

Phi 270 Foo test 4

Analyze the sentences below in as much detail as possible, providing a key to the non-logical vocabulary you use. *Notice the special instructions for 2.*

1. **Only necessary projects were funded.** [Different interpretations of the scope of *only* are possible here; any of them will do.]

answer

2. **Tom can solve the puzzle if anyone can.** [In this case, restate your answer using an unrestricted quantifier.]

answer

3. **No one received every vote**

answer

Use derivations to establish the validity of the following arguments. You may use attachment rules. English interpretations are suggested but remember that they play no role in derivations, and don't hesitate to ignore them if they don't help you think about the derivations.

4. $\forall x (Dx \rightarrow Mx)$
 $\forall x (\neg Ax \rightarrow \neg Mx)$

 $\forall x (Dx \rightarrow Ax)$

answer

A: [is an animal]; D: [is dog]; M: [is a mammal]

5. $\forall x \forall y ((Py \wedge Byx) \rightarrow Dyx)$

 $\forall x (Px \rightarrow \forall y (Bxy \rightarrow Dxy))$

answer

Everyone who has built anything is proud of it / Everyone is proud of everything he or she has built

6. Use a derivation to show that the following argument is not valid and describe a structure (by using either a diagram or tables) that divides one of the derivation's open gaps.

 $\forall x (Rxx \rightarrow \neg Fx)$ $\forall x Rxc$

 $\forall x \forall y (Fy \rightarrow \neg Rxy)$

answer

Phi 270 Foo test 4 answers

1. Only necessary projects were funded
 $(\forall x: \neg x \text{ was a necessary project}) \rightarrow x \text{ was funded}$
 $(\forall x: \neg (x \text{ was a project} \wedge x \text{ was necessary})) \rightarrow x \text{ was funded}$
 $(\forall x: \neg (Px \wedge Nx)) \rightarrow Fx$
 or: $(\forall x: Px \wedge \neg Nx) \rightarrow Fx$ —i.e., No unnecessary projects were funded;
 or: $(\forall x: Nx \wedge \neg Px) \rightarrow Fx$ —i.e., Among the necessities only projects were funded
 F: [_ was funded]; N: [_ was necessary]; P: [_ was a project]
2. Tom can solve the puzzle if anyone can
 $(\forall x: x \text{ is a person})$ Tom can solve the puzzle if x can
 $(\forall x: Px)$ (Tom can solve the puzzle \leftarrow x can solve the puzzle)
 $(\forall x: Px)$ (S Tom the puzzle \leftarrow S x the puzzle)
 $(\forall x: Px)$ (Stp \leftarrow Sxp) [or: $(\forall x: Px)$ (Sxp \rightarrow Stp)]
 $\forall x (Px \rightarrow (Stp \leftarrow Sxp))$ [or: $\forall x (Px \rightarrow (Sxp \rightarrow Stp))$]
 P: [_ is a person]; S: [_ can solve _]; p: the puzzle; t: Tom
3. No one received every vote
 $(\forall x: x \text{ is a person}) \rightarrow x \text{ received every vote}$
 $(\forall x: Px) \rightarrow x \text{ received every vote}$
 $(\forall x: Px) \rightarrow (\forall y: y \text{ is a vote}) x \text{ received } y$
 $(\forall x: Px) \rightarrow (\forall y: Vy) Rxy$
 P: [_ is a person]; R: [_ received _]; V: [_ is a vote]
 Incorrect answers:
 $(\forall x: Px) (\forall y: Vy) \rightarrow Rxy$ says No one received any vote
 $\neg (\forall x: Px) (\forall y: Vy) Rxy$ says Not everyone received every vote
 $(\forall y: Vy) \rightarrow (\forall x: Px) Rxy$ says No vote is such that everyone received it
- 4.
- | | | |
|---|---|-----|
| $\forall x (Dx \rightarrow Mx)$ | $\forall x (Dx \rightarrow Mx)$ | a:3 |
| $\forall x (\neg Ax \rightarrow \neg Mx)$ | $\forall x (\neg Ax \rightarrow \neg Mx)$ | a:5 |
| | ⊖ | |
| | ⓐ | |
| | Da | (4) |
| | — | |
| 3 UI | Da \rightarrow Ma | 4 |
| 4 MPP | Ma | (6) |
| 5 UI | $\neg Aa \rightarrow \neg Ma$ | 6 |
| 6 MTT | Aa | (7) |
| | • | |
| | — | |
| 7 QED | Aa | 2 |
| | — | |
| 2 CP | Da \rightarrow Aa | 1 |
| | — | |
| 1 UG | $\forall x (Dx \rightarrow Ax)$ | |

5.

	$\forall x \forall y ((Py \wedge Byx) \rightarrow Dyx)$	b:5
	<div style="border-left: 1px solid black; padding-left: 5px;"> (a) <div style="border-left: 1px solid black; padding-left: 5px;">Pa</div> </div>	(9)
	<div style="border-left: 1px solid black; padding-left: 5px;"> (b) <div style="border-left: 1px solid black; padding-left: 5px;">Bab</div> </div>	(10)
5 UI	$\forall y ((Py \wedge Byb) \rightarrow Dyb)$	a:6
6 UI	$(Pa \wedge Bab) \rightarrow Dab$	8
	<div style="border-left: 1px solid black; padding-left: 5px;">$\neg Dab$</div>	(8)
8 MTT	$\neg (Pa \wedge Bab)$	9
9 MPT	$\neg Bab$	(10)
10 Nc	\perp	7
7 IP	Dab	4
4 CP	$Bab \rightarrow Dab$	3
3 UG	$\forall y (Bay \rightarrow Day)$	2
2 CP	$Pa \rightarrow \forall y (Bay \rightarrow Day)$	1
1 UG	$\forall x (Px \rightarrow \forall y (Bxy \rightarrow Dxy))$	

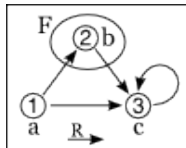
[This can be done without the *reductio* argument begun at stage 7 by using Adj to derive $Pa \wedge Bab$ in order to exploit $(Pa \wedge Bab) \rightarrow Dab$ for a]

6.

$\forall x (Rxx \rightarrow \neg Fx)$
 $\forall x Rxc$

b:4, c:9, a:11
 a:6, b:7, c:8

	(a)		
	(b)	Fb	(5)
4 UI		Rbb \rightarrow \neg Fb	5
5 MTT		\neg Rbb	
6 UI		Rac	
7 UI		Rbc	
8 UI		Rcc	(10)
9 UI		Rcc \rightarrow \neg Fc	10
10 MPP		\neg Fc	
11 UI		Raa \rightarrow \neg Fa	13
		Rab	
		\neg Raa	
		○	Fb, \neg Rbb, Rac, Rbc, Rcc, \neg Fc, Rab, \neg Raa $\Rightarrow \perp$
		⊥	14
14 IP		Raa	13
		\neg Fa	
		○	Fb, \neg Rbb, Rac, Rbc, Rcc, \neg Fc, Rab, \neg Fa $\Rightarrow \perp$
		⊥	13
13 RC		⊥	12
12 RAA		\neg Rab	3
3 CP		Fb \rightarrow \neg Rab	2
2 UG		$\forall y (Fy \rightarrow \neg Ray)$	1
1 UG		$\forall x \forall y (Fy \rightarrow \neg Rxy)$	



divides both open gaps