8.2. Uniform generality

8.2.0. Overview

Representation of the relative scope of existentials and universals was an important motivation in the development of logic, and we are now in a position to consider it directly.

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8.2.1. General and uniformly general exemplification

When first discussing quantifier phrases in 7.1.1 , we considered the ambiguity of sentences like

A reporter interviewed each juror.

Quantifiers were designed to represent the alternative interpretations of sentences like this, and we are now in a position to see how they provide an account of the ambiguity in this example.

Since this sentence contains two quantifier phrases, we have two places to begin its analysis; and two different logical forms can result.

A reporter interviewed each juror
A reporter is such that (he or she interviewed each juror)
($\exists$x: x *is a reporter*) x *interviewed each juror*
($\exists$x: x *is a reporter*) ($\forall$y: y *is a juror*) x *interviewed* y

($\exists$x: Rx) ($\forall$y: Jy) Ixy
$\exists$x (Rx $\wedge$ $\forall$y (Jy $\rightarrow$ Ixy))

A reporter interviewed each juror
Each juror is such that (a reporter interviewed him or her)
($\forall$y: y *is a juror*) *a reporter interviewed* y
($\forall$y: y *is a juror*) ($\exists$x: x *is a reporter*) x *interviewed* y

($\forall$y: Jy) ($\exists$x: Rx) Ixy
$\forall$y (Jy $\rightarrow$ $\exists$x (Rx $\wedge$ Ixy))

[I: λxy (x *interviewed* y); J: λx (x *is a juror*); R: λx (x *is a reporter*)]

Symbolically, the only difference in the analyses that use restricted quantifiers lies in the order of those quantifiers. The difference this makes can be seen best by looking at the second step in each analysis:

A reporter is such that he or she interviewed each juror
Each juror is such that a reporter interviewed him or her.

If we use terms that reflect the medieval theories of reference discussed in 7.1.1 , we can say that the difference is due to the fixed indefinite reference of *a reporter* in the first and its variably indefinite reference in the second. Since the latter sentence says

only that each juror was interviewed without claiming that any one reporter conducted all the interviews, it is entailed by the first but does not entail it. Thus the first of the claims is the stronger of the two.

An analysis of logical form using quantifiers is capable of much more than the simple dichotomy between fixed and variably indefinite reference, but a distinction between the sorts of claims represented by the sentences above will be useful in organizing the richer range of possibilities we now have available. In the terms we have been using recently, each of the two is both a generalization and a claim of exemplification. In each case, one of the two aspects is recognized as the overall form of the sentence while the other remains part of the quantified predicate. The first of these two sorts of claims, represented by the first interpretation of the original sentence, says that the property of interviewing each juror is exemplified. This is a general property, one whose predication is expressed by a generalization, so the first sort of claim says that a general property is exemplified. The second makes a generalization, but each instance of this generalization is a claim of exemplification that asserts that a particular juror was interviewed. We will describe this second sort of statement as a claim of **general exemplification**: it says that a relative property is exemplified generally with respect to some domain. In this case, the property of being an interviewing reporter is exemplified generally with respect to jurors; that is, an example of such a reporter can be found for each juror.

This way of looking at the two claims puts them in a parallel position, but we know that they do not stand on the same level as far as their content goes. The first implies the second but is not implied by it. In other words, the first adds information to the second: it says that the second is true in a special way. Let us capture this idea by saying that, while the second is a claim of general exemplification, the first is a claim of **uniformly general exemplification**. In the example above, the second claim says that an example of an interviewing reporter is available generally for jurors, and the first claims that this sort of example is not only available generally but can be chosen in a uniform way, the same reporter can serve as an example no matter what juror we consider. In symbolic terms, we have a claim of general

exemplification whenever a universal and existential have overlapping scope. If their scopes overlap the scope of one includes the other, and we have a claim of uniformly general exemplification when it is the existential that includes the universal.

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8.2.2. Quantifier scope ambiguities

One of the points of section 7.1.1 was that a simple dichotomy is not sufficient to account for the possible ambiguities when more than two quantifier phrases are present. So we need to extend the ideas developed in 8.2.1 to distinguish more than a single pair of claims. Consider the example cited in 7.1.1:

Every reporter asked a question of each juror.

