7.4. Multiple generality

7.4.0. Overview

A sentence that is not truth-functionally compound may contain more than one quantifier phrase and, when analyzing such a sentence, we will need to choose the one with widest scope to analyze first.

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7.4.1. Multiple generality

Frege suggested we understand the interaction of several quantifier phrases in a single sentence by thinking of them as operations that are applied to the sentence one at a time so that a sentence might already contain one quantifier phrase when another is applied to it. In such a case, the second phrase applied to the sentence would have the first in its scope, and the ambiguities of quantifiers in relation to one another could be understood as ambiguities regarding relative scope.

However, not all differences in scope make for differences in meaning and we will look first at some that do not. Consider, for example, the sentence *Everyone read each application*. We can analyze this as we have analyzed earlier examples—except that we will analyze quantifier phrases twice. If we take them in the order in which they appear in the sentence, the analysis will go as follows:

Everyone read each application
Everyone is such that (he or she read each application)
(∀x: x *is a person*) (x *read each application*)
(∀x: x *is a person*) (*each application is such that (*x *read it)*)
(∀x: x *is a person*) (∀y: y *is an application*) (x *read* y)

$(\forall x: Px)\ (\forall y: Ay)\ Rxy$
$\forall x\ (Px \to \forall y\ (Ay \to Rxy))$

[A: λx (x *is an application*); P: λx (x *is a person*); R: λxy (x *read* y)]

Before discussing the significance of this analysis, there is a technical point to be made. Notice that we chose a new variable when analyzing the second quantifier phrase. At that stage in the analysis, we were analyzing the formula x *read each application*. When we put this in expanded form, we had *each application is such that (*x *read it)*. In order to express this symbolically, we replaced the pronoun *it* with a variable. Using the variable x again would have gotten us the wrong antecedent because, while the abstract λy (x *read* y) expresses the property of being read by x, the abstract λx (x *read* x) expresses the property something has when it read itself. In more technical terms, the formula x *read each application* has a free occurrence of the variable x, so our symbolic version of this formula should also. And, while that it true of the formula (∀y: y *is an application*) x *read* y, the expression (∀x: x *is an application*) x *read* x has no free variables. Instead of being a formula that says something about an unspecified thing x, it is a complete sentence that says *every application read itself*. In short, when analyzing a formula that already contains a variable, you should choose a new variable for any quantifier phrase you analyze. In the example above, the variable y was chosen to analyze the quantifier phrase in the formula x *read each application*, but any variable other than x could have been used.

If we apply subject-predicate expansion to the above sentence while leaving it in English, we get something like *Every person is such that each application is such that he or she read it*. We could state this also as *Every person is such that each application is such that the former read the latter*, and the phrases *the former* and *the latter*, in this use of them, play much the same role here as the distinct variables x and y play in our symbolic analysis. When more than two independent references are needed, we can resort to *the first*, *the second*, etc. Like *the former* and *the latter*, these are definite descriptions in form but they describe what they refer to by way of earlier expressions in the sentence (as shorter forms of expressions like *the first thing referred to*). Consequently, they function like anaphoric pronouns in picking up their references from earlier material in the sentence. Other definite descriptions can be used in this way, too, and the sentence in expanded form might have been rendered as *Every person is such that each application is such that the person read the application*, where, for example, *the person* amounts to *the aforementioned person*.

Now suppose we had instead analyzed this sentence first as a generalization concerning applications. That would have led us to the following analysis:

Everyone read each application
Each application is such that (everyone it)
(∀y: y *is an application*) (*everyone read* y)
(∀y: y *is an application*) (*everyone is such that (he or she read* y*)*)
(∀y: y *is an application*) (∀x: x *is a person*) (x *read* y)
(∀y: Ay) (∀x: Px) Rxy

∀y (Ay → ∀x (Px → Rxy))

The variable y is chosen before x here only in order to facilitate comparison with the first analysis. The form we end up with is equivalent to the one we derived earlier, as can be seen by comparing subject-predicate expansions that correspond to the two analyses:

Every person is such that he or she read each application
Each application is such that every person read it

Either way, we state a double generalization, one that generalizes on the two dimensions of people and applications.

These equivalent forms are an example of a general principle we can state as follows (adapting the notation introduced in 7.3.2 to speak of either restricted or unrestricted quantifiers):

(∀x...) (∀y---) φ ⇔ (∀y---) (∀x...) φ.

Here φ can be any formula though it will normally contained free occurrences of both x and y. Dashes as well dots have been used in the notation for quantifiers to allow for the possibility that the quantifiers for the two variables have different restrictions (which must be brought along when their order is reversed) and to allow also for the possibility that the quantifier for one variable is restricted and the other unrestricted. To insure that no variables become unbound in the interchange, we must require that any restriction on a quantifier not contain free occurrences of the variable bound by the other quantifier. (An example where that restriction would not be met will be discussed below.)

Any generalization of the form displayed above can be described as a generalization over pairs. We can express it this way in English by using a subject-predicate expansion with a paired subject.

Every person and application are such that the former read the latter

It would not be difficult to extend our symbolic notation to get the same effect by using quantifiers that apply to many-place predicates. That is, the generalization at hand can be understood to say that the extension of the predicate λxy (x *is a person and* y *is an application*) is included in the extension of λxy (x *read* y), and we could capture this interpretation symbolically by an operation comparable to ∀ that applied to 2-place predicates. Other examples might lead us to consider quantifiers applying to predicates of 3 or more places. However, there are costs that attend the use of further notation, and we will not pay them here. We will continue to analyze double, triple, and other multiple generalizations by analyzing quantifier phrases in sequence. Still it will help to remember that when we find a sequence of universal quantifiers (with or without attached restrictions) the effect is the same as having a single quantifier over pairs, triples, or longer sequences.

