## 8.2.xa. Exercise answers

1. **a.** *Everyone has seen a bear*
   *Everyone is such that (he or she has seen a bear)*
   (∀x: x *is a person*) x *has seen a bear*
   (∀x: Px) *a bear is such that (*x *has seen it)*
   (∀x: Px) (∃y: y *is a bear*) x *has seen* y

   $$(∀x: Px) (∃y: By) Sxy$$
   $$∀x (Px → ∃y (By ∧ Sxy))$$

   [B: λx (x *is a bear*); P: λx (x *is a person*); S: λxy (x *has seen* y)]

   **b.** *Everyone was talking about a certain movie*
   *A certain movie is such that (everyone was talking about it)*
   (∃x: x *is a movie*) *everyone was talking about* x
   (∃x: Mx) *everyone is such that (he or she was talking about* x*)*
   (∃x: Mx) (∀y: y *is a person*) y *was talking about* x

   $$(∃x: Mx) (∀y: Py) Tyx$$
   $$∃x (Mx ∧ ∀y (Py → Tyx))$$

   [M: λx (x *is a movie*); P: λx (x *is a person*); T: λxy (x *was talking about* y)]

   **c.** *A capital was chosen by each state*
   *Each state is such that (a capital was chosen by it)*
   (∀x: x *is a state*) *a capital was chosen by* x
   (∀x: Sx) *a capital is such that (it was chosen by* x*)*
   (∀x: Sx) (∃y: y *is a capital*) y *was chosen by* x

   $$(∀x: Sx) (∃y: Cy) Hyx$$
   $$∀x (Sx → ∃y (Cy ∧ Hyx))$$

   [C: λx (x *is a capital*); H: λxy (x *was chosen by* y); S: λx (x *is a state*)]

   **d.** *There is a capital that was chosen by each state*
   *Something is a capital that was chosen by each state*
   *Something is such that (it is a capital that was chosen by each state)*
   ∃x x *is a capital that was chosen by each state*
   ∃x (x *is a capital* ∧ x *was chosen by each state*)
   ∃x (Cx ∧ *each state is such that (*x *was chosen by it))*

∃x (Cx ∧ (∀y: y *is a state*) x *was chosen by* y)

$$\exists x\ (Cx \land (\forall y: Sy)\ Hxy)$$
$$\exists x\ (Cx \land \forall y\ (Sy \to Hxy))$$

[C: λx (x *is a capital*); H: λxy (x *was chosen by* y); S: λx (x *is a state*)]

**e.** *Someone who no reporter knew leaked the information*
*Someone who no reporter knew is such that (he or she leaked the information)*
(∃x: x *is a person who no reporter knew*) x *leaked the information*
(∃x: x *is a person* ∧ *no reporter knew* x) Lxi
(∃x: Px ∧ *no reporter is such that (he or she knew x)*) Lxi
(∃x: Px ∧ (∀y: y *is a reporter*) ¬ y *knew* x) Lxi

$$(\exists x: Px \land (\forall y: Ry)\ \neg\ Kyx)\ Lxi$$
$$\exists x\ ((Px \land \forall y\ (Ry \to \neg\ Kyx)) \land Lxi)$$
*or:* $(\exists x: Px \land \neg\ (\exists y: Ry)\ Kyx)\ Lxi$

[K: λxy (x knew y); L: λx (x *leaked* y); P: λx (x *is a person*); R: λx (x *is a reporter*); i: *the information*]

**f.** *A head of a horse is the head of a mammal*
*Every head of a horse is such that (it is the head of a mammal)*
(∀x: x *is the head of a horse*) x *is the head of a mammal*
(∀x: *a horse is such that* (x *is the head of it*)) *a mammal is such that* (x *is the head of it*)
(∀x: (∃y: y *is a horse*) x *is the head of* y) (∃z: z *is a mammal*) x *is the head of* z
(∀x: (∃y:Hy) x = *the head of* y) (∃z: Mz) x = *the head of* z

$$(\forall x: (\exists y: Hy)\ x = hy)\ (\exists z: Mz)\ x = hz$$
$$\forall x\ (\exists y\ (Hy \land x = hy) \to \exists z\ (Mz \land x = hz))$$
*or:* $(\forall x: (\exists y: Hy)\ Dxy)\ (\exists z: Mz)\ Dxz$

[D: λxy (x *is a head of* y); H: λx (x *is a horse*); M: λx (x *is a mammal*); h: λx (*the head of* x)]

In this interpretation, which seems most natural given the content of the sentence, *a* is understood to indicate a generalization rather than a claim of exemplification. That is, it amounts to *any* in a use that is equivalent to *every* rather than in contrast with it. It appears in a location where *any* would not contrast with *every*, so if the sentence were understood to make a claim of exemplification, substituting

*any* for *a* would change the meaning.

