7.6. Arguments involving restricted universals

7.6.0. Overview

Since a restricted universal is equivalent to an unrestricted universal applied to a conditional formula, its logical properties are analogous in some respects to those of universals and in other respects to those of conditionals.

7.6.1. Principles for restricted universals

A restricted universal behaves like an indefinitely long conjunction of conditionals and the principles governing it derive from this fact.

7.6.2. Derivations for restricted universals

The principles for restricted universals can be implemented either by a group of four rules analogous to those for conditionals (but also reflecting rules for unrestricted universals) or by rules for restating restricted universals in unrestricted form or vice versa.

Glen Helman 25 Aug 2005

7.6.1. Principles for restricted universals

The laws for unrestricted universals as premises and as conclusions were based on the relation between a generalization and its instances. In the case of restricted universals, we will also be interested in instantiation but an unrestricted universal $(\forall x\colon \rho x)\ \theta x$ does not in general imply an instance $\theta\tau$. But since $(\forall x\colon \rho x)\ \theta x \Leftrightarrow \forall x\ (\rho x \rightarrow \theta x)$, the restricted universal will imply the sentences $\rho\tau \rightarrow \theta\tau$ that are the instances of its restatement in unrestricted form; we will call these its ***conditioned instances***. If we expand the language by the range **R** of a structure, an unrestricted universal $\forall x\ \theta x$ will be true in that structure if and only if all its instances are true and an unrestricted universal $(\forall x\colon \rho x)\ \theta x$ will be true if and only if all its conditioned instances are true. That means that a restricted universal behaves like a conjunction of its conditioned instances and its role in deductive reasoning is analogous in some respects to the role of an unrestricted universal and in other respects to the role of a conditional.

We can get laws for the restricted universal by restating such a universal in unrestricted form and and applying laws for both the unrestricted universal and the conditional. To get our ***law for the restricted universal as a conclusion***, we can reason as follows for any term a that does not appear in Γ or $(\forall x\colon \rho x)\ \theta x$:

$\Gamma \Rightarrow (\forall x\colon \rho x)\ \theta x$
if and only if
$\Gamma \Rightarrow \forall x\ (\rho x \rightarrow \theta x)$ — *by the equivalence of a restricted generalization with an unrestricted generalization of a conditional*
if and only if
$\Gamma \Rightarrow \rho a \rightarrow \theta a$ — *by the law for a universal as a conclusion*
if and only if
$\Gamma, \rho a \Rightarrow \theta a$ — *by the law for a conditional as a conclusion*

That is, we can conclude a restricted universal if and only if we can conclude its instance θa for a parametric term a, allowing ourselves to make the assumption that ρa—i.e., that the value of the term a is in the domain of the universal. The assumption here that ρa is comparable to the assumption that *ABC* is a triangle which is made when we wish to offer an argument about *ABC* as a basis for a generalization about all triangles.

We can approach the role of $(\forall x\colon \rho x)\ \theta x$ as a premise also by way of a restatement in unrestricted form. If we apply the law for an unrestricted universal premise and restate the result again using a restricted quantifier, we get, for any term τ,

$$\Gamma, (\forall x\colon \rho x)\ \theta x \Rightarrow \varphi \text{ if and only if } \Gamma, (\forall x\colon \rho x)\ \theta x, \rho\tau \rightarrow \theta\tau \Rightarrow \varphi$$

To go further, we need to take account of the conditional premise we have introduced. We have three ways of doing this, two detachment principles, each of which requires still another premise, and a principle for *reductio* arguments. Accordingly, we get three principles for a restricted universal premise each applying to a different sort of case:

$\Gamma, (\forall x\colon \rho x)\ \theta x, \rho\tau \Rightarrow \varphi$	if and only if $\Gamma, (\forall x\colon \rho x)\ \theta x, \rho\tau, \theta\tau \Rightarrow \varphi$
$\Gamma, (\forall x\colon \rho x)\ \theta x, \overline{\theta\tau} \Rightarrow \varphi$	if and only if $\Gamma, (\forall x\colon \rho x)\ \theta x, \overline{\theta\tau}, \overline{\rho\tau} \Rightarrow \varphi$
$\Gamma, (\forall x\colon \rho x)\ \theta x \Rightarrow \bot$	if and only if both $\Gamma, (\forall x\colon \rho x)\ \theta x \Rightarrow \rho\tau$ and $\Gamma, (\forall x\colon \rho x)\ \theta x, \theta\tau \Rightarrow \bot$

Each holds for any term τ.
The first two are based on the following valid forms of argument in the way that the detachment rules for conditionals are based on *modus ponens* and *modus tollens*.

