8.4.2. Definite descriptions as individual terms

Prior to 8.4.1, we had treated definite descriptions as individual terms, understanding them to have at least the nil value as a reference value. Historically, this approach is associated with Frege, who suggested that an actual object—for example, the number 0—be stipulated as the reference of definite descriptions that did not otherwise have one.

It is possible to retain the view that definite descriptions are individual terms and still go on to analyze them in a way that exposes the component descriptions; but, to do this, we need to introduce some further notation. This is a logical operation, a ***description operator***, that applies to a predicate abstract to form an individual term. Our notation will be a sans-serif capital I and we will abbreviate I[λx ρx] as Ix ρx. This notation might be read in English as *the thing* x *such that* ρx. Notice that this is a noun phrase rather than a sentence, so, although the description operator looks like an unrestricted quantifier, its reading does not involve a verb.

The reference value of Ix ρx is stipulated to be the one value in the extension of ρ if contains just one value and to be the nil value otherwise. We do not distinguish the nil value from others in a referential range in any other way, so the stipulation of it as the default value of Ix ρx is somewhat limited in its significance. But this stipulation does entail that definite descriptions that fail to uniquely describe an object all have the same reference value. The description *the rational number whose square is 2* thus has the same reference value as *the planet whose orbit lies between the Earth and Venus.*

If we use the description operator to analyze *The house Jack built still stands*, we get

The house Jack built *still stands*
S *the house Jack built*
S(Ix (x *is a house Jack built*))
S(Ix (x *is a house* ∧ *Jack built* x))

S(Ix (Hx ∧ Bjx))

[B: λxy (x *built* y); H: λx (x *is house*); S: λx (x *still stands*); j: *Jack*]

The parentheses surrounding the whole definite description in this analysis are not needed to avoid ambiguity in our notation, but they make it easier to read.

This analysis does more than use different notation from Russell's analysis; it offers a different interpretation of the sentence. While the simpler notation may be pleasing, the interpretation may not be, so we should consider it more closely. To compare the two interpretations, it will help to give Russell's in a different but equivalent form. Since on Russell's analysis *The* C *is such that (… it …)* entails both *Some* C *is such that (… it …)* and that at most one thing is a C, it can be restated somewhat redundantly as the conjunction

There is exactly one C ∧ *some* C *is such that (… it …)*

That is, Russell interprets *The house Jack built still stands* as *There is exactly one house that Jack built and some house that Jack built still stands*.

On the other hand, if we analyze *The* C *is such that (... it ...)* using the description operator, we interpret it as saying that the predicate λx (... x ...) is true of the reference value of *the* C. Now, what that reference value is depends on whether *There is exactly one* C is true. If there is exactly one C, the value of *the* C is the one and only C. Otherwise, the value of *the* C is the nil value. For example, if Jack did build exactly one house, the sentence *The house Jack built still stands* is true just in case this house still stands. But if Jack built no house or more than one, this sentence is true if and only if the predicate λx (x *still stands*) is true of the nil value.

To make it easier to express this interpretation in English, let's fix an individual term whose reference is bound to be nil and read it in English as *the nil*. Since the extension of λx ⊥ is bound to be empty, the definite description ℩x ⊥ could play this role, but it will be convenient to have a special symbol, for which we will use ∗ (known as the ***asterisk operator***).

Then we can express the content of the analysis using the description operator as follows:

(*there is exactly one* C ∧ *some* C *is such that* (... *it* ...))
∨ (¬ *there is exactly one* C ∧ ... ∗ ...)

Comparison with the expression of Russell's analysis given above will show that this interpretation is weaker, having been hedged by an added disjunct. It could be expressed equivalently as follows:

If there is exactly one C, *then some* C *is such that* (... *it* ...);
otherwise, ... ∗ ...

where the English *if* φ *then* ψ*; otherwise* χ expresses the form (φ → ψ) ∧ (¬ φ → χ), which we have called a branching conditional. This is equivalent to the form (φ ∧ ψ) ∨ (¬ φ ∧ χ) that was used above because each form has the same truth value as ψ when φ is true and the same value as χ when φ is false. While, the formulation of the content of this analysis using the branching conditional makes the comparison with Russell's analysis a little less direct, it is probably the more natural way of thinking about the significance of this approach to definite descriptions in its own right.

So, when we use the description operator, we interpret *The house Jack built still stands* as either of the following equivalent claims:

Either there is exactly one house that Jack built and some house that Jack built still stands; or there is not exactly one and the nil still stands

If there is exactly one house that Jack built then some house that Jack built still stands; otherwise the nil still stands

This interpretation has both fortunate and unfortunate consequences.

