Phi 270 F98 test 3

(questions 1-6 are from quiz 3 and 7-10 are from quiz 4 out of 6 quizzes—these two quizzes addressed the part of the course your test is designed to cover)

Analyze the sentences below in as much detail as possible *without* going below the level of sentences (i.e., without recognizing individual terms and predicates). Be sure that the unanalyzed components of your answer are complete and independent sentences and that you respect any grouping in the English. You may use right-to-left arrows to reflect English word order but you should then also restate your symbolic analysis with arrows running left to right and, in any case, you should restate it using English notation.

1. If our message got there, they should be on their way

answer

2. Unless we make reservations, we'll get a table only if it is a slow night

answer

3. Check the following for validity using derivations; you *may use* attachment rules and detachment rules. If the derivation fails, present a counterexample that divides the premises from the conclusion.

$A \rightarrow (B \rightarrow (C \vee D))$

$\neg C \rightarrow (A \rightarrow \neg B)$

answer

4. [This question was on a topic not covered in F06] Use replacement by equivalence to put the following sentence into disjunctive normal form. Show how you reach your result; you may combine uses of associativity and commutativity with other principles in a single step but there should be no more than one use of De Morgan's laws or distributivity in each step.

$$\neg ((A \vee \neg B) \wedge (C \wedge A))$$

answer

5. Analyze the sentence below in as much detail as possible, continuing the analysis when there are no more connectives by identifying predicates, functors, and individual terms. Be sure that the unanalyzed expressions in your answer are independent and that you respect any grouping in the English. (You need not state the result in English notation.)

If Sam is the winner of the trip, then the winner of the grand prize presented it to him

answer

6. Give two different expansions (using predicate abstracts) of the sentence below as a one-place predicate applied to a term:

$$Pb \wedge Rab$$

answer

7. Draw a diagram which presents the same interpretation as the following tables:

range: 1, 2, 3

a	c	g
2	3	2

τ	Fτ
1	T
2	F
3	T

τ	Gτ
1	F
2	T
3	T

R	1	2	3
1	T	F	T
2	T	F	F
3	F	T	T

answer

8. Describe a structure (i.e., an assignment of extensions to the non-logical vocabulary) which makes the following sentences all true. (You may present the structure either using tables or, were possible, using diagrams.)

$$fa = b,\ b = c,\ Pb,\ \neg\ Pa,\ Ra(fa),\ R(fb)(fc),\ \neg\ Rbc$$

answer

Check each of the arguments below for validity using derivations. You need *not* present counterexamples to gaps that reach dead ends.

9.

$fa = c$

Rbc

$a = b \to Ra(fa)$

answer

10.

$Rab \vee Rcb$

$a = b \wedge gb = gc$

$Rbc \to Rcb$

answer

Phi 270 F98 test 3 answers

1. If our message got there, they should be on their way

our message got there → they should be on their way

$$M \to W$$

if M then W

M: our message got there; W: they should be on their way

2. ¬ we will make reservations → we'll get a table only if it is a slow night

¬ we will make reservations → (¬ we'll get a table ← ¬ it will be a slow night)

$$\neg R \rightarrow (\neg T \leftarrow \neg S) \text{ or: } \neg R \rightarrow (\neg S \rightarrow \neg T)$$

if not R then if not S then not T

R: we will make reservations; S: it will be a slow night; T: we'll get a table

3.

```
        │A → (B → (C ∨ D))        4
        ├─
        │ │¬ C                    (6)
        │ ├─
        │ │ │A                    (4)
        │ │ ├─
        │ │ │ │B                  (5)
        │ │ │ ├─
4 MPP   │ │ │ │B → (C ∨ D)        5
5 MPP   │ │ │ │C ∨ D              6
6 MTP   │ │ │ │D
        │ │ │ │○                  A, B, ¬ C, D ⇏ ⊥
        │ │ │ ├─
        │ │ │ │⊥
        │ │ ├─
3 RAA   │ │ │¬ B                  2
        │ ├─
2 CP    │ │A → ¬ B                1
        ├─
1 CP    │¬ C → (A → ¬ B)
```

