7.7.4. Effectuality

All that remains in our argument for completeness is to show that any fully developing path is divided by an interpretation. This is in some ways like an argument that could be made for systems of earlier chapters. For them it can be shown that any dead-end open gap is divided by an interpretation that also divides all ancestors of the gap. But in the system we are looking at now, while a fully developing path might end with a dead-end gap, it might instead develop infinitely; and a path that develops infinitely is quite a different beast from a gap that has reached a dead end. And their differences will affect the way we argue for the existence of structures dividing them.

We need some new ways of talking about resources and goals. The ***accumulated resources*** of a path include all sentences that ever appear as active resources in the course of its development. Its ***ultimate resources*** are the accumulated resources that are not exploited (not even partially) at any stage in its development. In a fully developing path, the ultimate resources will consist solely of atomic sentences and negated atomic sentences. The ***accumulated goals*** of a path are all the sentences that ever appear as goals in the course of its development. In a fully developing gap, any such goal, apart from $\bot$, will eventually be planned for. Since a structure divides a path if and only if it divides all gaps in the path, a path-dividing structure makes all of the accumulated resources of a path true and all of the accumulated goals false.

There are two parts to the argument that any fully developing path is divided. One involves considerations used to establish sufficiency in the old sense, and the other involves considerations related to the safety of rules. Specifically, we will show first that, given any fully developing path, we can find some structure that (i) makes the ultimate resources all true and (ii) assigns each value in its referential range to some term appearing in the ultimate resources. Secondly, we will show that such a structure divides the path. The first of these arguments really involves nothing we did not see already in 6.4.3. The concrete calculations we carried out there may no longer be possible since we may be dealing with infinitely many terms, but the definitions continue to apply and the arguments are essentially unchanged. However, we must make one stipulation that was left open there: each value of the referential range we set up must correspond to one of the alias sets derived from the ultimate resources. This handles our requirement (ii) that the structure assign each value in its range to some term—or, more briefly, that it associate a name with each value in the range.

There is also little that is new in the second part of the argument although the form is different. Instead of arguing for the truth values a structure assigns at one stage from those it assigns at the next one, we argue for the truth values it assigns to a sentence from the truth values it assigns to the components (or instances) of the sentence. However, the chief difference between the resources and goal of one stage and those of the next lies in the introduction of components or instances at the new stage to replace or add to compounds that appear at the old one, so the arguments both end up concerning the semantic relations between compounds and components and we will not look at the new argument in much detail.

Why then do we need a new argument at all? One reason lies in the form. Suppose we have a structure making the ultimate resources of a path all true. We need to show that it divides the path. The old way was to begin with the final stage of the gap and work our way back stage by stage, with each step of this argument using the safety of the rules. The new way is to begin with the ultimate resources and work our way up to more and more complex sentences. The considerations will be much the same at each step. We have changed only the overall form of the argument, and we have changed it only because we have to: we have ultimate components to start from but there may not be a final stage to the path.

There is one exception to the analogy between the two forms of argument, and it concerns the only part of the new argument we will consider. A universal resource is not exploited by a single stage in the development of a path, so the relation between a universal and its instances is not replicated by a transition from one stage of development to the next. So suppose we are arguing in the new way; that is, we have a structure making the ultimate resources of a path true and we are moving step by step from components (or instances) to compounds in order to show that this structure divides the path. How do we know that we can make the step we need to in the case of a universal ∀x θx appearing among the accumulated resources?

Let us collect what we know (setting aside for the moment the possibility of non-trivial alias sets). Since the path is fully developing, the universal has been exploited for each term τ appearing in the gap. And this means that each instance θτ for such a term will appear among the accumulated resources. Moreover, in our step-by-step climb to more and more complex sentences, we will have already shown that the structure makes each of the instances θτ true. Now the structure assigns each value in its range to some term τ. So, since the structure makes every instance θτ true, it must assign θ an extension that includes the whole of the referential range, and that means the structure will make ∀x θx true. Now, notice that, for the structure to make ∀x θx true, it is really only necessary that it make true an instance θτ for at least one term τ from each alias set, and that means that a fully developing gap need have only this many instances among is accumulated resources. Although it has been convenient for the purposes of these general arguments to think of fully developing gaps as exploiting universals for all terms appearing in them, this is not necessary to insure that the gap is divisible, and there is no need to render universals inactive for every term when constructing actual derivations.

It *is* crucial for this argument that the referential range of the structure dividing the gap contain no reference values beyond those used as the extensions of terms. That is why we limit the range to values that correspond to alias sets. And the reason for this is not at all mysterious. We can now state logical forms that are true only in ranges of limited size. To take an extreme case, the sentence ∀x ∀y x = y (i.e., *Everything is identical to everything*) is true if and only if the referential range has just one member. If this sentence is among the resources of a gap, the gap can be divided only by a structure whose range has a population of 1.

This makes it harder to duplicate structures by intensional interpretations and possible worlds. Clearly, we cannot always choose the actual world if the range of reference values must be severely limited, and it may not be clear what the extensions of ordinary English vocabulary are like in possible worlds that have very limited ranges. So it is hard to tell whether this undermines the argument from the existence of a dividing structure to the failure of formal validity. If it does undermine that argument, we could redefine entailment so that we speak not simply of all possible worlds but of all worlds and all ways of choosing a referential range from each world. The device mentioned in 6.4.3 of regarding structures as partial accounts of a possible world would then be usable also in accounts of entailment for generalizations.

Glen Helman 25 Aug 2005