8.5.xa. Exercise answers

1. Some of the derivations below are given in two forms, one that does not use EG and REG and another that does. It is also possible to use the rules REP and REC along with the rules for unrestricted existentials. Such answers are not shown but the they can constructed from the answers that are given by using the substitutions of rules shown in the following table:

rule	*alternative approach using REP and REC*
PRCh	REP, PCh, Ext
RNcP	REC, NcP (with later uses SB, SC, and MCR replaced by UI and either MPT or CR)
REG	Adj, EG, REC

a.

```
        │∃x Fx              1
        │∀x (Fx → Gx)       a:2
        ├─
        │ⓐ
        ││Fa                (3)
        │├─
2 UI    ││Fa → Ga           3
3 MPP   ││Ga
        │││∀x ¬ Gx          a:5
        ││├─
5 UI    │││¬ Ga             (6)
        │││●
        ││├─
6 Nc    │││⊥
        │├─
4 NcP   ││∃x Gx             1
        ├─
1 PCh   │∃x Gx
```

```
        │∃x Fx              1
        │∀x (Fx → Gx)       a:2
        ├─
        │ⓐ
        ││Fa                (3)
        │├─
2 UI    ││Fa → Ga           3
3 MPP   ││Ga                (4)
4 EG    ││∃x Gx             X, (5)
        ││●
        │├─
5 QED   ││∃x Gx             1
        ├─
1 PCh   │∃x Gx
```

b.

```
         │(∃x: Fx) Gx          1
         │(∀x: Gx) Hx          a:2
         ├─
         │ⓐ
         ││Fa                  (4)
         ││Ga                  (2)
         │├─
2 SB     ││Ha                  (5)
         │││(∀x: Fx) ¬ Hx      a:4
         ││├─
4 SB     │││¬ Ha               (5)
         │││●
         ││├─
5 Nc     │││⊥                  3
         │├─
3 RNcP   ││(∃x: Fx) Hx         1
         ├─
1 PRCh   │(∃x: Fx) Hx
```

```
         │(∃x: Fx) Gx       1
         │(∀x: Gx) Hx       a:2
         ├─
         │ⓐ
         ││Fa               (3)
         ││Ga               (2)
         │├─
2 SB     ││Ha               (3)
3 REG    ││(∃x: Fx) Hx      X, (4)
         ││●
         │├─
4 QED    ││(∃x: Fx) Hx      1
         ├─
1 PRCh   │(∃x: Fx) Hx
```

c.

	│ ∀x (Fx → Ga)	b:3
	│ │ ∃x Fx	2
	│ │ ⓑ │ Fb	(4)
3 UI	│ │ │ Fb → Ga	4
4 MPP	│ │ │ Ga	(5)
	│ │ │ •	
5 QED	│ │ │ Ga	2
2 PCh	│ │ Ga	1
1 CP	│ ∃x Fx → Ga	

	│ ∃x Fx → Ga	4
	│ ⓑ │ │ Fb	(8)
	│ │ │ │ ¬ Ga	(4)
4 MTT	│ │ │ │ ¬ ∃x Fx	5
	│ │ │ │ │ │ ∀x ¬ Fx	b:7
7 UI	│ │ │ │ │ │ ¬ Fb	(8)
	│ │ │ │ │ │ •	
8 Nc	│ │ │ │ │ │ ⊥	6
6 NcP	│ │ │ │ │ ∃x Fx	5
5 CR	│ │ │ │ ⊥	3
3 IP	│ │ │ Ga	2
2 CP	│ │ Fb→Ga	1
1 UG	│ ∀x (Fx → Ga)	

	│ ∃x Fx → Ga	4
	│ ⓑ │ │ Fb	(3)
3 EG	│ │ │ ∃x Fx	X, (4)
4 MPP	│ │ │ Ga	(5)
	│ │ │ •	
5 QED	│ │ │ Ga	2
2 CP	│ │ Fb → Ga	1
1 UG	│ ∀x (Fx → Ga)	

d.

	│ Fa	(2)
	│ │ (∀x: x = a) ¬ Fx	a:2
2 SC	│ │ ¬ a = a	(3)
	│ │ •	
3 DC	│ │ ⊥	1
1 RNcP	│ (∃x: x = a) Fx	

	│ Fa	(2)
1 EC	│ a = a	(2)
2 REG	│ (∃x: x = a) Fx	(3)
	│ •	
3 QED	│ (∃x: x = a) Fx	

	(∃x: x = a) Fx	1
	ⓑ	
	b = a	a—b
	Fb	(2)
	●	
2 QED=	Fa	1
1 PRCh	Fa	

e.

