

7.7.xa. Exercise answers

These answers handle restricted quantifiers using the rules RUG, SB, SC, and MCR rather than RUP and RUC; see [7.6.xa] for advice for converting them into derivations that instead use the latter rules along with rules for conditionals.

1.

Fa <hr style="border: 0; border-top: 1px solid black; margin: 0;"/> \textcircled{b} $\neg Fb$ \circ \perp	$Fa, \neg Fb \Rightarrow \perp$ 2	
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> Fb	1	
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> $\forall x Fx$	1 UG	
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/>	2 IP	

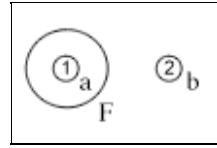
2.

$\forall x Rxx$	a:1, b:3	
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> Raa	1 UI	
\textcircled{b} Rbb	3 UI	
$\neg Rba$ \circ \perp	$Raa, Rbb, \neg Rba \Rightarrow \perp$ 4	
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> Rba	4 IP	
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> $\forall x Rxa$	2 UG	

3.

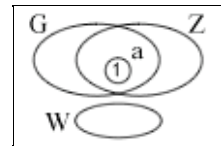
$\forall x \neg Fx$	a:3
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> $\forall x Fx$	a:2
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> Fa	2 UI (4)
$\neg Fa$	3 UI (4)
\bullet \perp	4 Nc
<hr style="border: 0; border-top: 1px solid black; margin: 0;"/> $\neg \forall x Fx$	1 RAA

	$\neg \forall x Fx$				
	(a)	Fa			
		(b)	$\neg Fb$		
			o	$Fa, \neg Fb \Rightarrow \perp$	
			\perp	5	
5 IP		Fb		4	
4 UG		$\forall x Fx$		3	
3 CR		\perp		2	
2 RAA		$\neg Fa$		1	
1 UG		$\forall x \neg Fx$			



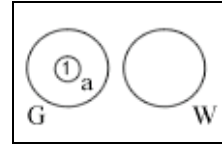
4.

	$(\forall x: Wx) \neg Gx$				
	$(\forall x: Zx) \neg Wx$				a:4
		(a)	Za		(2)
2 SB			$\neg Wa$		
			Ga		(4)
4 SC			$\neg Wa$		
			o	$Za, \neg Wa, Ga \Rightarrow \perp$	
			\perp	3	
3 RAA		$\neg Ga$			
1 RUG		$(\forall x: Zx) \neg Gx$			

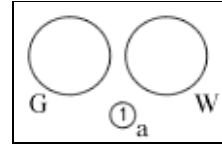


5.

	$(\forall x: Wx) \neg Gx$	a:2
	$(\forall x: Wx) Gx$	a:4,a:6
	$\neg Wa$	
	$\neg Wa$	
	\circ	$\neg Wa \not\Rightarrow \perp$
	\perp	5
5 IP	Wa	4
	Ga	
	\circ	$\neg Wa, Ga \not\Rightarrow \perp$
	\perp	4
4 MCR	\perp	3
3 IP	Wa	2
	$\neg Ga$	(6)
	$\neg Wa$	
	\circ	$\neg Wa, \neg Ga \not\Rightarrow \perp$
	\perp	2
2 MCR	\perp	1
1 RAA	$\neg (\forall x: Wx) Gx$	

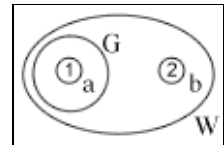


divides the 1st and 2nd gaps

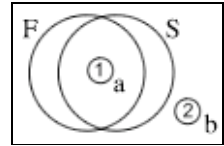


divides the 1st and 3rd gaps

	$\neg (\forall x: Wx) Gx$	
	\textcircled{a}	
	Wa	
	Ga	
	\textcircled{b}	
	Wb	
	$\neg Gb$	
	\circ	$Wa, Ga, Wb, \neg Gb \not\Rightarrow \perp$
	\perp	5
5 IP	Gb	4
4 RUG	$(\forall x: Wx) Gx$	3
3 CR	\perp	2
2 RAA	$\neg Ga$	1
1 RUG	$(\forall x: Wx) \neg Gx$	



6.	$\forall x (Fx \vee \neg Sx)$	a:4, b:9
	$\neg \forall x Fx$	
	(a)	
	Sa	(5)
4 UI	Fa $\vee \neg Sa$	5
5 MTP	Fa	
	(b)	
	$\neg Fb$	(10)
9 UI	Fb $\vee \neg Sb$	10
10 MTP	$\neg Sb$	
	o	Sa, Fa, $\neg Fb, \neg Sb \Rightarrow \perp$
	\perp	8
8 IP	Fb	7
7 UG	$\forall x Fx$	6
6 CR	\perp	3
3 RAA	$\neg Sa$	2
2 UG	$\forall y \neg Sy$	1
1 PE	$\forall x Fx \vee \forall y \neg Sy$	



7.	$\neg \forall x \neg \forall y Rxy$	
	(a)	
	$\forall y \neg Ray$	a:3, b:6
3 UI	$\neg Raa$	
	(b)	
6 UI	$\neg Rab$	
	$\forall y Rby$	a:8, b:9
8 UI	Rba	
9 UI	Rbb	
	o	$\neg Raa, \neg Rab, Rba, Rbb \Rightarrow \perp$
	\perp	7
7 RAA	$\neg \forall y Rby$	5
5 UG	$\forall x \neg \forall y Rxy$	4
4 CR	\perp	2
2 RAA	$\neg \forall y \neg Ray$	1
1 UG	$\forall x \neg \forall y \neg Rxy$	

