

## Phi 270 F02 test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

1. Only bears performed.

answer

2. If everyone cheered, the elephant bowed. [In this case, restate your answer using an unrestricted quantifier.]

answer

3. No one laughed at any performers except clowns.

answer

Synthesize an English sentence with the following logical form:

4.  $(\forall x: Px \wedge Cxt) Ctx$

C: [ \_ called \_ ]; P: [ \_ is a person ]; t: Tom

answer

Use derivations to establish the validity of the following arguments. You may use attachment rules.

5.  $\forall x Fx$

$\forall x \neg (Fx \wedge Gx)$

—————  
 $\forall x \neg Gx$

answer

6.  $\forall x \forall y (Fy \rightarrow Rxy)$

—————  
 $\forall x (Fx \rightarrow \forall y Ryx)$

answer

Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

7.  $\forall x Rax$

$\forall x (Rbx \rightarrow \neg Rxa)$

—————  
 $\forall x \neg Rbx$

answer

---

## Phi 270 F02 test 4 answers

1. Only bears performed

$(\forall x: \neg x \text{ is a bear}) \neg x \text{ performed}$

$(\forall x: \neg Bx) \neg Px$

B: [ \_ is a bear ]; P: [ \_ performed ]

2. If everyone cheered, the elephant bowed  
 everyone cheered  $\rightarrow$  the elephant bowed  
 $(\forall x: x \text{ is a person}) x \text{ cheered} \rightarrow \text{the elephant bowed}$   
 $(\forall x: Px) Cx \rightarrow Be$   
 $\forall x (Px \rightarrow Cx) \rightarrow Be$

B: x bowed; C: x cheered; P: x is a person; e: the elephant

Incorrect:

$(\forall x: Px) (Cx \rightarrow Be)$  or:  $\forall x (Px \rightarrow (Cx \rightarrow Be))$

these say: If anyone cheered, the elephant bowed

3. No one laughed at any performers except clowns  
 all performers except clowns are such that (no one laughed at them)

$(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown}) \text{ no one laughed at } x$

$(\forall x: x \text{ is a performer} \wedge \neg x \text{ is a clown}) (\forall y: y \text{ is a person}) \neg y$   
 laughed at x

$(\forall x: Fx \wedge \neg Cx) (\forall y: Py) \neg Lyx$

C: [ \_ is a clown]; F: [ \_ is a performer]; P: [ \_ is a person]; L: [ \_  
 laughed at \_ ]

Incorrect:

$(\forall y: Py) \neg (\forall x: Fx \wedge \neg Cx) Lyx$

says: No one laughed at all performers who weren't clowns

4.  $(\forall x: x \text{ is a person} \wedge x \text{ called Tom}) \text{ Tom called } x$   
 $(\forall x: x \text{ is a person who called Tom}) \text{ Tom called } x$   
 everyone who called Tom is such that (Tom called him or her)  
 Tom called everyone who called him

5.	$\forall x Fx$	a:2
	$\forall x \neg (Fx \wedge Gx)$	a:3
	⊖	
	ⓐ	
2 UI	Fa	(4)
3 UI	$\neg (Fa \wedge Ga)$	4
4 MPT	$\neg Ga$	(5)
	●	
	⊖	
5 QED	$\neg Ga$	1
	⊖	
1 UG	$\forall x \neg Gx$	

6.	$\forall x \forall y (Fy \rightarrow Rxy)$	b:4
	<div style="border-left: 1px solid black; padding-left: 5px;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">a</span> </div>	
	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa</math> </div>	(6)
	<div style="border-left: 1px solid black; padding-left: 5px;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">b</span> </div>	
4 UI	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y (Fy \rightarrow Rby)</math> </div>	a:5
5 UI	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa \rightarrow Rba</math> </div>	6
6 MPP	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rba</math> </div>	(7)
	<div style="border-left: 1px solid black; padding-left: 5px;"> <p style="text-align: center;">•</p> </div>	
7 QED	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rba</math> </div>	3
3 UG	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y Rya</math> </div>	2
2 CP	<div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa \rightarrow \forall y Rya</math> </div>	1
1 UG	$\forall x (Fx \rightarrow \forall y Ryx)$	

7.  $\forall x Rax$  a:3,b:4,c:5  
 $\forall x (Rbx \rightarrow \neg Rxa)$  c:6,a:8,b:10

	⊙			
		Rbc		(7)
3 UI		Raa		(9)
4 UI		Rab		
5 UI		Rac		
6 UI		$Rbc \rightarrow \neg Rca$		7
7 MPP		$\neg Rca$		
8 UI		$Rba \rightarrow \neg Raa$		9
9 MTT		$\neg Rba$		
10 UI		$Rbb \rightarrow \neg Rba$		11
		$\neg Rbb$		
		○		$Rbc, Raa, Rab, Rac, \neg Rca, \neg Rba, \neg Rbb \Rightarrow \perp$
		⊥		12
12 IP		Rbb		11
		$\neg Rba$		
		○		$Rbc, Raa, Rab, Rac, \neg Rca, \neg Rba \Rightarrow \perp$
		⊥		11
11 RC		⊥		2
2 RAA		$\neg Rbc$		1
1 UG		$\forall x \neg Rbx$		

Counterexample presented by tables

range: 1, 2, 3	a b c	R	1	2	3
	1 2 3	1	T	T	T
		2	F	F	T
		3	F	F	F

Grayed values are not required to divide either gap;  
the value for R22 is not required to divide the 2nd gap

Counterexample presented by a diagram