**g.** *Everyone who has seen a rainbow has seen a rainstorm*

*Everyone who has seen a rainbow is such that (he or she has seen a rainstorm)*

($\forall$x: x *is person who has seen a rainbow*) x *has seen a rainstorm*

($\forall$x: x *is person* $\land$ x *has seen a rainbow*) *a rainstorm is such that (x has seen it)*

($\forall$x: x *is person* $\land$ *a rainbow is such that (x has seen it)*) ($\exists$z: z *is a rainstorm*) x *has seen* z

($\forall$x: x *is person* $\land$ ($\exists$y: y *is a rainbow*) x *has seen* y) ($\exists$z: Rz) Sxz

$$(\forall x: Px \land (\exists y: Ry)\ Sxy)\ (\exists z: Tz)\ Sxz$$
$$\forall x\ (\ (Px \land \exists y\ (Ry \land Sxy)) \to \exists z\ (Tz \land Sxz)\ )$$

[P: $\lambda$x (x *is a person*); R: $\lambda$x (x *is a rainbow*); S: $\lambda$xy (x *has seen* y); T: $\lambda$x (x *is a rainstorm*)]

**h.** *Every child was given a toy by each Santa*

*Every child is such that (he or she was given a toy by each Santa)*

($\forall$x: x *is a child*) x *was given a toy by each Santa*

($\forall$x: Cx) *each Santa is such that (x was given a toy by him or her)*

($\forall$x: Cx) ($\forall$y: y *is a Santa*) x *was given a toy by* y

($\forall$x: Cx) ($\forall$y: Sy) *a toy is such that (x was given it by y)*

($\forall$x: Cx) ($\forall$y: Sy) ($\exists$z: z *is a toy*) x *was given* z *by* y

$$(\forall x: Cx)\ (\forall y: Sy)\ (\exists z: Tz)\ Gxzy$$
$$\forall x\ (Cx \to \forall y\ (Sy \to \exists z\ (Tz \land Gxzy)))$$

[C: $\lambda$x (x *is a child*); G: $\lambda$xyz (x *was given* y *by* z); S: $\lambda$x (x *is a Santa*); T: $\lambda$x (x *is a toy*)]

Notice that, in spite of the capitalization, *Santa* is not used here as a proper name but instead as a sort of job title. As a result it is represented not by an individual term but instead by a predicate. For representation by an individual term to be appropriate, it would have to be possible to paraphrase the sentence using *each thing that is Santa* rather than *each thing that is a Santa*.

**i.** *There is a toy that every child was given by each Santa*

*Something is a toy that every child was given by each Santa*

$\exists$x x *is a toy that every child was given by each Santa*

∃x (x *is a toy* ∧ *every child was given* x *by each Santa*)
∃x (Tx ∧ *every child is such that (he or she was given* x
  *by each Santa)*)
∃x (Tx ∧ (∀y: y *is a child*) y *was given* x *by each Santa*)
∃x (Tx ∧ (∀y: Cy) *each Santa is such that (y was given* x
  *by him or her)*)
∃x (Tx ∧ (∀y: Cy) (∀z: z *is a Santa*) y *was given* x *by* z)

$$\exists x \, (Tx \wedge (\forall y: Cy)(\forall z: Sz) \, Gyxz)$$
$$\exists x \, (Tx \wedge \forall y \, (Cy \rightarrow \forall z \, (Sz \rightarrow Gyxz)))$$

[C: λx (x *is a child*); G: λxyz (x *was given* y *by* z); S: λx (x
*is a Santa*); T: λx (x *is a toy*)]