This could be said to generalize along two dimensions (reporters and jurors) and the exemplification (of a question that was asked) might be claimed to be uniform in either or both of these dimensions. That is, the exemplification may be set forth as doubly uniform (the same question could be used as an example in all cases) or as uniform in one dimension only (e.g., we might have to vary the question cited as an example from reporter to reporter but would not need to vary it from juror to juror provided we keep the reporter fixed). This means that there are four interpretations here: the basic claim of doubly general exemplification and three stronger claims, citing uniformity in or the other dimension or in both of them.

With three quantifier phrases, there will be six different symbolic representations of this sentence since there are three choices for the first quantifier phrase to be analyzed and, for each of these, two orders in which the remaining two can be analyzed. The results of these choices are shown below in a way that reflects their logical relations, with stronger claims lower on the page and equivalent claims grouped together.

$$(\forall x{:} Rx)\ (\forall y{:} Jy)\ (\exists z{:} Qz)\ Axzy$$
$$(\forall y{:} Jy)\ (\forall x{:} Rx)\ (\exists z{:} Qz)\ Axzy)$$

[no uniformity claimed]

$(\forall x{:} Rx)\ (\exists z{:} Qz)\ (\forall y{:} Jy)\ Axzy$ $(\forall y{:} Jy)\ (\exists z{:} Qz)\ (\forall x{:} Rx)\ Axzy$

[claims uniformity with respect to jurors]   [claims uniformity with respect to reporters]

$$(\exists z{:} Qz)\ (\forall x{:} Rx)\ (\forall y{:} Jy)\ Axzy$$
$$(\exists z{:} Qz)\ (\forall y{:} Jy)\ (\forall x{:} Rx)\ Axzy)$$

[claims uniformity with respect to both jurors and reporters]

[A: λxyz (x *asked* y *of* z); J: λx (x *is a juror*); Q: λx (x *is a question*);
R: λx (x *is a reporter*)]

Two pairs of these six forms are equivalent, and the distinguishing features of the forms that are not equivalent is the location of the existential quantifier used to represent *a question*— whether it is outside the scope of one or the other of the universal quantifier phrases, outside the scope of both, or outside the scope of neither. For, when the two universals are side by side (and neither binds variables in the restriction of the other), we can interchange them without altering the proposition expressed. The four non-equivalent symbolic possibilities shown above correspond to the four possibilities of uniformity we have noticed.

This brings us to one of the chief lessons of this section. When a claim of general exemplification is uniform with respect to a given dimension of generality, the existential quantifier representing the claim of exemplification should have wider scope than the universal quantifier corresponding to the relevant dimension of generality. When you are faced with choosing the order in which to represent several quantifier phrases and you wonder what effect the order you choose will have on the meaning, you can proceed as follows. First, identify the quantifier phrases making existential claims and the quantifier phrases that generalize on one or another dimension. Then ask, for each existential quantifier phrase and each generalizing one, whether the existential claims that exemplification is uniform on the dimension referred to by the generalizing phrase. If the existential makes this claim, it should be dealt with first; if it does not, the generalizing quantifier phrase should be given wider scope. The answers to these questions will settle the relative order of treatment for each pair consisting of an existential and a universal.

This approach may not settle all questions about the order in which quantifier phrases are to be treated in claims of general exemplification, but the remaining questions can be settled arbitrarily without any effect on the meaning ascribed to the sentence. For example, if *a question* is held to claim an exemplification that is uniform with respect to both reporters and jurors, we know the existential quantifier phrase must be treated first. Nothing is implied about the order in which we go on to handle *every reporter* and *each juror*, but that order also has no affect on the content of the result.

The language we have been using to speak about the process of settling the relative scope of quantifier phrases is open to one sort

of misinterpretation. Although there is no way to arrange overlapping scopes to claim uniformity in each of two dimensions without claiming uniformity in both, this does not mean that claims of uniformity in each of two dimensions together entail a claim of uniformity in both. Since there might be different examples exhibiting each sort of uniformity, there can be situations where both sorts of partial uniformity occur without full uniformity. For example, it may be that reporters had favorite questions and also that there was an obvious question for each juror while still there was no one question that appeared in all interviews. In short, while the conjunction

(∀x: Rx) (∃z: Qz) (∀y: Jy) Axzy ∧ (∀y: Jy) (∃z: Qz) (∀x: Rx) Axzy

says more than either of its conjuncts, it still says less than a claim of doubly uniform exemplification.

