Appendix B. Laws for relative exhaustiveness

Atomic sentences

The first of the following laws is stated only for unanalyzed sentences because laws of the same form for equations and other predications are special cases of the second and third laws:

$\Gamma, A \Rightarrow A, \Sigma$
$\Gamma \Rightarrow \tau = \upsilon, \Sigma$ (where τ and υ are co-aliases relative to the equations in Γ)
$\Gamma, P \tau_1 \ldots \tau_n \Rightarrow P \upsilon_1 \ldots \upsilon_n, \Sigma$ (where τ_i and υ_i, for i from 1 to n, are co-aliases relative to the equations in Γ)

Non-atomic sentences

For each logical constant which forms non-atomic sentences, there are two laws, one for its appearances among the assumptions and one for its among the alternatives that are claimed to be exhaustive relative to the assumptions.

Constant	*As an assumption*	*As an alternative*
$\top$	$\Gamma, \top \Rightarrow \Sigma$ if and only if $\Gamma \Rightarrow \Sigma$	$\Gamma \Rightarrow \top, \Sigma$
$\bot$	$\Gamma, \bot \Rightarrow \Sigma$	$\Gamma \Rightarrow \bot, \Sigma$ if and only if $\Gamma \Rightarrow \Sigma$
$\neg$	$\Gamma, \neg \phi \Rightarrow \Sigma$ if and only if $\Gamma \Rightarrow \phi, \Sigma$	$\Gamma \Rightarrow \neg \phi, \Sigma$ if and only if $\Gamma, \phi \Rightarrow \Sigma$
$\wedge$	$\Gamma, \phi \wedge \psi \Rightarrow \Sigma$ if and only if $\Gamma, \phi, \psi \Rightarrow \Sigma$	$\Gamma \Rightarrow \phi \wedge \psi, \Sigma$ if and only if both $\Gamma \Rightarrow \phi, \Sigma$ and $\Gamma \Rightarrow \psi, \Sigma$
$\vee$	$\Gamma, \phi \vee \psi \Rightarrow \Sigma$ if and only if both $\Gamma, \phi \Rightarrow \Sigma$ and $\Gamma, \psi \Rightarrow \Sigma$	$\Gamma \Rightarrow \phi \vee \psi, \Sigma$ if and only if $\Gamma \Rightarrow \phi, \psi, \Sigma$
$\rightarrow$	$\Gamma, \phi \rightarrow \psi \Rightarrow \Sigma$ if and only if both $\Gamma \Rightarrow \phi, \Sigma$ and $\Gamma, \psi \Rightarrow \Sigma$	$\Gamma \Rightarrow \phi \rightarrow \psi, \Sigma$ if and only if $\Gamma, \phi \Rightarrow \psi, \Sigma$
$\forall$	$\Gamma, \forall x\ \theta x \Rightarrow \Sigma$ if and only if $\Gamma, \forall x\ \theta x, \theta\ \tau \Rightarrow \Sigma$	$\Gamma \Rightarrow \forall x\ \theta x, \Sigma$ if and only if $\Gamma \Rightarrow \theta a, \Sigma$
$\exists$	$\Gamma, \exists x\ \theta x \Rightarrow \Sigma$ if and only if $\Gamma, \theta a \Rightarrow \Sigma$	$\Gamma \Rightarrow \exists x\ \theta x, \Sigma$ if and only if $\Gamma \Rightarrow \theta\ \tau, \exists x\ \theta x, \Sigma$
where τ is any term and a is parametric in the sense that it does not appear in θ, Γ, or Σ		

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