Singular Barbara	***Singular Camestres***
$(\forall x\colon \rho x)\ \theta x$	$(\forall x\colon \rho x)\ \theta x$
$\rho\tau$	$\overline{\theta\tau}$
$\theta\tau$	$\overline{\rho\tau}$

The first is a principle of ***restricted universal instantiation***. It is perhaps the most widely recognized pattern of argument; an instance of it was our first example of a valid argument in 1.1.2 . In the logical tradition, it and the second pattern (which stands to it as *modus tollens* stands to *modus ponens*) were often not distinguished from certain patterns of argument studied in the theory of syllogisms whose second premises and conclusions contain quantifier phrases rather than individual terms, and the names used here are adapted from a medieval system of nomenclature for such syllogistic arguments. (Notice that vowels in the names are the first vowels appearing in the English verbs *affirm* and *negate*, which happen to be cognate to the corresponding Latin terms, and that these vowels mark the affirmative or negative character of the premises and conclusion taken in order. Many of the consonants are also significant, but their significance lies in connections among various patterns of argument in the theory of syllogisms proper.)

The third principle for restricted universal conclusions says that we can reduce $(\forall x\colon \rho x)\ \theta x$ to absurdity given Γ if and only if the two together enable us to do two things in the case of any term τ: (i) show that the value of τ is in the domain of the generalization and (ii) reduce to absurdity the claim that the attribute of the generalization is true of the value of τ. More briefly, Γ entails the falsity of $(\forall x\colon \rho x)\ \theta x$ (and thus the existence of a counterexample to it) if and only if the two together entail that any term τ is a counterexample. (The *only-if* part of this may seem an odd claim; but its truth can be seen by reflecting that, if Γ does entail the falsity of $(\forall x\colon \rho x)\ \theta x$, the premises and the universal together form an inconsistent set and entail every sentence.) Because this law is sweeping enough to pull in terms that are innocent bystanders at the failure of a generalization (so we do not need a further premise as in the first two principles), we will count it as our ***law for the restricted universal as a premise***.

To summarize, we have the following basic principles for the restricted universal:

Law for the restricted universal as a premise. For any term τ, we have:
$\Gamma, (\forall x\colon \rho x)\ \theta x \Rightarrow \bot$ if and only if both $\Gamma, (\forall x\colon \rho x)\ \theta x \Rightarrow \rho\tau$ and $\Gamma, (\forall x\colon \rho x)\ \theta x, \theta\tau \Rightarrow \bot$.

Law for the unrestricted universal as a conclusion. For any unanalyzed term a appearing in neither Γ nor $(\forall x\colon \rho x)\ \theta x$, we have:

$$\Gamma \Rightarrow (\forall x\colon \rho x)\ \theta x \text{ if and only if } \Gamma, \rho a \Rightarrow \theta a.$$

Although these two principles suffice to capture the logical properties of restricted universals, rules implementing singular Barbara and singular Camestres will play an important role in our system of derivations.

Glen Helman 25 Aug 2005

7.6.2. Derivations for restricted universals

The rules for restricted quantifiers will resemble those for conditionals as well as those for restricted universals. Figure 7.6.2-1 shows the planning rule, ***Restricted Universal Generalization*** (RUG).

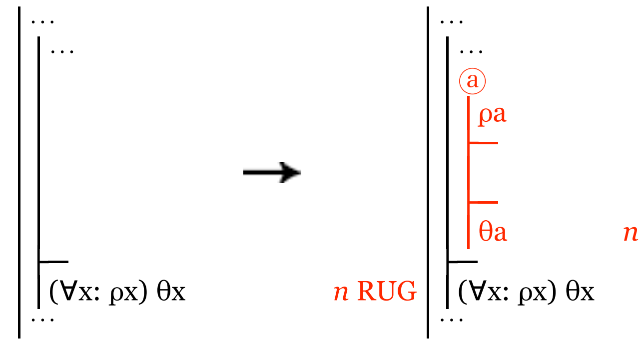


Fig. 7.6.2-1. Developing a derivation at stage *n* by planning for a restricted universal.

The notation $(\forall x\colon \rho x)\ \theta x$ is more convenient here than (∀x: ...x...) ---x--- but remember that an expression like ρx has the same significance as one like ...x...; that is, it stands for a formula, with the variable x standing for all free occurrences of that variable in it. And the expression ρa stands for the result of putting the parameter a in place of all free occurrences of x in ρx.

As with the planning rule for an unrestricted universal, the parameter a must be new to the derivation. Notice that the general argument begins with a supposition. This is like beginning a general argument concerning triangles by saying, "Suppose *ABC* is a triangle. . . ," so it recognizes one more function for suppositions: to place restrictions on the arbitrary choice of a reference value for a parametric term in an argument that is both general and hypothetical. It is the hypothetical aspect of the argument that forces its conclusion to be restricted: if we have supposed that *ABC* is a triangle, we cannot claim that a property we establish for *ABC* holds of everything, only that it holds of all triangles.

The principles singular Barbara and singular Camestres bear enough similarities to the idea detachment introduced in 4.3.1 that we have extended that term to them. However, the principles for universals do have one difference from the principles for truthfunctional compounds: their conclusions do not imply either premise. What they represent is really a sort of partial detachment since the conclusion in each case does imply the conditioned instance of (∀x: ρx) θx for the term τ. This is enough for them to justify a partial exploitation of universals, and that is the most we could expect.