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8.2.3. Controlling ambiguity

Although ambiguity is hard to avoid entirely in English when claims of exemplification are combined with generalization, there are some indicators that tend to make a given interpretation more likely. There are, of course, many contextual indicators of the correct interpretation, and these can be quite strong, but explicit verbal indication is rarely conclusive.

Probably the most important verbal indication of relative scope is the simplest, word order. All things being equal, the first quantifier phrase is understood to be the main one. This means that the passive voice plays an important role in indicating the sort of claim we wish to make since it enables us to alter word order and promote a given quantifier phrase to subject position. Subject-predicate expansion can help in the same way when it is stylistically acceptable. But the effect is subject-predicate expansion is due also to another syntactic indicator: quantifier phrases within relative clauses are usually understood as having narrower scope than those outside them. In the case of existentials, use of the *there-is* form, which typically also involves a relative clause, will accomplish the same thing as an expanded form, and usually with better style. For example, *There is* [or *was*] *a reporter who interviewed each juror* definitely claims uniformity.

In addition to such syntactic indicators, word choice can play a role. The words *each*, *every*, and *all* (and *any* when it is grammatically possible) used to express generalizations and the words *some* and *a* used to express existential claims lend varying degrees of force to a quantifier phrase's claim to a wide scope. Perhaps these words never overcome the effects of word order, but they can moderate it, as may been seen with the following four restatements of our original sentence:

Some reporter interviewed every juror

A reporter interviewed each juror

Every juror was interviewed by some reporter

Each juror was interviewed by a reporter

The guiding idea here is that the word *some* marks a stronger claim to wide scope than the word *a* does and that the word *each*

marks a stronger claim than the word *every*. The sentence at the upper left is the most likely to be understood as a claim of uniformly general exemplification and the one at the lower right is the least likely; the other two cases are intermediate, with word order probably beating out word choice so that the sentence at the upper right is the second most likely to be understood to involve a claim of uniformity.

But, while the choices of wording mentioned so far are perhaps never enough to overcome the effects of word order, there are other words choices that are. There is a use of the word *certain* that seems to function only to mark an existential quantifier phrase as having wide scope. If we add this word to the existential quantifier phrase in the sentence at the lower right, we get *Each juror was interviewed by a certain reporter* and this sentence stands a very good chance of being interpreted as a claim of uniform exemplification in spite of word order and other choices of wording. On the other hand, if we add *or other* to the existential in the top left sentence, we get *Some reporter or other interviewed every juror*, which is less likely to carry an implication of uniformity. Context also plays a role in the effect of *or other*. For example, if we were discussing the events surrounding a *sort* of sensational trial that was typical of some historical era (rather than discussing a particular example of such a trial), there would be an implicit generalization concerning such trials in what we said. The use of *or other* might then simply cancel a claim of uniformity with respect to trials while allowing it to be maintained with respect to jurors.

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8.2.s. Summary

English sentences that involve both generalization and claims of exemplification are often ambiguous, and the differences between interpretations can be expressed in analyses of them by the relative scope of universal and existential quantifiers. We will refer to a sentence that mixes generalization and with a claim of exemplification as a claim of general exemplification . One in which the existential has wider scope than the universal can be thought of as a claim of uniformly general exemplification because it asserts that a single example can be given that suffices for all instances of the generalization.

When more than two quantifier phrases are present, an existential may be classified as making or not making a claim of uniformity with respect to each universal, giving rise to a variety of uniformity claims that a sentence may be understood to make. The issue of quantifier scope can thus be addressed by asking, for each of the dimensions of generality with which a claim of exemplification is asserted, whether the exemplification is claimed to uniform in that dimension; this settles the relative scope of each existential with respect to each universal, and the relative scope of contiguous universals and contiguous existentials does not matter.

The ambiguity in sentences involving both existentials and universals is hard to eliminate, but syntax and word choice can help. The first quantifier phrase is usually understood to have widest scope, and a quantifier phrase in a relative clause usually has its scope limited to that clause (a fact that makes the *there-is* form useful). The choice of quantifier words can counteract the effect of word order to some extent, and the use of the special quantifier phrases *a certain* X and *some* X *or other* will strongly tend, respectively, to advance or to renounce a claim of uniformity.

