fa	(6)
2 Ext		Ga	(5)
		∀x ¬ (Gx ∧ Fx)	a:4
4 UI		¬ (Ga ∧ Fa)	5
5 MPT		¬ Fa	(6)
		•	
6 Nc		⊥	3
3 NcP		∃x (Gx ∧ Fx)	1
1 PCh		∃x (Gx ∧ Fx)	

6.

		∃x ∃y Rxy	1
	ⓑ		
		∃y Rby	2
	ⓒ		
		Rbc	
		∀x ¬ Rax	a:4, b:5, c:6
4 UI		¬ Raa	
5 UI		¬ Rab	
6 UI		¬ Rac	
		○	Rbc,¬Raa, ¬Rab,¬Rac ⇏ ⊥
		⊥	3
3 NcP		∃x Rax	2
2 PCh		∃x Rax	1
1 PCh		∃x Rax	

7. A set Γ entails a sentence φ if and only if there is no possible world

in which every member of Γ is true but φ is false (*or:* if and only if φ is true in every possible world in which all members of Γ are true)

8.

A	B	C	D	(A → B)	∧	¬	(C	∨	¬	D)
T	F	F	T	F	Ⓕ	T		F		F

9. Up to the choice of variables, the possibilities are the following:

$$[\text{Rabx}]_x a, \ [\text{Rxba}]_x a, \ [\text{Rxbx}]_x a, \ [\text{Raxa}]_x b$$