Figures 7.6.2-2 and 7.6.2-3 show rules—named ***Singular Barbara*** (SB) and ***Singular Camestres*** (SC)—that are based on these principles.

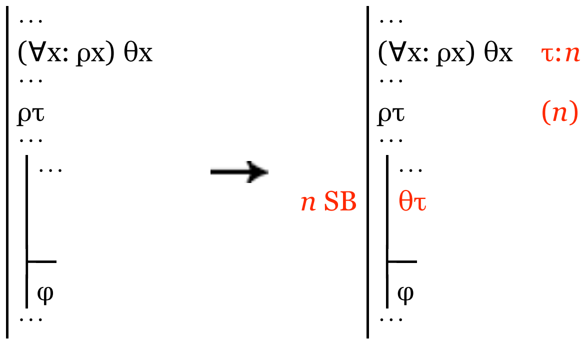


Fig. 7.6.2-2. Developing a derivation at stage *n* by exploiting for a term τ a restricted universal whose restricting predicate is applied to τ in an active resource.

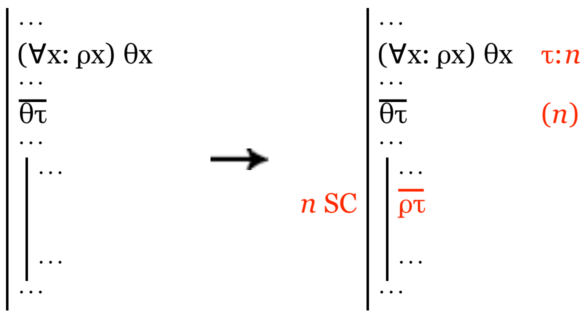


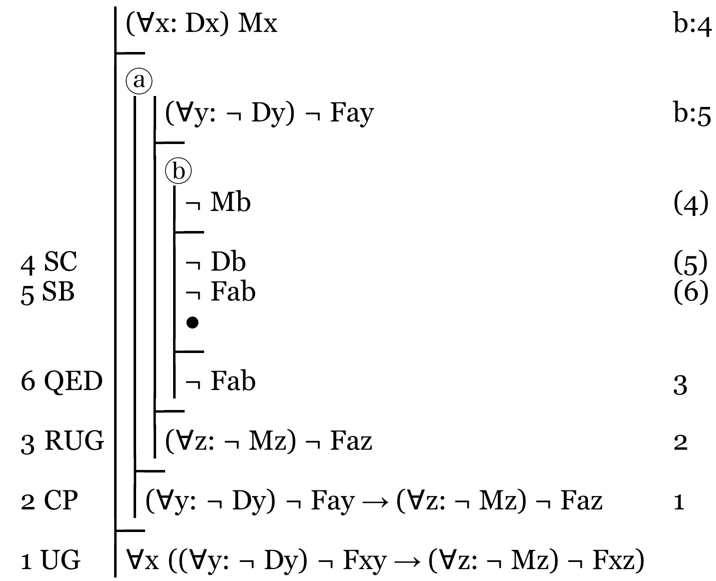
Fig. 7.6.2-3. Developing a derivation at stage *n* by exploiting for a term τ a restricted universal whose quantified predicate is denied of τ in an active resource.

These two are the rules that we will use most often to exploit restricted universals. The fullest use of the first will come by using attachment rules, for the sentence ρτ may be a complex sentence that is entailed by our resources even though it does not appear among them.

The derivation shown below combines these rules with planning rules for both sorts of universal. It establishes the following entailment:

All dogs are mammals

If anything affects only dogs, then it affects only mammals



In giving a symbolic version of the conclusion, the quantifier phrase *anything* was dealt with first. This made it possible to analyze the body of its quantified predicate as the conditional *if* x *affects only dogs, then* x *affects only mammals* and to analyze the two components further. The derivation begins at stage 1 with planning for the unrestricted universal conclusion. At stage 2 we plan for the new conditional goal and at stage 3 for a restricted universal, introducing a new parameter and supplying a supposition that begins exploitation of the two restricted universal resources in stages 4 and 5. Notice that the argument for ¬ Fab is doubly general. It uses no special assumption about the reference of the tem a and assumes about the term b only that it does not refer to a mammal.

The following example shows a typical use of attachment rules with instantiation for restricted universals:

Everything important that was broken was repaired
If anything broken was important, it was repaired

	(∀x: Ix ∧ Bx) Rx	b:4
	ⓐ	
	Ba	(3)
	Ia	(3)
3 Adj	Ia ∧ Ba	X,(4)
4 SB	Ra	(5)
	•	
5 QED	Ra	2
2 CP	Ia → Ra	1
1 RUG	(∀x: Bx) (Ix → Rx)	

After stage 2, our resources entail that any object referred to by the parameter a is in the domain of the generalization (∀x: Ix ∧ Bx) Rx. But we do not have a resource that actually applies the restricting predicate of this generalization to the term a, so we cannot immediately apply the rule for singular Barbara. To put ourselves in a position to do so, we use attachment at stage 3 to add the required sentence to our inactive resources and then instantiate the premise at stage 4.

