8.3.xa. Exercise answers

1. **a.** *If Oswald didn't shoot Kennedy, someone else did*
Oswald didn't shoot Kennedy → *someone other than Oswald shot Kennedy*
¬ *Oswald shot Kennedy* → (∃x: x *is a person other than Oswald*) x *shot Kennedy*
¬ Sok → (∃x: x *is a person* ∧ x *is other than Oswald*) x *shot Kennedy*
¬ Sok → (∃x: x *is a person* ∧ ¬ x = *Oswald*) x *shot Kennedy*

$$\neg \text{Sok} \rightarrow (\exists x\text{: Px} \land \neg\, x = o)\ \text{Sxk}$$

$$\neg \text{Sok} \rightarrow \exists x\ ((\text{Px} \land \neg\, x = o) \land \text{Sxk})$$

[P: λx (x *is a person*); S: λxy (x *shot* y); k: *Kennedy*; o: *Oswald*]

b. *No one but Frank saw Sue*
¬ *someone other than Frank saw Sue*
¬ (∃x: x *is a person* ∧ ¬ x = *Frank*) x *saw Sue*

$$\neg\, (\exists x\text{: Px} \land \neg\, x = f)\ \text{Sxs}$$

$$\neg\, \exists x\ ((\text{Px} \land \neg\, x = f) \land \text{Sxs})$$

or:

No one but Frank saw Sue
(∀x: x *is a person other than Frank*) ¬ x *saw Sue*
(∀x: x *is a person* ∧ ¬ x = *Frank*) ¬ x *saw Sue*

$$(\forall x\text{: Px} \land \neg\, x = f)\ \neg\, \text{Sxs}$$

$$\forall x\ ((\text{Px} \land \neg\, x = f) \rightarrow \neg\, \text{Sxs})$$

[P: λx (x *is a person*); S: λxy (x *saw* y); f: *Frank*; s: *Sue*]

c. *Ed and only Ed was awake*
Ed was awake ∧ *only Ed was awake*
Ed was awake ∧ (∀x: ¬ x *is Ed*) ¬ x *was awake*

$$\text{Ae} \land (\forall x\text{: } \neg\, x = e)\ \neg\, \text{Ax}$$

$$\text{Ae} \land \forall x\ (\neg\, x = e \rightarrow \neg\, \text{Ax})$$

[A: λx (x *was awake*); e: *Ed*]

d. *Everyone except Tom, Dick, and Harry arrived early*
(∀x: x *is a person* ∧ x *is other than Tom, Dick, and Harry*) x *arrived early*
(∀x: x *is a person* ∧ (¬ x = *Tom* ∧ ¬ x = *Dick* ∧ ¬ x = *Harry*)) x *arrived early*

$$(\forall x\text{: Px} \land (\neg\, x = t \land \neg\, x = d \land \neg\, x = h))\ \text{Ex}$$

$$\forall x\ (\ (\text{Px} \land (\neg\, x = t \land \neg\, x = d \land \neg\, x = h)) \rightarrow \text{Ex}\ )$$

[E: λx (x *arrived early*); P: λx (x *is a person*); d: *Dick*; h: *Harry*; t: *Tom*]

e. *Adam and another officer thanked everyone else*
(∃x: x *is a officer other than Adam*) *Adam and* x *thanked everyone else*
(∃x: x *is a officer* ∧ x *is other than Adam*) *everyone other than Adam and* x *is such that (Adam and* x *thanked him or her)*
(∃x: Ox ∧ ¬ x = *Adam*) (∀y: y *is a person other than Adam and* x) *Adam and* x *both thanked* y
(∃x: Ox ∧ ¬ x = *Adam*) (∀y: y *is a person* ∧ y *is other than Adam and* x) (*Adam thanked* y ∧ x *thanked* y)
(∃x: Ox ∧ ¬ x = a) (∀y: Py ∧ (¬ y = *Adam* ∧ ¬ y = x)) (Tat ∧ Txy)

$$(\exists x\text{: Ox} \land \neg\, x = a)\ (\forall y\text{: Py} \land (\neg\, y = a \land \neg\, y = x))\ (\text{Tay} \land \text{Txy})$$

$$\exists x\ (\ (\text{Ox} \land \neg\, x = a) \land \forall y\ (\ (\text{Py} \land (\neg\, y = a \land \neg\, y = x)) \rightarrow (\text{Tay} \land \text{Txy})\ )\ )$$

