8.4.xa. Exercise answers

1. **a.** *using Russell's analysis:*

Sam guessed the winning number

the winning number is such that (Sam guessed it)

(∃x: x *is a winning number* ∧ *only* x *is a winning number*) *Sam guessed* x

(∃x: Wx ∧ (∀y: ¬ y = x) ¬ y *is a winning number*) Gsx

$$(\exists x{:}\ Wx \land (\forall y{:}\ \neg\, y = x)\ \neg Wy)\ Gsx$$

$$\exists x\,(Wx \land \forall y\,(\neg\, y = x \rightarrow \neg Wy) \land Gsx)$$

or:

$$(\exists x{:}\ Wx \land (\forall y{:}\ Wy)\ x = y)\ Gsx$$

$$\exists x\,(Wx \land \forall y\,(Wy \rightarrow x = y) \land Gsx)$$

[G: λxy (x *guessed* y); W: λx (x *is a winning number*); s: *Sam*]

[*Note:* λx (x *is a winning number*) might be open to further analysis as λx (x *is a number* ∧ x *won*)]

with the description operator:

Sam guessed the winning number

G *Sam the winning number*

Gs(℩x x *is a winning number*)

$$Gs(\iota x\ Wx)$$

b. *using Russell's analysis:*

The winner who spoke to Tom was well-known

The winner who spoke to Tom is such that (he or she was well-known)

(∃x: x *is a winner who spoke to Tom* ∧ *only* x *is a winner who spoke to Tom*) x *was well-known*

(∃x: (x *is a winner* ∧ x *spoke to Tom*) ∧ (∀y: ¬ y = x) ¬ (y *is a winner* ∧ y *spoke to Tom*)) Kx

$$(\exists x{:}\ (Wx \land Sxt) \land (\forall y{:}\ \neg\, y = x)\ \neg\,(Wy \land Syt))\ Kx$$

$$\exists x\,((Wx \land Sxt) \land \forall y\,(\neg\, y = x \rightarrow \neg\,(Wy \land Syt)) \land Kx)$$

or:

$$(\exists x{:}\ (Wx \land Sxt) \land (\forall y{:}\ Wy \land Syt)\ x = y)\ Kx$$

$$\exists x\,((Wx \land Sxt) \land \forall y\,((Wy \land Syt) \rightarrow x = y) \land Kx)$$

K: λx (x *was well-known*); S: λxy (x spoke to y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:

The winner who spoke to Tom was well-known

The winner who spoke to Tom was well-known

K *the winner who spoke to Tom*

K(℩x (x *is a winner who spoke to Tom*))

K(℩x (x *is a winner* ∧ x *spoke to Tom*))

$$K(\iota x\ (Wx \land Sxt))$$

c. *using Russell's analysis:*

The winner, who spoke to Tom, was well-known.

The winner is such that (he or she, who spoke to Tom, was well-known).

(∃x: x *is a winner* ∧ *only* x *is a winner*) x, *who spoke to Tom, was well-known*)

(∃x: x *is a winner* ∧ (∀y: ¬ y = x) ¬ y *is a winner*) (x *spoke to Tom* ∧ x *was well-known*)

$$(\exists x\colon Wx \wedge (\forall y\colon \neg\, y = x)\, \neg\, Wy)\, (Sxt \wedge Kx)$$

$$\exists x\, (Wx \wedge \forall y\, (\neg\, y = x \rightarrow \neg\, Wy) \wedge (Sxt \wedge Kx))$$

or:

$$(\exists x\colon Wx \wedge (\forall y\colon Wy)\, x = y)\, (Sxt \wedge Kx)$$

$$\exists x\, (Wx \wedge \forall y\, (Wy \rightarrow x = y) \wedge (Sxt \wedge Kx))$$

[K: λx (x *was well-known*); S: λxy (x *spoke to* y); W: λx (x *is a winner*); t: *Tom*]

with the description operator:

The winner, who spoke to Tom, was well-known.

The winner *spoke to* *Tom* ∧ *the winner* *was well-known*

S *the winner* Tom ∧ K *the winner*

S(℩x x *is a winner*)t ∧ K(℩x x *is a winner*)

$$S(\iota x\, Wx)t \wedge K(\iota x\, Wx)$$

d. *using Russell's analysis:*

Every number greater than one is greater than its positive square root

(∀x: x *is a number greater than one*) x *is greater than its positive square root*

(∀x: x *is a number* ∧ x *is greater than one*) x *is greater than the positive square root of* x

(∀x: Nx ∧ Gxo) *the positive square root of* x *is such that (*x *is greater than it)*

(∀x: Nx ∧ Gxo) (∃y: y *is a positive square root of* x ∧ *only* y *is a positive square root of* x) x *is greater than* y

(∀x: Nx ∧ Gxo) (∃y: (y *is positive* ∧ y *is a square root of* x) ∧ (∀z: ¬ z = y) ¬ (z *is positive* ∧ z *is a square root of* x)) Gxy

$$(\forall x\colon Nx \wedge Gxo)\, (\exists y\colon (Py \wedge Syx) \wedge (\forall z\colon \neg\, z = y)\, \neg\, (Pz \wedge Szx))\, Gxy$$

$$\forall x\, (\, (Nx \wedge Gxo) \rightarrow \exists y\, ((Py \wedge Syx) \wedge \forall z\, (\neg\, z = y \rightarrow \neg\, (Pz \wedge Szx)) \wedge Gxy)\,)$$

or:

$$(\forall x\colon Nx \wedge Gxo)\, (\exists y\colon (Py \wedge Syx) \wedge (\forall z\colon Pz \wedge Szx)\, y = z)\, Gxy$$

$$\forall x\, (\, (Nx \wedge Gxo) \rightarrow \exists y\, ((Py \wedge Syx) \wedge \forall z\, ((Pz \wedge Szx) \rightarrow y = z) \wedge Gxy)\,)$$

[G: λx (x *is greater than* y); N: λx (x *is a number*); P: λx (x *is positive*); S: λxy (x *is a square root of* y)] *with the description operator:*

Every number greater than one is greater than its positive square root

($\forall$x: x *is a number* $\wedge$ x *is greater than one*) <u>x</u> *is greater than* <u>*the positive square root of* x</u>

($\forall$x: Nx $\wedge$ Gxo) G <u>x</u> <u>*the positive square root of* x</u>

($\forall$x: Nx $\wedge$ Gxo) Gx(Ιy y *is a positive square root of* x)

($\forall$x: Nx $\wedge$ Gxo) Gx(Ιy (y *is a positive* $\wedge$ y *is a square root of* x))

($\forall$x: Nx $\wedge$ Gxo) Gx(Ιy (Py $\wedge$ Syx))

$\forall$x ((Nx $\wedge$ Gxo) $\rightarrow$ Gx(Ιy (Py $\wedge$ Syx)))

2. a. ($\exists$x: x *owns Spot* $\wedge$ ($\forall$y: $\neg$ y = x) $\neg$ y *owns Spot*) x *called*

($\exists$x: x *owns Spot* $\wedge$ *only* x *owns Spot*) x *called*

The owner of Spot is such that (it called)

Spot's owner called

b. *John found* (Ιx (x *is a photographer* $\wedge$ x *enlarged* (Ιy y *is a picture of John*)))

John found (Ιx (x *is a photographer* $\wedge$ x *enlarged the picture of John*))

John found (Ιx (x *is a photographer who enlarged the picture of John*))

John found the photographer who enlarged the picture of him