In some cases, even the use of attachment rules will not enable us to introduce the auxiliary premises we would need to exploit a restricted universal by the detachment rules. In such cases, we need to resort to a rule for exploiting restricted universals that implements the law for the restricted universal as a premise. It is shown in Figure 7.5.4-7:

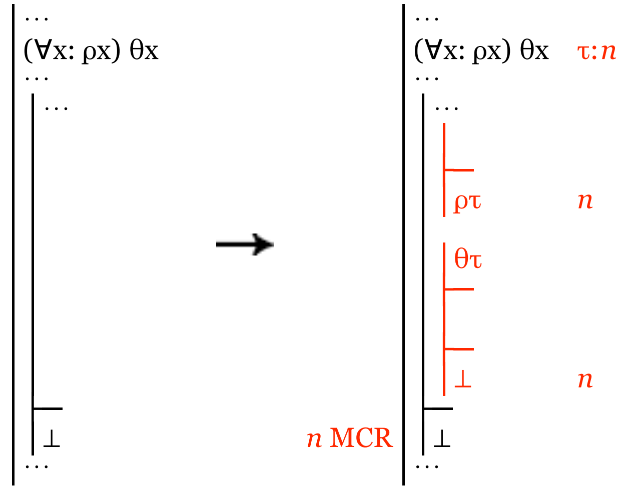


Fig. 7.6.2-4. Developing a *reductio* at stage *n* by exploiting a restricted universal for the term τ.

We can exploit a restricted universal to complete a *reductio* by showing that other resources entail its denial. Here we plan to do this by producing a counterexample, specifically by showing that τ is in the domain of the generalization and reducing to absurdity the claim θτ that it has the attribute. This rule will be named ***Making a Counterexample for Reductio*** (MCR). Of course, the term τ is not the only possible counterexample so the content of the universal is not limited to the claim that τ in particular is not a counterexample. That is why this counts as an exploitation of the universal only for the term τ; our resources may conflict with the universal even if they do not entail that this term is a counterexample to it, so the universal has further potential to contribute to a reduction of our resources to absurdity. A restricted universal (∀x: ρx) θx can be thought of as a general inference ticket that will get us from ρτ to θτ for any term τ. This feature is used in the rule to link the goal of the first gap and the supposition of the second, enabling us to complete the *reductio* if we can close these two gaps. The special virtue of the restricted universal as opposed to a conditional is that it supports an indefinite number of links between gaps rather than merely one.

The comments made about the exploitation rule for unrestricted universals apply to these three exploitation rules as well. First, a universal is never completely exploited though it may be rendered inactive for certain terms. Also, the exploitation of universals should be limited to terms already appearing in the derivation whenever there are such terms. And, indeed, it may be limited to a single term from each alias set.

Another approach to the logical properties of restricted universals simply uses their restatements using unrestricted quantifiers. That is, we take as the basic principle the equivalence

$$(\forall x\colon \rho x)\ \theta x \Leftrightarrow \forall x\ (\rho x \rightarrow \theta x)$$

and implement this by two rules, one each for exploiting and planning for a restricted universal simply replace the restricted universal (as an active resource or a goal) by its restatement:

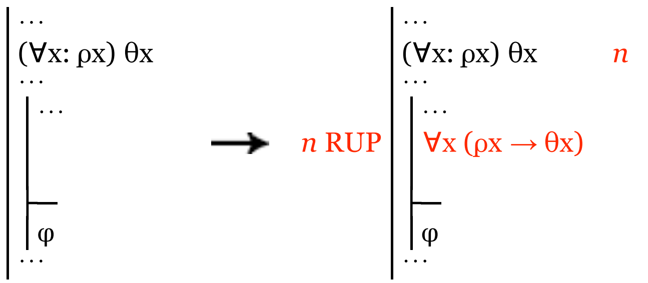


Fig. 7.6.2-5. Developing a derivation at stage *n* by restating a restricted universal resource.

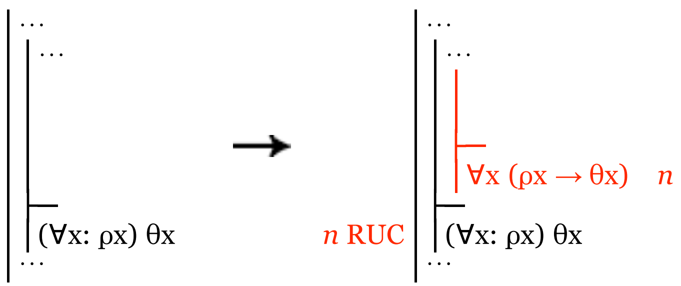
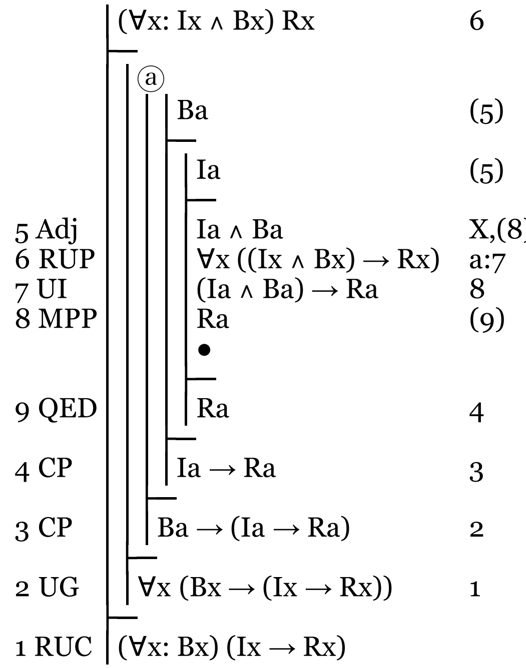