[O: λx (x *is an officer*); P: λx (x *is a person*); T: λxy (x *thanked* y); a: *Adam*] *or:*

Adam and another officer thanked everyone else

Adam thanked everyone else

∧ *an officer other than Adam thanked everyone else*

everyone other than Adam is such that (Adam thanked him or her)

∧ (∃x: x *is a officer other than Adam*) x *thanked everyone else*

(∀y: y *is a person other than Adam*) *Adam thanked* y

∧ (∃x: Ox ∧ ¬ x = *Adam*) *everyone other than* x *is such that (*x *thanked him or her)*

(∀y: Py ∧ ¬ y = *Adam*) Tay

∧ (∃x: Ox ∧ ¬ x = a) (∀y: y *is a person other than* x) x *thanked* y

$$(\forall y{:}\ Py \land \neg\, y = a)\ Tay \land (\exists x{:}\ Ox \land \neg\, x = a)\ (\forall y{:}\ Py \land \neg\, y = x)\ Txy$$

$$\forall y\ ((Py \land \neg\, y = a) \rightarrow Tay) \land \exists x\ (\ (Ox \land \neg\, x = a) \land \forall y\ ((Py \land \neg\, y = x) \rightarrow Txy)\ )$$

The logical forms produced by these two analyses are not equivalent. It could be said that the first interprets *else* as referring to Adam and the other officer collectively while the second interprets it as referring to them individually. The latter interpretation produces a pair of generalizations each of whose domains excludes only one of the two rather than both together. That means that the second together with the assumption that Adam and the other office are both people entails that they thanked each other.

f. *At least two things went wrong*

∃x (∃y: ¬ y = x) (x *and* y *went wrong*)

∃x (∃y: ¬ y = x) (x *went wrong* ∧ y *went wrong*)

$$\exists x\ (\exists y{:}\ \neg\, y = x)\ (Wx \land Wy)$$

$$\exists x\ \exists y\ (\neg\, y = x \land (Wx \land Wy))$$

[W: λx (x *went wrong*)]

g. *Bill spoke to at most one person*

¬ *Bill spoke to at least two people*

¬ *at least two people are such that (Bill spoke to them)*

¬ (∃x: x *is a person*) (∃y: y *is a person* ∧ ¬ y = x) (*Bill spoke to* x *and* y)

¬ (∃x: Px) (∃y: Py ∧ ¬ y = x) (*Bill spoke to* x ∧ *Bill spoke to* y)

$$\neg\ (\exists x{:}\ Px)\ (\exists y{:}\ Py \land \neg\, y = x)\ (Sbx \land Sby)$$

$$\neg\ \exists x\ (Px \land \exists y\ ((Py \land \neg\, y = x) \land (Sbx \land Sby)))$$

[S: λxy (x *spoke to* y); b: *Bill*]

h. *At least one thing will do* ∧ *at most one thing will do*

∃x x *will do* ∧ ¬ *at least 2 things will do*

∃x Dx ∧ ¬ ∃x (∃y: ¬ y = x) (x *and* y *will do*)

∃x Dx ∧ ¬ ∃x (∃y: ¬ y = x) (x *will do* ∧ y *will do*)

$$\exists x\ Dx \land \neg\ \exists x\ (\exists y{:}\ \neg\, y = x)\ (Dx \land Dy)$$

$$\exists x\ Dx \land \neg\ \exists x\ \exists y\ (\neg\, y = x \land (Dx \land Dy))$$

[D: λx (x *will do*)]

or:

∃x (x *will do* ∧ *nothing other than* x *will do*)

∃x (Dx ∧ (∀y: ¬ y = x) ¬ y *will do*)

$$\exists x\ (Dx \land (\forall y{:}\ \neg\, y = x)\ \neg\, Dy)$$

$$\exists x\ (Dx \land \forall y\ (\neg\, y = x \rightarrow \neg\, Dy))$$

or:

∃x (x *will do* ∧ x *is all that will do*)

∃x (Dx ∧ *everything that will do is such that (*x *is it)*)