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8.2.x. Exercise questions

1. Analyze the following in as much detail as possible. Since it is difficult to completely avoid ambiguity in English sentences that both generalize and make existential claims, alternative non-equivalent analyses are possible in some cases. You should choose an analysis that captures the most likely interpretation (or one of the most likely ones). The answers will represent my own judgment about this.

   a. *Everyone has seen a bear*.
   b. *Everyone was talking about a certain movie*.
   c. *A capital was chosen by each state*.
   d. *There is a capital that was chosen by each state*.
   e. *Someone who no reporter knew leaked the information*.
   f. *A head of a horse is the head of a mammal*.
   g. *Everyone who has seen a rainbow has seen a rainstorm*.
   h. *Every child was given a toy by each Santa*.
   i. *There is a toy that was given to every child by each Santa*.

2. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below by the intensional interpretations that are provided for each group.

   a. ∀x ∃y Dxy   [D: λxy (x *depends on* y)]
   b. ∃x ∀y Dxy
   c. ∀x ∃y Dyx
   d. ∃x ∀y Dyx

   e. (∀x: Px ∧ Hx) (∃y: Py) Axy   [A: λxy (x *admires* y);
   f. (∃y: Py) (∀x: Px ∧ Hx) Axy   H: λx (x *is humble*);
   g. ¬ (∀x: Px ∧ (∃y: Py) Axy) Hx   P: λx (x *is a person*)]

   h. ¬ (∃x: Px) (∀y: Py ∧ Syx) Sxy   [P: λx (x *is a person*);
                                        S: λxy (x *has seen* y)]

   i. ¬ (∃x: Px ∧ (∀y: ¬ (Py ∧ Syx)) ¬ Sxy) Ex
      [E: λx (x *is an extrovert*); P: λx (x *is a person*); S: λxy (x *has spoken to* y)]

8.2.xa. Exercise answers

1. a. *Everyone has seen a bear*
      Everyone is such that (he or she has seen a bear)
      (∀x: x *is a person*) x *has seen a bear*
      (∀x: Px) *a bear is such that (*x *has seen it)*
      (∀x: Px) (∃y: y *is a bear*) x *has seen* y

$$(∀x: Px) (∃y: By) Sxy$$
$$∀x (Px → ∃y (By ∧ Sxy))$$

   [B: λx (x *is a bear*); P: λx (x *is a person*); S: λxy (x *has seen* y)]

   b. *Everyone was talking about a certain movie*
      A certain movie is such that (everyone was talking about it)
      (∃x: x *is a movie*) *everyone was talking about* x
      (∃x: Mx) *everyone is such that (he or she was talking about* x*)*
      (∃x: Mx) (∀y: y *is a person*) y *was talking about* x

$$(∃x: Mx) (∀y: Py) Tyx$$
$$∃x (Mx ∧ ∀y (Py → Tyx))$$

   [M: λx (x *is a movie*); P: λx (x *is a person*); T: λxy (x *was talking about* y)]

   c. *A capital was chosen by each state*
      Each state is such that (a capital was chosen by it)
      (∀x: x *is a state*) *a capital was chosen by* x
      (∀x: Sx) *a capital is such that (it was chosen by* x*)*
      (∀x: Sx) (∃y: y *is a capital*) y *was chosen by* x

$$(∀x: Sx) (∃y: Cy) Hyx$$
$$∀x (Sx → ∃y (Cy ∧ Hyx))$$

   [C: λx (x *is a capital*); H: λxy (x *was chosen by* y); S: λx (x *is a state*)]

   d. *There is a capital that was chosen by each state*
      Something is a capital that was chosen by each state
      Something is such that (it is a capital that was chosen by each state)
      ∃x x *is a capital that was chosen by each state*
      ∃x (x *is a capital* ∧ x *was chosen by each state*)
      ∃x (Cx ∧ *each state is such that (*x *was chosen by it))*

∃x (Cx ∧ (∀y: y *is a state*) x *was chosen by* y)

$$\exists x\ (Cx \land (\forall y: Sy)\ Hxy)$$
$$\exists x\ (Cx \land \forall y\ (Sy \to Hxy))$$

[C: λx (x *is a capital*); H: λxy (x *was chosen by* y); S: λx (x *is a state*)]

e. *Someone who no reporter knew leaked the information*
Someone who no reporter knew is such that (he or she leaked the information)
(∃x: x *is a person who no reporter knew*) x *leaked the information*
(∃x: x *is a person* ∧ *no reporter knew* x) Lxi
(∃x: Px ∧ *no reporter is such that (he or she knew* x*)*) Lxi
(∃x: Px ∧ (∀y: y *is a reporter*) ¬ y *knew* x) Lxi

$$(\exists x: Px \land (\forall y: Ry)\ \neg\ Kyx)\ Lxi$$
$$\exists x\ ((Px \land \forall y\ (Ry \to \neg\ Kyx)) \land Lxi)$$
$$or: (\exists x: Px \land \neg\ (\exists y: Ry)\ Kyx)\ Lxi$$