Fig. 7.6.2-6. Developing a derivation at stage *n* by restating a restricted universal goal.

These rules are named ***Restricted Universal Premise*** (RUP) and ***Restricted Universal Conclusion*** (RUC), respectively. Whether they are useful to you will depend on the way you organize your thinking: using them instead of the earlier four rules would mean fewer rules to remember but they will usually need to be combined with two further rules to get the effect of the earlier rules (e.g., RUC with UG and CP to get the effect of RUG). Here is a derivation for the second of the examples above using these rules:



This has been constructed so that each of the rules for restricted universals is replaced by a series of three steps, a use of rule for the unrestricted quantifier preceded by RUP or RUC and followed by a rule for the conditional. Whether you find this approach to restricted quantifiers easier or harder than the use of the four earlier rules, it is a legitimate one: the rules RUP and RUC are certainly sound and safe since they replace a resource or goal by an equivalent sentence and, while they are not direct since they restate rather than decomposing, they are progressive if we measure distance from the end of a gap in part by the number of restricted universal quantifiers appearing in resources or the goal.

7.6.s. Summary

While the logical properties of an unrestricted universal may be tied to those of its instances, in the case of a restricted universal, we use the idea of a conditioned instance, an application of the quantified predicate to a term made conditional on an application of the restricting predicate to the same term. Laws for restricted universals then combine the ideas used to capture the role of unrestricted universals with ideas used to capture the role of conditionals. The law for the restricted universal as a conclusion requires that we conclude an instance for a parameter but allows us to add a supposition that applies the restricting predicate of the universal to the parameter. For restricted universal resources, we have two detachment arguments, singular *Barbara* and singular *Camestres*, named after similar syllogistic patterns. These principles are analogous to uses of *modus ponens* and *modus tollens* for a conditioned instance of the universal and the first amounts to a sort of restricted universal instantiation. There is also a principle for the restricted universal as a premise in *reductio* arguments that again reflects the role of its conditioned instances.

The most characteristic derivation rules for restricted universals — Restricted Universal Generalization (RUG), Singular Barbara (SB), Singular Camestres (SC), and Making a Counterexample for *Reductio* (MCR) —are rules that implement these principles. But it is also possible to capture the properties of restricted universals by rules, Restricted Universal Premise (RUP) and Restricted Universal Conclusion (RUC), that support restating restricted universal resources and goals using the unrestricted universal so that rules for that quantifier and conditionals may be applied.

7.6.x. Exercise questions

1. Use the system of derivations to establish valdity in each of the following cases. You may use detachment and attachment rules. Exercises **h** and **i** are contrived to require the rule MCR for exploiting restricted universals; any restricted universals in the others may be exploited using the detachment rules.

- **a.** $\forall x\, \forall y\, Rxy,\ (\forall x\!: Rxx)\, Gx \Rightarrow Ga$
- **b.** $(\forall x\!: Fx)\, Gx \Leftrightarrow \forall x\, (Fx \rightarrow Gx)$
- **c.** $Fa \Leftrightarrow (\forall x\!: x = a)\, Fx$
- **d.** $\forall x\, \forall y\, (Rxy \rightarrow \neg Ryx) \Rightarrow \forall x\, (\forall y\!: \neg x = y)\, \neg (Rxy \wedge Ryx)$
- **e.** $\forall x\, (\forall y\!: \neg x = y)\, \neg (Rxy \wedge Ryx)$
  $\forall x\, \neg Rxx$
  ———
  $\forall x\, \forall y\, (Rxy \rightarrow \neg Ryx)$
- **f.** *Everyone loves everyone who loves anyone*
  ———
  If anyone loves anyone, then everyone loves everyone
- **g.** $\forall x\, (\forall y\!: gx = y)\, Fy \Rightarrow \forall x\, F(g(hx))$
- **h.** $\forall x\, \forall y\, Rxy,\ (\forall x\!: \forall y\, Ryx)\, (Fx \rightarrow Gx) \Rightarrow (\forall x\!: Fx)\, Gx$
- **i.** *Al said everything he remembered*
  Al is a person who said nothing
  Anyone who remembered nothing forgot everything
  ———
  Al forgot everything

2. In the following, certain alternative expressions are enclosed in brackets and separated by vertical bars. Choose one of each alternative pair of premises and one of each alternative pair of words or phrases in the conclusion so as to make a valid argument; then analyze the premises and conclusion and construct a derivation to show that the argument is valid. You may use detachment and attachment rules.