A	B	C	D	A	→	(B	→	(C ∨ D))	/	¬	C	→	(A	→	¬ B)
T	T	F	T		Ⓣ		T	T		T		Ⓕ		F	F

4. [This question was on a topic not covered in F06]

$$\neg ((A \vee \neg B) \wedge (C \wedge A))$$
$$\Leftrightarrow$$
$$\neg (A \vee \neg B) \vee \neg (C \wedge A)$$
$$\Leftrightarrow$$
$$(\neg A \wedge B) \vee \neg (C \wedge A)$$
$$\Leftrightarrow$$
$$(\neg A \wedge B) \vee \neg C \vee \neg A$$

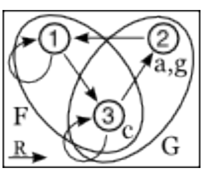
[However, that problem was a typo;
I had really intended something along these lines:]

$$\neg ((A \vee \neg B) \vee (C \wedge \neg A))$$
$$\Leftrightarrow$$
$$\neg (A \vee \neg B) \wedge \neg (C \wedge \neg A)$$
$$\Leftrightarrow$$
$$(\neg A \wedge B) \wedge \neg (C \wedge \neg A)$$
$$\Leftrightarrow$$
$$(\neg A \wedge B) \wedge (\neg C \vee A)$$
$$\Leftrightarrow$$

(¬ A ∧ B ∧ ¬ C) ∨ (¬ A ∧ B ∧ A)
[which could, but need not, be continued as follows:
⇔
(¬ A ∧ B ∧ ¬ C) ∨ (¬ A ∧ A)
⇔
¬ A ∧ B ∧ ¬ C]

5. If Sam is the winner of the trip, then the winner of the grand prize presented it to him
Sam is the winner of the trip → the winner of the grand prize presented the trip to Sam
s = the winner of the trip → [_ presented _ to _]the winner of the grand prize the trip Sam
s = [the winner of _] the trip → P(the winner of the grand prize)ts
s = nt → P([the winner of _] the grand prize)ts
s = nt → P(ng)ts
P: [_ presented _ to _]; g: the grand prize; n: [the winner of _]; s: Sam; t: the trip

6. The following are the possibilities; in the last, τ may be any term:
$[Pb \wedge Rxb]_x a$, $[Px \wedge Rab]_x b$, $[Pb \wedge Rax]_x b$, $[Px \wedge Rax]_x b$,
$[Pb \wedge Rab]_x \tau$

7.


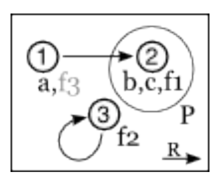
8. range: 1, 2, 3

a	b	c
1	2	2

τ	fτ
1	2
2	3
3	1

τ	Pτ
1	F
2	T
3	F

R	1	2	3
1	F	T	F
2	F	F	F
3	F	F	T



(The diagram above provides a complete answer, and so do the tables to its left. The tables below show a way of arriving at these answers.)

alias sets	*IDs*	*values*
a	1	a: 1
fa	2	f1: 2
b		b: 2
c		c: 2
fb	3	f2: 3
fc		f2: 3

resources	*values*
Pb	P2: **T**
¬ Pa	P1: **F**
Ra(fa)	R12: **T**
R(fb)(fc)	R33: **T**
¬ Rbc	R22: **F**

9.

	fa = c	a, b, fa-c, fb
	Rbc	(2)
	a = b	a-b,fa-fb-c
	•	
2 QED=	Ra(fa)	1
1 CP	a = b → Ra(fa)	

10.

	Rab ∨ Rcb	4
	a = b ∧ gb = gc	2
	Rbc	
2 Ext	a = b	a-b,c,gb,gc
2 Ext	gb = gc	a-b,c,gb-gc
	¬ Rcb	(4)
4 MTP	Rab	
	○	a=b,gb=gc,Rbc,¬Rcb,Rab ⇏ ⊥
	⊥	3
3 CP	Rcb	1
1 CP	Rbc → Rcb	