	(∃x: Fx) ∀y Rxy	2
	ⓐ	
	ⓑ	
	Fb	(4)
	∀y Rby	b:5
	(∀y: Fy) ¬ Rya	b:4
4 SB	¬ Rba	(6)
5 UI	Rba	(6)
	●	
6 Nc	⊥	3
3 RNcP	(∃y: Fy) Rya	2
2 PRCh	(∃y: Fy) Rya	1
1 UG	∀x (∃y: Fy) Ryx	

	(∃x: Fx) ∀y Rxy	2
	ⓐ	
	ⓑ	
	Fb	(4)
	∀y Rby	b:3
3 UI	Rba	(4)
4 REG	(∃y: Fy) Rya	X, (5)
	●	
5 QED	(∃y: Fy) Rya	2
2 PRCh	(∃y: Fy) Rya	1
1 UG	∀x (∃y: Fy) Ryx	

f.

	(∃x: Gx) Fx	1
	¬ Fa	(4)
	ⓑ	
	Gb	(3)
	Fb	(4)
	(∀x: ¬ x = a) ¬ Gx	b:3
3 SC	b = a	a—b
	●	
4 Nc=	⊥	2
2 RNcP	(∃x: ¬ x = a) Gx	1
1 PRCh	(∃x: ¬ x = a) Gx	

g.

	$\forall x\,(Fx \to Ga)$	c:3
	$\forall x\,(Ga \to Fx)$	b:5
	$\exists x\,Fx$	2
	ⓑ	
	ⓒ	
	Fc	(4)
3 UI	$Fc \to Ga$	4
4 MPP	Ga	(6)
5 UI	$Ga \to Fb$	6
6 MPP	Fb	(7)
	●	
7 QED	Fb	2
2 PCh	Fb	1
1 UG	$\forall x\,Fx$	

h. *Everyone loves everyone who loves someone*
Someone loves someone

	$(\forall x{:}\ Px)\,(\forall y{:}\ Py \wedge (\exists z{:}\ Pz)\,Lyz)\,Lxy$	b:5, a:9
	$(\exists x{:}\ Px)\,(\exists y{:}\ Py)\,Lxy$	3
	ⓐ	
	Pa	(9)
	ⓑ	
	Pb	(5), (11)
	ⓒ	
	Pc	(7), (10)
	$(\exists y{:}\ Py)\,Lcy$	4
	ⓓ	
	Pd	(6)
	Lcd	(6)
5 SB	$(\forall y{:}\ Py \wedge (\exists z{:}\ Pz)\,Lyz)\,Lby$	c:8
6 REG	$(\exists z{:}\ Pz)\,Lcz$	X, (7)
7 Adj	$Pc \wedge (\exists z{:}\ Pz)\,Lcz$	X, (8)
8 SB	Lbc	(10)
9 SB	$(\forall y{:}\ Py \wedge (\exists z{:}\ Pz)\,Lyz)\,Lay$	b:12
10 REG	$(\exists z{:}\ Pz)\,Lbz$	X, (11)
11 Adj	$Pb \wedge (\exists z{:}\ Pz)\,Lbz$	X, (12)
12 SB	Lab	(13)
	●	
13 QED	Lab	4
4 PRCh	Lab	3
3 PRCh	Lab	2
2 RUG	$(\forall y{:}\ Py)\,Lay$	1
1 RUG	$(\forall x{:}\ Px)\,(\forall y{:}\ Py)\,Lxy$	

Everyone loves everyone

Note that stages 4 and 6 serve only to move us from $(\exists y{:}\ Py)\,Lcy$ to $(\exists z{:}\ Pz)\,Lcz$—i.e., to change a bound variable. If sentences that differ only in the choice of a letter for a bound variable are regarded as the same, (∃y: Py) Lcy could be used as a premise for Adj at stage 7 and the use of PRCh at stage 4 would not be needed.

i. *Something is such that nothing other than it is done*

[When *nothing* is analyzed using a negative generalization, a derivation like that below but without stages 6 and 7 could be used.]

```
           │∃x ¬ (∃y: ¬ y = x) Dy                 2
           ├─
           ││∃x (∃y: ¬ y = x) (Dx ∧ Dy)           3
           │├─
           ││ⓐ
           │││¬ (∃y: ¬ y = a) Dy                  6
           ││├─
           │││ⓑ
           ││││(∃y: ¬ y = b) (Db ∧ Dy)            4
           │││├─
           ││││ⓒ
           │││││¬ c = b                           (10)
           │││││Db ∧ Dc                           5
           ││││├─
5 Ext      │││││Db                                (8)
5 Ext      │││││Dc                                (9)
           ││││││ │(∀y: ¬ y = a) ¬ Dy             b:8, c:9
           ││││││ ├─
8 SC       ││││││ │b = a                           a—b, c
9 SC       ││││││ │c = a                           a—b—c, (10)
           ││││││ │●
           ││││││ ├─
10 Nc=     ││││││ │⊥                               7
           │││││├─
7 RNcP     ││││││(∃y: ¬ y = a) Dy                  6
           ││││├─
6 CR       │││││⊥                                  4
           │││├─
4 PRCh     ││││⊥                                   3
           ││├─
3 PCh      │││⊥                                    2
           │├─
2 PCh      ││⊥                                     1
           ├─
1 RAA      │¬ ∃x (∃y: ¬ y = x) (Dx ∧ Dy)
```

At most one thing is done

At stage 10, the conclusion ⊥ could also be justified as coming by DC from ¬ c = b alone since c = a serves to make b and c co-aliases.