- **a.** *Every road sign was colored*
  [*Every stop sign was a road sign* | *Every road sign was a traffic marker***]**
  [*If anything was red, it was colored* | *If anything was colored, it was painted***]**
  ———
  Every **[***stop sign* | *traffic marker***]** *was* **[***red* | *painted***]**

b. *No road sign was colored*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]

No [*stop sign* | *traffic marker*] *was* [*red* | *painted*]

c. *Only road signs were colored*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]

Only [*stop signs* | *traffic markers*] *were* [*red* | *painted*]

d. *Among road signs all except colored ones were replaced*
[*Every stop sign was a road sign* | *Every road sign was a traffic marker*]
[*If anything was red, it was colored* | *If anything was colored, it was painted*]

Among [*stop signs* | *traffic markers*] *all except* [*red* | *painted*] *ones were replaced*

e. *Everyone watched every snake*
[*Every cobra is a snake* | *Every snake is a reptile*]

Everyone watched every [*cobra* | *reptile*]

f. *No one watched every snake*
[*Every cobra is a snake* | *Every snake is a reptile*]

No one watched every [*cobra* | *reptile*]

g. *No one watched any snake*
[*Every cobra is a snake* | *Every snake is a reptile*]

No one watched any [*cobra* | *reptile*]

h. *Everyone who likes every snake was pleased*
[*Every cobra is a snake* | *Every snake is a reptile*]

Everyone who likes every [*cobra* | *reptile*] *was pleased*

i. *Everyone who likes a snake was pleased*
[*Every cobra is a snake* | *Every snake is a reptile*]

Everyone who likes a [*cobra* | *reptile*] *was pleased*

7.6.xa. Exercise answers

These answers use the rules RUG, SB, SC, and MCR rather than RUP and RUC. Derivations using the latter rules can be constructed from them by replacing each use of one of the first four rules by a series of three steps as shown in the following table:

basic rule	*alternative approach using RUP and RUC*
RUG	RUC, UG, CP
SB	RUP, UI, MPP
SC	RUP, UI, MTT
MCR	RUP, UI, RC

1. a.

```
          │ ∀x ∀y Rxy        a:1
          │ (∀x: Rxx) Gx     a:3
          ├──
1 UI      │ ∀y Ray           a:2
2 UI      │ Raa              (3)
3 SB      │ Ga               (4)
          │ •
          ├──
4 QED     │ Ga
```

b.

```
          │ (∀x: Fx) Gx        a:3
          ├──
          │ ⓐ
          │ │ Fa               (3)
          │ ├──
3 SB      │ │ Ga               (4)
          │ │ •
          │ ├──
4 QED     │ │ Ga               2
          ├──
2 CP      │ Fa → Ga            1
          ├──
1 UG      │ ∀x (Fx → Gx)
```

```
          │ ∀x (Fx → Gx)       a:2
          ├──
          │ ⓐ
          │ │ Fa               (3)
          │ ├──
2 UI      │ │ Fa → Ga          3
3 MPP     │ │ Ga               (4)
          │ │ •
          │ ├──
4 QED     │ │ Ga               1
          ├──
1 RUG     │ (∀x: Fx) Gx
```

c.

```
          │ Fa                 (2)
          ├──
          │ ⓑ
          │ │ b = a            a—b
          │ ├──
          │ │ •
          │ ├──
2 QED=    │ │ Fb               1
          ├──
1 RUG     │ (∀x: x = a) Fx
```

```
          │ (∀x: x = a) Fx     a:2
          ├──
          │ │ ¬ Fa             (2)
          │ ├──
2 SC      │ │ ¬ a = a          (3)
          │ │ •
          │ ├──
3 DC      │ │ ⊥                1
          ├──
1 IP      │ Fa
```

d.

	∀x ∀y (Rxy → ¬ Ryx)	a:5
	ⓐ	
	ⓑ	
	¬ a = b	
	Rab ∧ Rba	4
4 Ext	Rab	(7)
4 Ext	Rba	(8)
5 UI	∀y (Ray → ¬ Rya)	b:6
6 UI	Rab → ¬ Rba	7
7 MPP	¬ Rba	(8)
	•	
8 Nc	⊥	3
3 RAA	¬ (Rab ∧ Rba)	2
2 RUG	(∀y: ¬ a = y) ¬ (Ray ∧ Rya)	1
1 UG	∀x (∀y: ¬ x = y) ¬ (Rxy ∧ Ryx)	

e.