[K: λxy (x knew y); L: λx (x *leaked* y); P: λx (x *is a person*); R: λx (x *is a reporter*); i: *the information*]

f. *A head of a horse is the head of a mammal*
Every head of a horse is such that (it is the head of a mammal)
(∀x: x *is the head of a horse*) x *is the head of a mammal*
(∀x: *a horse is such that (x is the head of it)*) *a mammal is such that (x is the head of it)*
(∀x: (∃y: y *is a horse*) x *is the head of* y) (∃z: z *is a mammal*) x *is the head of* z
(∀x: (∃y:Hy) x = *the head of* y) (∃z: Mz) x = *the head of* z

$$(\forall x: (\exists y: Hy)\ x = hy) (\exists z: Mz)\ x = hz$$
$$\forall x\ (\exists y\ (Hy \land x = hy) \to \exists z\ (Mz \land x = hz))$$
$$or: (\forall x: (\exists y: Hy)\ Dxy) (\exists z: Mz)\ Dxz$$

[D: λxy (x *is a head of* y); H: λx (x *is a horse*); M: λx (x *is a mammal*); h: λx (*the head of* x)]

In this interpretation, which seems most natural given the content of the sentence, *a* is understood to indicate a generalization rather than a claim of exemplification. That is, it amounts to *any* in a use that is equivalent to *every* rather than in contrast with it. It appears in a location where *any* would not contrast with *every*, so if the sentence were understood to make a claim of exemplification, substituting

any for *a* would change the meaning.

g. *Everyone who has seen a rainbow has seen a rainstorm*
Everyone who has seen a rainbow is such that (he or she has seen a rainstorm)
(∀x: x *is person who has seen a rainbow*) x *has seen a rainstorm*
(∀x: x *is person* ∧ x *has seen a rainbow*) *a rainstorm is such that (x has seen it)*
(∀x: x *is person* ∧ *a rainbow is such that (x has seen it)*) (∃z: z *is a rainstorm*) x *has seen* z
(∀x: x *is person* ∧ (∃y: y *is a rainbow*) x *has seen* y) (∃z: Rz) Sxz

$$(\forall x: Px \land (\exists y: Ry)\ Sxy) (\exists z: Tz)\ Sxz$$
$$\forall x\ (\ (Px \land \exists y\ (Ry \land Sxy)) \to \exists z\ (Tz \land Sxz)\ )$$

[P: λx (x *is a person*); R: λx (x *is a rainbow*); S: λxy (x *has seen* y); T: λx (x *is a rainstorm*)]

h. *Every child was given a toy by each Santa*
Every child is such that (he or she was given a toy by each Santa)
(∀x: x *is a child*) x *was given a toy by each Santa*
(∀x: Cx) *each Santa is such that (x was given a toy by him or her)*
(∀x: Cx) (∀y: y *is a Santa*) x *was given a toy by* y
(∀x: Cx) (∀y: Sy) *a toy is such that (x was given it by y)*
(∀x: Cx) (∀y: Sy) (∃z: z *is a toy*) x *was given* z *by* y

$$(\forall x: Cx) (\forall y: Sy) (\exists z: Tz)\ Gxzy$$
$$\forall x\ (Cx \to \forall y\ (Sy \to \exists z\ (Tz \land Gxzy)))$$

[C: λx (x *is a child*); G: λxyz (x *was given* y *by* z); S: λx (x *is a Santa*); T: λx (x *is a toy*)]

Notice that, in spite of the capitalization, *Santa* is not used here as a proper name but instead as a sort of job title. As a result it is represented not by an individual term but instead by a predicate. For representation by an individual term to be appropriate, it would have to be possible to paraphrase the sentence using *each thing that is Santa* rather than *each thing that is a Santa*.

i. *There is a toy that every child was given by each Santa*
Something is a toy that every child was given by each Santa
∃x x *is a toy that every child was given by each Santa*

∃x (x *is a toy* ∧ *every child was given* x *by each Santa*)

∃x (Tx ∧ *every child is such that (he or she was given* x *by each Santa)*)

∃x (Tx ∧ (∀y: y *is a child*) y *was given* x *by each Santa*)