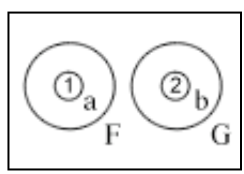
At most one thing is done

	¬ ∃x (∃y: ¬ y = x) (Dx ∧ Dy)	(9)
	∀x (∃y: ¬ y = x) Dy	a:2, b:4
2 UI	(∃y: ¬ y = a) Dy	3
	ⓑ ¬ b = a	
	Db	(6)
4 UI	(∃y: ¬ y = b) Dy	5
	ⓒ ¬ c = b	(7)
	Dc	(6)
6 Adj	Db ∧ Dc	X, (7)
7 REG	(∃y: ¬ y = b) (Db ∧ Dy)	X, (8)
8 EG	∃x (∃y: ¬ y = x) (Dx ∧ Dy)	X, (9)
	●	
9 Nc	⊥	5
5 PRCh	⊥	3
3 PRCh	⊥	1
1 NcP	∃x ¬ (∃y: ¬ y = x) Dy	

Something is such that nothing other than it is done

2. a.

	∃x Fx	1
	∃x Gx	2
	ⓐ Fa	(5)
	ⓑ Gb	(7)
	∀x ¬ (Fx ∧ Gx)	a:4, b:6
4 UI	¬ (Fa ∧ Ga)	5
5 MPT	¬ Ga	
6 UI	¬ (Fb ∧ Gb)	7
7 MPT	¬ Fb	
	○	Fa,¬ Fb,¬ Ga,Gb ⇏ ⊥
	⊥	3
3 NcP	∃x (Fx ∧ Gx)	2
2 PCh	∃x (Fx ∧ Gx)	1
1 PCh	∃x (Fx ∧ Gx)	



b.

	│ (∃x: Fx) Gx	1
	│ (∃x: Fx) Hx	2
	│ (∀x: Fx) (∀y: Fy) x = y	a:3
	├─	
	│ ⓐ	
	│ │ Fa	
	│ │ Ga	(7)
	│ ├─	
	│ │ ⓑ	
	│ │ │ Fb	
	│ │ │ Hb	(8)
	│ │ ├─	
3 SB	│ │ │ (∀y: Fy) a = y	b:4
4 SB	│ │ │ a = b	a—b
	│ │ │ │ ∀x ¬ (Gx ∧ Hx)	a:6
	│ │ │ ├─	
6 UI	│ │ │ │ ¬ (Ga ∧ Ha)	7
7 MPT	│ │ │ │ ¬ Ha	(8)
	│ │ │ │ ●	
	│ │ │ ├─	
8 Nc=	│ │ │ │ ⊥	5
	│ │ ├─	
5 NcP	│ │ │ ∃x (Gx ∧ Hx)	2
	│ ├─	
2 PRCh	│ │ ∃x (Gx ∧ Hx)	1
	├─	
1 PRCh	│ ∃x (Gx ∧ Hx)	

3. a. *Some road sign was colored*
Every road sign was a traffic marker
If anything was colored, it was painted

	│ (∃x: Sx) Cx	1
	│ (∀x: Sx) Tx	a:2
	│ ∀x (Cx → Px)	a:3
	├─	
	│ ⓐ	
	│ │ Sa	(2)
	│ │ Ca	(4)
	│ ├─	
2 SB	│ │ Ta	(5)
3 UI	│ │ Ca → Pa	4
4 MPP	│ │ Pa	(5)
5 REG	│ │ (∃x: Tx) Px	X, (6)
	│ │ ●	
	│ ├─	
6 QED	│ │ (∃x: Tx) Px	1
	├─	
1 PRCh	│ (∃x: Tx) Px	

Some traffic marker was painted

b. *Someone who owns a snake was pleased*
Every snake is a reptile

```
          │(∃x: Px ∧ (∃y: Sy) Oxy) Dx          1
          │(∀x: Sx) Rx                         b:4
          ├──
          │ⓐ
          ││Pa ∧ (∃y: Sy) Oay                  2
          ││Da                                 (7)
          │├──
2 Ext     ││Pa                                 (6)
2 Ext     ││(∃y: Sy) Oay                       3
          ││ⓑ
          │││Sb                                (4)
          │││Oab                               (5)
          ││├──
4 SB      │││Rb                                (5)
5 REG     │││(∃y: Ry) Oay                      X, (6)
6 Adj     │││Pa ∧ (∃y: Ry) Oay                 X, (7)
7 REG     │││(∃x: Px ∧ (∃y: Ry) Oxy) Dx        X, (8)
          │││●
          ││├──
8 QED     │││(∃x: Px ∧ (∃y: Ry) Oxy) Dx        3
          │├──
3 PRCh    ││(∃x: Px ∧ (∃y: Ry) Oxy) Dx         1
          ├──
1 PRCh    │(∃x: Px ∧ (∃y: Ry) Oxy) Dx
```

Someone who owns a reptile was pleased