	∀x (∀y: ¬ x = y) ¬ (Rxy ∧ Ryx)	a:5
	∀x ¬ Rxx	a:8
	ⓐ	
	ⓑ	
	Rab	(6),(9)
	Rba	(6)
5 UI	(∀y:¬ a = y) ¬(Ray ∧ Rya)	b:7
6 Adj	Rab ∧ Rba	X,(7)
7 SC	a = b	a—b
8 UI	¬ Raa	(9)
	•	
9 Nc=	⊥	4
4 RAA	¬ Rba	3
3 CP	Rab → ¬ Rba	2
2 UG	∀y (Ray → ¬ Rya)	1
1 UG	∀x ∀y (Rxy → ¬ Ryx)	

f.

	(∀x: Px) (∀y: Py) (∀z: Pz ∧ Lzx) Lyz	b:6,a:10
	ⓐ	
	Pa	(8), (10)
	ⓑ	
	Pb	(6)
	Lab	(8)
	ⓒ	
	Pc	(11)
	ⓓ	
	Pd	(7), (12)
6 SB	(∀y: Py) (∀z: Pz ∧ Lzb) Lyz	d:7
7 SB	(∀z: Pz ∧ Lzb) Ldz	a:9
8 Adj	Pa ∧ Lab	X, (9)
9 SB	Lda	(12)
10 SB	(∀y: Py) (∀z: Pz ∧ Lza) Lyz	c:11
11 SB	(∀z: Pz ∧ Lza) Lcz	d:13
12 Adj	Pd ∧ Lda	X, (13)
13 SB	Lcd	(14)
	•	
14 QED	Lcd	5
5 RUG	(∀w: Pw) Lcw	4
4 RUG	(∀z: Pz) (∀w: Pw) Lzw	3
3 CP	Lab → (∀z: Pz) (∀w: Pw) Lzw	2
2 RUG	(∀y: Py) (Lay → (∀z: Pz) (∀w: Pw) Lzw)	1
1 RUG	(∀x:Px) (∀y:Py) (Lxy → (∀z:Pz) (∀w:Pw) Lzw)	

g.

	∀x (∀y: gx = y) Fy	ha:2
	ⓐ	
2 UI	(∀y: g(ha) = y) Fy	g(ha):4
3 EC	g(ha) = g(ha)	X, (4)
4 SB	F(g(ha))	(5)
	•	
5 QED	F(g(ha))	1
1 UG	∀x F(g(hx))	

h.

	∀x ∀y Rxy	b:5
	(∀x: ∀y Ryx) (Fx → Gx)	a:3
	ⓐ	
	Fa	(8)
	¬ Ga	(9)
	ⓑ	
5 UI	∀y Rby	a:6
6 UI	Rba	(7)
	•	
7 QED	Rba	4
4 UG	∀y Rya	3
	Fa → Ga	8
8 MPP	Ga	(9)
	•	
9 Nc	⊥	3
3 MCR	⊥	2
2 IP	Ga	1
1 RUG	(∀x: Fx) Gx	

i.

	(∀x: Rax) Sax	c:9
	Pa ∧ ∀x ¬ Sax	1
	(∀x: Px ∧ ∀y ¬ Rxy) ∀z Fxz	a:4
1 Ext	Pa	(6)
1 Ext	∀x ¬ Sax	c:8
	ⓑ	
	¬ Fab	
	•	
6 QED	Pa	5
	ⓒ	
8 UI	¬ Sac	(9)
9 SC	¬ Rac	(10)
	•	
10 QED	¬ Rac	7
7 UG	∀y ¬ Ray	5
5 Cnj	Pa ∧ ∀y ¬ Ray	4
	∀z Faz	b:9
	Fab	
	•	
	⊥	4
4 MCR	⊥	3
3 IP	Fab	2
2 UG	∀z Faz	

2. **a.** *Every road sign was colored*
Every stop sign was a road sign
If anything was colored, it was painted
Every stop sign was painted

	(∀x: Dx) Cx	a:3
	(∀x: Sx) Dx	a:2
	∀x (Cx → Px)	a:4
	ⓐ	
	Sa	(2)
2 SB	Da	(3)
3 SB	Ca	(5)
4 UI	Ca → Pa	
5 MPP	Pa	(6)
	•	
6 QED	Pa	1
1 RUG	(∀x: Sx) Px	

b. *No road sign was colored*
Every stop sign was a road sign
If anything was red, it was colored
No stop sign was red

	(∀x: Dx) ¬ Cx	a:3
	(∀x: Sx) Dx	a:2
	∀x (Rx → Cx)	a:4
	ⓐ	
	Sa	(2)
2 SB	Da	(3)
3 SB	¬ Ca	(5)
4 UI	Ra → Ca	5
5 MTT	¬ Ra	(6)
	•	
6 QED	¬ Ra	1
1 RUG	(∀x: Sx) ¬ Rx	

c. *Only road signs were colored*
Every road sign was a traffic marker
If anything was red, it was colored
Only traffic markers were red