∃x (Dx ∧ (∀y: y *will do*) x *is* y)

$$\exists x\ (Dx \land (\forall y{:}\ Dy)\ x = y)$$

$$\exists x\ (Dx \land \forall y\ (Dy \rightarrow x = y))$$

i. *Ann saw more than one assassin*
Ann saw at least two assassins
At least two assassins are such that (Ann saw them)
(∃x: x *is an assassin*) (∃y: y *is an assassin* ∧ ¬ y = x) (*Ann saw* x *and* y)
(∃x: Ax) (∃y: Ay ∧ ¬ y = x) (*Ann saw* x ∧ *Ann saw* y)

$$(\exists x: Ax)\ (\exists y: Ay \land \neg\, y = x)\ (Sax \land Say)$$
$$\exists x\ (Ax \land \exists y\ ((Ay \land \neg\, y = x) \land (Sax \land Say)))$$

[A: λx (x *is an assassin*); S: λxy (x *saw* y); a: *Ann*]

j. *Ann saw exactly two assassins*
Exactly two assassins are such that (Ann saw them)
Two assassins are such that (Ann saw them and no other assassins)
(∃x: x *is an assassin*) (∃y: y *is an assassin* ∧ ¬ y = x) (*Ann saw* x *and* y *and no other assassins*)
(∃x: Ax) (∃y: Ay ∧ ¬ y = x) (*Ann saw* x ∧ *Ann saw* y ∧ *Ann saw no assassin other than* x *and* y)
(∃x: Ax) (∃y: Ay ∧ ¬ y = x) ((Sax ∧ Say) ∧ *no assassin other than* x *and* y *is such that (Ann saw him or her)*)
(∃x: Ax) (∃y: Ay ∧ ¬ y = x) ((Sax ∧ Say) ∧ (∀z: z *is an assassin* ∧ (¬ z = x ∧ ¬ z = y)) ¬ *Ann saw* z)

$$(\exists x: Ax)\ (\exists y: Ay \land \neg\, y = x)\ ((Sax \land Say) \land (\forall z: Az \land (\neg\, z = x \land \neg\, z = y))\ \neg\, Saz)$$
$$\exists x\ (Ax \land \exists y\ ((Ay \land \neg\, y = x) \land ((Sax \land Say) \land \forall z\ ((Az \land (\neg\, z = x \land \neg\, z = y)) \rightarrow \neg\, Saz))))$$

[A: λx (x *is an assassin*); S: λxy (x *saw* y); a: *Ann*]

or:

$$(\exists x: Ax)\ (\exists y: Ay \land \neg\, y = x)\ ((Sax \land Say) \land (\forall z: Az \land Saz)\ (x = z \lor y = z))$$

The formula (∀z: Az ∧ Saz) (x = z ∨ y = z)) used here amounts to x *and* y *together account for all the assassins Ann saw*.

2. a. *Tom found Tom's hat* ∧ (∃x: ¬ x = *Tom's hat*) *Tom lost* x
Tom found his hat ∧ (∃x: x *is other than Tom's hat*) *Tom lost* x
Tom found his hat ∧ *something other than Tom's hat is such that (Tom lost it)*
Tom found his hat ∧ *Tom lost something other than his hat*
Tom found his hat but he lost something else

b. (∃x: x *is a person*) (∃y: y *is a person* ∧ ¬ y = x) x *spoke to* y
(∃x: x *is a person*) (∃y: y *is a person* ∧ y *is other than* x) x *spoke to* y
(∃x: x *is a person*) (∃y: y *is a person other than* x) x *spoke to* y
(∃x: x *is a person*) *someone other than* x *is such that* (x *spoke to him or her)*
(∃x: x *is a person*) x *spoke to someone else*
Someone is such that (he or she spoke to someone else)
Someone spoke to someone else

c. (∀x: x *is a person* ∧ ¬ x = *Mary*) ¬ *Sam recognized* x
(∀x: x *is a person* ∧ x *is other than Mary*) ¬ *Sam recognized* x
(∀x: x *is a person other than Mary*) ¬ *Sam recognized* x
No one other than Mary is such that (Sam recognized him or her)
Sam recognized no one other than Mary
or: Sam didn't recognize anyone other than Mary

d. (∃x: x *is a store*) x *was open* ∧ ¬ (∃x: x *is a store*) (∃y: y *is a store* ∧ ¬ y = x) (x *was open* ∧ y *was open*)

At least one store was open ∧ ¬ (∃x: x *is a store*) (∃y: y *is a store* ∧ ¬ y = x) (x *and* y *were open*)

At least one store was open ∧ ¬ *at least two stores are such that (they were open)*

At least one store was open ∧ ¬ *at least 2 stores were open*

At least one store was open ∧ *at most 1 store was open*

Just one store was open

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