∃x (Tx ∧ (∀y: Cy) *each Santa is such that (y was given* x *by him or her)*)

∃x (Tx ∧ (∀y: Cy) (∀z: z *is a Santa*) y *was given* x *by* z)

∃x (Tx ∧ (∀y: Cy) (∀z: Sz) Gyxz)

∃x (Tx ∧ ∀y (Cy → ∀z (Sz → Gyxz)))

[C: λx (x *is a child*); G: λxyz (x *was given* y *by* z); S: λx (x *is a Santa*); T: λx (x *is a toy*)]

2. a. ∀x ∃y x *depends on* y

∀x *something is such that (* x *depends on it)*

∀x x *depends on something*

Everything is such that (it depends on something)

Everything depends on something

b. ∃x ∀y x *depends on* y

∃x *everything is such that (* x *depends on it)*

∃x x *depends on everything*

Something is such that (it depends on everything)

Something depends on everything

c. ∀x ∃y y *depends on* x

∀x *something is such that (it depends on* x*)*

∀x *something depends on* x

Everything is such that something depends on it
or: Everything has something depending on it
or (perhaps): Something or other depends on each thing

d. ∃x ∀y y *depends on* x

∃x *everything is such that (it depends on* x*)*

∃x *everything depends on* x

Something is such that everything depends on it
or: Something has everything depending on it
or: There is something that everything depends on
or (perhaps): All things depend on a certain thing

e. (∀x: x *is a person* ∧ x *is humble*) (∃y: y *is a person*) x *admires* y

(∀x: x *is a humble person*) *someone is such that (*x *admires him or her)*

(∀x: x *is a humble person*) x *admires someone*

Every humble person is such that (he or she admires someone)

Every humble person admires someone
or: Everyone who is humble admires someone

f. (∃y: y *is a person*) (∀x: x *is a person* ∧ x *is humble*) x *admires* y

(∃y: y *is a person*) (∀x: x *is a humble person*) x *admires* y

(∃y: y *is a person*) *every humble person is such that (he or she admires* y*)*

(∃y: y *is a person*) *every humble person admires* y

Someone is such that every humble person [or: everyone who is humble] admires him or her
or: Someone has every humble person admiring him or her
or: There is someone [or: a person] who every humble person admires
or (perhaps): All who are humble admire a certain person

g. ¬ (∀x: x *is a person* ∧ (∃y: y *is a person*) x *admires* y) x *is humble*

¬ (∀x: x *is a person* ∧ *someone is such that (*x *admires him or her)*) x *is humble*

¬ (∀x: x *is a person* ∧ x *admires someone*) x *is humble*

¬ (∀x: x *is a person who admires someone*) x *is humble*

¬ *everyone who admires someone is such that (he or she is humble)*

¬ *everyone who admires someone is humble*

Not everyone who admires someone is humble
or: Not everyone who admires anyone is humble

h. ¬ (∃x: x *is a person*) (∀y: y *is a person* ∧ y *has seen* x) x *has seen* y

¬ (∃x: x *is a person*) (∀y: y *is a person who has seen* x) x *has seen* y

¬ (∃x: x *is a person*) *everyone who has seen* x *is such that (*x *has seen him or her)*

¬ (∃x: x *is a person*) x *has seen everyone who has seen* x

¬ *someone is such that (he or she has seen everyone who has seen him or her)*

¬ *someone has seen everyone who has seen him or her*

No one has seen everyone who has seen him or her

i.   ¬ (∃x: x *is a person* ∧ (∀y: ¬ (y *is a person* ∧ y *has spoken to* x)) ¬ x *has spoken to* y) x *is an extrovert*

¬ (∃x: x *is a person* ∧ (∀y: ¬ y *is a person who has spoken to* x) ¬ x *has spoken to* y) x *is an extrovert*

¬ (∃x: x *is a person* ∧ *only people who have spoken to* x *are such that*(x *has spoken to them)*) x *is an extrovert*

¬ (∃x: x *is a person* ∧ x *has spoken only to people who have spoken to* x) x *is an extrovert*

¬ (∃x: x *is a person who has spoken only to people who have spoken to him or her*) x *is an extrovert*

¬ *someone who has spoken only to people who have spoken to him or her is such that (he or she is an extrovert)*

¬ *someone who has spoken only to people who have spoken to him or her is an extrovert*

No one who has spoken only to people who have spoken to him or her is an extrovert