	(∀x: ¬ Dx) ¬ Cx	a:3
	(∀x: Dx) Mx	a:2
	∀x (Rx → Cx)	a:4
	ⓐ	
	¬ Ma	(2)
2 SC	¬ Da	(3)
3 SB	¬ Ca	(5)
4 UI	Ra → Ca	5
5 MTT	¬ Ra	(6)
	•	
6 QED	¬ Ra	1
1 RUG	(∀x: ¬ Mx) ¬ Rx	

d. *Among road signs, all except colored ones were replaced*
Every stop sign was a road sign
If anything was colored, it was painted
Among stop signs, all except painted ones were replaced

	(∀x: Dx ∧ ¬ Cx) Lx	a:7
	(∀x: Sx) Dx	a:3
	∀x (Cx → Px)	a:4
	ⓐ	
	Sa ∧ ¬ Pa	2
2 Ext	Sa	(3)
2 Ext	¬ Pa	(5)
3 SB	Da	(6)
4 UI	Ca → Pa	5
5 MTT	¬ Ca	(6)
6 Adj	Da ∧ ¬ Ca	X, (7)
7 SB	La	(8)
	•	
8 QED	La	1
1 RUG	(∀x: Sx ∧ ¬ Px) Lx	

e. *Everyone watched every snake*
Every cobra is a snake
Everyone watched every cobra

	(∀x: Px) (∀y: Sy) Wxy	a:3
	(∀x: Cx) Sx	b:4
	ⓐ	
	Pa	(3)
	ⓑ	
	Cb	(4)
3 SB	(∀y: Sy) Way	b:5
4 SB	Sb	(5)
5 SB	Wab	(6)
	•	
6 QED	Wab	2
2 RUG	(∀y: Cy) Way	1
1 RUG	(∀x: Px) (∀y: Cy) Wxy	

f. *No one watched every snake*
Every snake is a reptile

No one watched every reptile

```
         | (∀x: Px) ¬ (∀y: Sy) Wxy      a:2
         | (∀x: Sx) Rx                  b:6
         |—
         | ⓐ
         | | Pa                         (2)
         | |—
2 SB     | | ¬ (∀y: Sy) Way             4
         | |
         | | | (∀y: Ry) Way             b:7
         | | |—
         | | | | ⓑ
         | | | | | Sb                   (6)
         | | | | |—
6 SB     | | | | | Rb                   (7)
7 SB     | | | | | Wab                  (8)
         | | | | | ●
         | | | | |—
8 QED    | | | | | Wab                  5
         | | | |—
5 RUG    | | | | (∀y: Sy) Way           4
         | | |—
4 CR     | | | ⊥                        3
         | |—
3 RAA    | | ¬ (∀y: Ry) Way             1
         |—
1 RUG    | (∀x: Px) ¬ (∀y: Ry) Wxy
```

g. *No one watched any snake*
Every cobra is a snake

No one watched any cobra

```
         | (∀y: Sy) (∀x: Px) ¬ Wxy      a:3
         | (∀x: Cx) Sx                  a:2
         |—
         | ⓐ
         | | Ca                         (2)
         | |—
2 SB     | | Sa                         (3)
3 SB     | | (∀x: Px) ¬ Wxa             (4)
         | | ●
         | |—
4 QED    | | (∀x: Px) ¬ Wxa             1
         |—
1 RUG    | (∀y: Cy) (∀x: Px) ¬ Wxy
```

h. *Everyone who likes every snake was pleased*
Every snake is a reptile

Everyone who likes every reptile was pleased

```
         | (∀x: Px ∧ (∀y: Sy) Lxy) Dx   a:4
         | (∀x: Sx) Rx                  b:8
         |—
         | ⓐ
         | | Pa ∧ (∀y: Ry) Lay          2
         | |—
2 Ext    | | Pa                         (5)
2 Ext    | | (∀y: Ry) Lay               b:9
         | |
         | | | ¬ Da                     (4)
         | | |—
4 SC     | | | ¬ (Pa ∧ (∀y: Sy) Lay)   5
5 MPT    | | | ¬ (∀y: Sy) Lay          6
         | | |
         | | | | ⓑ
         | | | | | Sb                   (8)
         | | | | |—
8 SB     | | | | | Rb                   (9)
9 SB     | | | | | Lab                  (10)
         | | | | | ●
         | | | | |—
10 QED   | | | | | Lab                  7
         | | | |—
7 RUG    | | | | (∀y: Sy) Lay           6
         | | |—
6 RC     | | | ⊥                        3
         | |—
3 IP     | | Da                         1
         |—
1 RUG    | (∀x: Px ∧ (∀y: Ry) Lxy) Dx
```

i. *Everyone who likes a snake was pleased*
Every cobra is a snake

Everyone who likes a cobra was pleased

```
         | (∀x: Sx) (∀y: Py ∧ Lyx) Dy   a:3
         | (∀x: Cx) Sx                  a:2
         |—
         | ⓐ
         | | Ca                         (2)
         | |—
2 SB     | | Sa                         (3)
3 SB     | | (∀y: Py ∧ Lya) Dy         (4)
         | | ●
         | |—
4 QED    | | (∀y: Py ∧ Lya) Dy         1
         |—
1 RUG    | (∀x: Cx) (∀y: Py ∧ Lyx) Dy
```