8.5.x. Exercise questions

1. Use the system of derivations to establish each of the following:

a. $\exists x\ Fx,\ \forall x\ (Fx \rightarrow Gx) \Rightarrow \exists x\ Gx$

b. $(\exists x\text{: } Fx)\ Gx,\ (\forall x\text{: } Gx)\ Hx \Rightarrow (\exists x\text{: } Fx)\ Hx$

c. $\forall x\ (Fx \rightarrow Ga) \Leftrightarrow \exists x\ Fx \rightarrow Ga$

d. $Fa \Leftrightarrow (\exists x\text{: } x = a)\ Fx$

e. $(\exists x\text{: } Fx)\ \forall y\ Rxy \Rightarrow \forall x\ (\exists y\text{: } Fy)\ Ryx$

f. $(\exists x\text{: } Gx)\ Fx,\ \neg\ Fa \Rightarrow (\exists x\text{: } \neg\ x = a)\ Gx$

g. $\forall x\ (Fx \rightarrow Ga), \forall x\ (Ga \rightarrow Fx),\ \exists x\ Fx \Rightarrow \forall x\ Fx$

h. *Everyone loves everyone who loves anyone*, *Someone loves someone* ⇒ *Everyone loves everyone*

i. *Something is such that nothing other than it is done* ⇔ *At most one thing is done*

2. Use derivations to check each of the claims below; if a derivation indicates that a claim fails, describe a structure that divides an open gap. You need not worry about infinite derivations.

a. $\exists x\ Fx,\ \exists x\ Gx \Rightarrow \exists x\ (Fx \wedge Gx)$

b. $(\exists x\text{: } Fx)\ Gx,\ (\exists x\text{: } Fx)\ Hx,\ (\forall x\text{: } Fx)\ (\forall y\text{: } Fy)\ x = y \Rightarrow \exists x\ (Gx \wedge Hx)$

3. In the following, choose one of each bracketed pair of premises and one each bracketed pair of words or phrases in the conclusion so as to make a valid argument; then analyze the premises and conclusion and construct a derivation to show that the argument is valid.

a. *Some road sign was colored*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker***]**
[*If anything was red, it was colored* | *If anything was colored, it was painted***]**

Some **[***stop sign* | *traffic marker***]** *was* **[***red* | *painted***]**

b. *Someone who owns a snake was pleased*
[*Every cobra is a snake* | *Every snake is a reptile***]**

Someone who owns a **[***cobra* | *reptile***]** *was pleased*

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