First the bad news. Because the analysis using the description operator hedges the claim it makes with the possibility that there is not exactly one house that Jack built, it can be true if he built no house or more than one. So we must ask whether we would count the original sentence as true in this sort of case. In answering this question, it is important to remember that the analysis will be true in such a case only if the predicate λx (x *still stands*) is true of the nil reference value. The truth value yielded by properties when they are applied to the nil value is something that we have left open. (More precisely, this is true in the case of unanalyzed predicates; $\lambda x\ x = x$, for example, is bound to be true of the nil value because it is true of all reference values.) So when we analyze definite descriptions using the description operator, we do not specify the truth value of *The house Jack built still stands* in cases where *the house Jack built* does not refer. But on Russell's account the value is definitely **F** in these cases. If the discussion of the issue throughout the course of the last century has shown anything, it has shown that there is no consensus on this matter among the community of English speakers.

That's the bad news. The good news is that the analysis using the description operator removes any room for ambiguity concerning the relative scope of definite descriptions and negation. That much is clear just from the notation. The definite description operator forms terms and to deny that a predicate applies to a term is the same thing as to apply a negative predicate. That is, $\neg\ \theta\tau \Leftrightarrow [\lambda x\ \neg\ \theta x]\tau$. (Indeed, we really have more than an equivalence here since we regard these symbolic forms as notation for the same sentence.)

We can see this lack of ambiguity also by exploring the interpretation given by the second analysis. First, let us look a little more closely at the ambiguity exhibited by *The present king of France is not bald* on Russell's analysis. Consider the following restatements and partial analyses of a pair of sentences:

The present king of France is such that (he is bald)
There is at present one and only one king of France
$\wedge$ *some present king of France is such that (he is bald)*

$$O \wedge (\exists x\!: Kx)\ Bx$$

The present king of France is such that (he is not bald)
There is at present one and only one king of France
$\wedge$ *some present king of France is such that (he is not bald)*

$$O \wedge (\exists x\!: Kx)\ \neg\ Bx$$

[B: λx (x *is bald*); K: λx (x *is at present king of France*);
O: *there is at present one and only one king of France*]

If O is true, at least one of these is true because there is some king of France at present who must be either bald or not, and at most one is true because there is no more than one present king of France so being bald and not being bald cannot both be exemplified by present kings of France. But, if O is not true, both of the sentences above are false; and therefore they are not contradictory. Now, on Russell's analysis, *The present king of France is not bald* might be interpreted as equivalent to either $\neg\ (O \wedge (\exists x\!: Kx)\ Bx)$, the denial of the first sentence above, or $O \wedge (\exists x\!: Kx)\ \neg\ Bx$, the second sentence. And these two interpretations are not equivalent because the two sentences above are not contradictory.

On the other hand if we consider the same two sentences but restate them in the way corresponding to the semantics of the definite description operator we get this:

The present king of France is such that (he is bald)

(O ∧ *some present king of France is such that (he is bald)*)
  ∨ (¬ O ∧ *the nil is bald*)

$$(O \wedge (\exists x\!: Kx)\, Bx) \vee (\neg\, O \wedge B*)$$

The present king of France is such that (he is not bald)
(O ∧ *some present king of France is such that (he is not bald)*)
  ∨ (¬ O ∧ *the nil is not bald*)

$$(O \wedge (\exists x\!: Kx) \neg\, Bx) \vee (\neg\, O \wedge \neg\, B*)$$

Now, we have already seen that, if O is true, the left disjunct of exactly one of these is true and, since the right disjuncts are both false when O is true, exactly one of the disjunctions will be true in such a case. And, when O is false, the left disjuncts are both false and exactly one of the right disjuncts is true. So again exactly one the disjunctions is true, and these sentences are contradictory. Thus, the denial of the first of these sentences is equivalent to the second; and taking *The present king of France is not bald* to be a negation leads to the same interpretation as we would get by supposing that it applies the negative predicate λx (x *is not bald*) to the individual term *the present king of France*.

In an analysis using the description operator, both of the sentences we have been considering are given weaker interpretations than Russell would give them, and these interpretations are weaker in different ways. In particular, in a case where O is false, one of the hedges is true and the other is not. Which is which depends on whether λx (x *is bald*) is true or false of the nil value, but we do not care which hedge is true and which false. What is important is that, when the sentence O is false and thus both of the logical forms derived from Russell's analysis are false, one and only one of the weaker pair of forms is true.