**2.** **a.** ∀x ∃y x *depends on* y
  ∀x *something is such that (* x *depends on it)*
  ∀x x *depends on something*
  *Everything is such that (it depends on something)*

  *Everything depends on something*

**b.** ∃x ∀y x *depends on* y
  ∃x *everything is such that (* x *depends on it)*
  ∃x x *depends on everything*
  *Something is such that (it depends on everything)*

  *Something depends on everything*

**c.** ∀x ∃y y *depends on* x
  ∀x *something is such that (it depends on* x*)*
  ∀x *something depends on* x

  *Everything is such that something depends on it*
  *or: Everything has something depending on it*
  *or (perhaps): Something or other depends on each thing*

**d.** ∃x ∀y y *depends on* x
  ∃x *everything is such that (it depends on* x*)*
  ∃x *everything depends on* x

  *Something is such that everything depends on it*
  *or: Something has everything depending on it*
  *or: There is something that everything depends on*
  *or (perhaps): All things depend on a certain thing*

**e.** (∀x: x *is a person* ∧ x *is humble*) (∃y: y *is a person*) x
  *admires* y
  (∀x: x *is a humble person*) *someone is such that (x
  admires him or her)*

(∀x: x *is a humble person*) x *admires someone*
*Every humble person is such that (he or she admires*
  *someone)*

*Every humble person admires someone*
*or: Everyone who is humble admires someone*

**f.**   (∃y: y *is a person*) (∀x: x *is a person* ∧ x *is humble*) x
  *admires* y
  (∃y: y *is a person*) (∀x: x *is a humble person*) x *admires* y
  (∃y: y *is a person*) *every humble person is such that (he*
    *or she admires* y*)*
  (∃y: y *is a person*) *every humble person admires* y

*Someone is such that every humble person* [*or: everyone*
*who is humble*] *admires him or her*
*or: Someone has every humble person admiring him or*
*her*
*or: There is someone* [*or: a person*] *who every humble*
*person admires*
*or (perhaps): All who are humble admire a certain*
*person*

**g.**   ¬ (∀x: x *is a person* ∧ (∃y: y *is a person*) x *admires* y) x *is*
  *humble*
  ¬ (∀x: x *is a person* ∧ *someone is such that (x admires*
    *him or her)*) x *is humble*
  ¬ (∀x: x *is a person* ∧ x *admires someone*) x *is humble*
  ¬ (∀x: x *is a person who admires someone*) x *is humble*
  ¬ *everyone who admires someone is such that (he or she*
    *is humble)*
  ¬ *everyone who admires someone is humble*

*Not everyone who admires someone is humble*
*or: Not everyone who admires anyone is humble*

**h.**   ¬ (∃x: x *is a person*) (∀y: y *is a person* ∧ y *has seen* x) x
  *has seen* y
  ¬ (∃x: x *is a person*) (∀y: y *is a person who has seen* x) x
  *has seen* y
  ¬ (∃x: x *is a person*) *everyone who has seen* x *is such that*
  (x *has seen him or her)*
  ¬ (∃x: x *is a person*) x *has seen everyone who has seen* x
  ¬ *someone is such that (he or she has seen everyone who*
    *has seen him or her)*

*¬ someone has seen everyone who has seen him or her*

*No one has seen everyone who has seen him or her*

**i.** ¬ (∃x: x *is a person* ∧ (∀y: ¬ (y *is a person* ∧ y *has spoken to* x)) ¬ x *has spoken to* y) x *is an extrovert*

¬ (∃x: x *is a person* ∧ (∀y: ¬ y *is a person who has spoken to* x) ¬ x *has spoken to* y) x *is an extrovert*

¬ (∃x: x *is a person* ∧ *only people who have spoken to* x *are such that*(x *has spoken to them)*) x *is an extrovert*

¬ (∃x: x *is a person* ∧ x *has spoken only to people who have spoken to* x) x *is an extrovert*

¬ (∃x: x *is a person who has spoken only to people who have spoken to him or her*) x *is an extrovert*

¬ *someone who has spoken only to people who have spoken to him or her is such that (he or she is an extrovert)*

¬ *someone who has spoken only to people who have spoken to him or her is an extrovert*

*No one who has spoken only to people who have spoken to him or her is an extrovert*

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