## Phi 270 Foo test 2

1.   Define (logical) relative inconsistency by completing the following: φ is inconsistent with Γ (i.e., Γ, φ ⇒ ) if and only if ... . (Your answer need not replicate the wording of the text's definitions, but it should define equivalence in terms of truth values and possible worlds.)

   answer

Analyze the sentences below in as much detail as possible, presenting the result in both symbolic and English notation. Be sure that the unanalyzed components of your answer are complete and independent sentences; also try to respect any grouping in the English.

2.   Sam didn't eat his cake and keep it, too, but he wasn't disappointed.

   answer

3.   Either the intruder woke neither the cat nor the dog or it was someone they both knew.

   answer

Use derivations to check whether each of the entailments below holds. You may use detachment and attachment rules. If an entailment fails, present a counterexample that divides an open gap.

4.   C ∧ D ⇒ ¬ (B ∧ ¬ C)

   answer

5.   (A ∧ C) ∨ (B ∧ D) ⇒ B ∨ C

   answer

6.   ¬ (A ∨ B), A ∨ D, ¬ (C ∧ D) ⇒ C

   answer

7.   [This question was on a topic not covered in F06] Use replacement principles to put the following sentence into disjunctive normal form (in which there are no negated compounds and no conjunction has a disjunction as a component):

   A ∧ ¬ (B ∧ ¬ C)

   answer

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## Phi 270 Foo test 2 answers

1.   Γ, φ ⇒ if and only if there is no possible world in which φ is true along with and all members of Γ.

**2.** Sam didn't eat his cake and keep it ∧ Sam wasn't disappointed

¬ Sam ate his cake and kept it ∧ Sam wasn't disappointed

¬ (Sam ate his cake ∧ Sam kept his cake) ∧ ¬ Sam was disappointed

$$\neg\,(A \wedge K) \wedge \neg\,D$$

both not both J and S and not D

A: Sam ate his cake; D: Sam was disappointed; K: Sam kept his cake

**3.** the intruder woke neither the cat nor the dog ∨ the intruder was someone the cat and the dog both knew

¬ the intruder woke either the cat or the dog ∨ (the intruder was someone the cat knew ∧ the intruder was someone the dog knew)

¬ (the intruder woke the cat ∨ the intruder woke the dog ∨ (the intruder was someone the cat knew ∧ the intruder was someone the dog knew)

$$\neg\,(C \vee D) \vee (K \wedge N)$$

either not either C or D or both K and N

C: the intruder woke the cat; D:the intruder woke the dog; K: the intruder was someone the cat knew; N: the intruder was someone the dog knew

**4.**

| | | |
|---|---|---|
| | C ∧ D | 2 |
| | B ∧ ¬ C | 3 |
| 2 Ext | C | (4) |
| 2 Ext | D | |
| 3 Ext | B | |
| 3 Ext | ¬ C | (4) |
| | • | |
| 4 Nc | ⊥ | 1 |
| 1 RAA | ¬ (B ∧ ¬ C) | |

**5.**

$$(A \wedge C) \vee (B \wedge D) \qquad 1$$

$$A \wedge C \qquad 2$$

2 Ext  $\quad$ A
2 Ext  $\quad$ C  $\qquad$ (4)

$$\neg\, B$$

$$\bullet$$

4 QED  $\quad$ C  $\qquad$ 3

3 PE  $\quad$ B ∨ C  $\qquad$ 1

$$B \wedge D \qquad 5$$

5 Ext  $\quad$ B  $\qquad$ (7)
5 Ext  $\quad$ D

$$\neg\, C$$

$$\bullet$$

7 QED  $\quad$ B  $\qquad$ 6

6 PE  $\quad$ B ∨ C  $\qquad$ 1

1 PC  $\quad$ B ∨ C

**6.** This answer illustrates the use of detachment rules; other, longer, derivations are possible without them. IP is used at the first stage in order to make it possible to exploit the first premise by CR, the only rule available for exploiting negated disjunctions.

$$\neg\, (A \vee B) \qquad 2$$
$$A \vee D \qquad 4$$
$$\neg\, (C \wedge D) \qquad 5$$

$$\neg\, C$$

$$\neg\, A \qquad (4)$$

4 MTP  $\quad$ D  $\qquad$ (5)
5 MPT  $\quad$ ¬ C

$$\neg\, B$$

$$\circ \qquad \neg A,\ \neg B,\ \neg C,\ D \Rightarrow\!\!\!/ \ \bot$$

$$\bot \qquad 6$$

6 IP  $\quad$ B  $\qquad$ 3

3 PE  $\quad$ A ∨ B  $\qquad$ 2

2 CR  $\quad$ ⊥  $\qquad$ 1

1 IP  $\quad$ C

| A | B | C | D | ¬ (A ∨ B) | , | A ∨ D | , | ¬ (C ∧ D) | / | C |
|---|---|---|---|---|---|---|---|---|---|---|
| F | F | F | T | Ⓣ   F | | Ⓣ | | Ⓣ   F | | Ⓕ |

**7.** [This question was on a topic not covered in F06]

$$A \wedge \neg (B \wedge \neg C)$$
$$\Leftrightarrow$$
$$A \wedge (\neg B \vee C)$$
$$\Leftrightarrow$$
$$(A \wedge \neg B) \vee (A \wedge C)$$