There is one type of case where our approach to such sentences will make analyses a little awkward. Consider the sentence *Not every employer and employee get along*. This is the denial of a generalization over pairs, so we can expect it to be analyzed as the negation of a sentence that begins with a pair of universal quantifiers. However, this case is unlike the one we considered above in that the two universal quantifications are not restricted in independent ways. The generalization is not over all pairs consisting of someone who is a employer and someone who is an employee but rather over pairs consisting of someone who is an employer and someone who is *his or her* employee. That is, the universal quantification is restricted to pairs whose members stand in the employer-employee relation. So we must ask how to represent such a restriction when we use two separate quantifiers. The answer is that we need not restrict the first, outer, quantifier at all, but we must restrict the second, inner, quantifier with reference to the outer one. This is illustrated in the following analysis:

¬ *every employer and employee get along*
¬ ∀x x *and every employee of* x *get along*
¬ ∀x *every employee of* x *is such that (*x *and he or she get along)*
¬ ∀x (∀y: y *is an employee of* x) x *and* y *get along*
¬ ∀x (∀y: x *employs* y) Gxy

¬ ∀x (∀y: Exy) Gxy

[E: λxy (x *employs* y); G: λxy (x *and* y *get along*)]

(The formula y *is an employee of* x has been restated as x *employs* y to make it easier to compare this example with the next one.) Notice the pattern of binding in this form.

¬ ∀x (∀y: Exy) Gxy

We cannot simply reverse the expressions ∀x and (∀y: Exy) (as we did with the quantifiers in the earlier example) because the variable x in the restricting predicate of the second would be moved outside the scope of ∀x and would no longer be bound.

¬ (∀y: Exy)∀x Gxy

On the other hand, if we were to analyze the two quantifiers in the other order we would get the following:

¬ *every employer and employee get along*
¬ ∀y *every employer of* y *and* y *get along*
¬ ∀y *every employer of* y *is such that (he or she and* y *get along)*
¬ ∀y (∀x: x *is an employer of* y) x *and* y *get along*
¬ ∀y (∀x: x *employs* y) x *and* y *get along*

¬ ∀y (∀x: Exy) Gxy

Again the first quantifier in the analysis is unrestricted and the second is restricted in a way that refers back to it. This asymmetry is the compensation we must pay for using an asymmetric notation to represent an essentially symmetric claim. The asymmetry is mitigated if we use unrestricted quantification, for then we have the following two symbolic forms:

¬ ∀x ∀y (Exy → Gxy)
¬ ∀y ∀x (Exy → Gxy)

Here the only difference is in the order of the expressions ∀x and ∀y, and the predicate E can be seen to restrict both of them together.

7.4.2. Judging the scope of quantifier phrases

In the examples we have just been looking at, we were free to choose the order in which we analyzed quantifier phrases; but that is not always possible. A change in the order of analysis will change the relative scopes assigned to quantifiers, and this will often change the claim made by a sentence. We saw the examples in 7.1.1 where such changes corresponded to different possible interpretations of ambiguous sentences. Ambiguity is less pronounced with the limited range of quantifier phrases we are dealing with in this chapter, so certain ways of choosing the order of analysis will be definitely wrong.

One example where two interpretations do seem to be possible is the sentence *Only teenagers went to each showing*. As in the examples of 7.1.1, the two interpretations can be brought out by applying subject-predicate expansion in two different ways:

Only teenagers are such that (they went to each showing)

Each showing is such that (only teenagers went to it)

The first says that, if you can find people who went back for each showing, they are all teenagers while the second says that the audience at each showing (if there was any) consisted solely of teenagers. Unlike the ambiguous examples of 7.1.1, neither of these claims implies the other.

The corresponding two analyses are the following:

Only teenagers are such that (they went to each showing)
(∀x: ¬ Tx) ¬ x *went to each showing*
(∀x: ¬ Tx) ¬ *each showing is such that (*x *went to it)*
(∀x: ¬ Tx) ¬ (∀y: Sy) x *went to* y

(∀x: ¬ Tx) ¬ (∀y: Sy) Wxy
∀x (¬ Tx → ¬ ∀y (Sy → Wxy))

Each showing is such that (only teenagers went to it)
(∀y: Sy) *only teenagers went to* y
(∀y: Sy) *only teenagers are such that (they went to* y*)*
(∀y: Sy) (∀x: ¬ Tx) ¬ x *went to* y

(∀y: Sy) (∀x: ¬ Tx) ¬ Wxy
∀y (Sy → ∀x (¬ Tx → ¬ Wxy))

[S: λx (x *is a showing*); T: λx (x *is a teenager*); W: λxy (x *went to* y)]

The first denies the generalization x *went to each showing* in any case where x is a not a teenager. The second says of each showing that non-teenagers stayed away.

In other cases, there is less room for alternative interpretations. Since two of the kinds of generalization we are considering are negative, decisions about the relative scope of quantifier phrases are often at the same time decisions about the relative scope of negations and quantifier phrases, and English tends to be more unambiguous in that regard. We saw in 7.3.3 that the word *any* can be used to indicate that a sentence containing negation is not the denial of a generalization but rather the assertion of a generalization whose attribute is negative. For example, compare *No one saw everything* with *No one saw anything*. The first says of each person, x, that the generalization x *saw everything* is false while the second asserts of each thing that no one saw it. That is, the proper analyses of the two are the following:

No one saw everything
No one is such that (he or she saw everything)
(∀x: x *is a person*) ¬ x *saw everything*
(∀x: Px) ¬ *everything is such that (*x *saw it)*
(∀x: Px) ¬ ∀y x *saw* y

(∀x: Px) ¬ ∀y Sxy
∀x (Px → ¬ ∀y Sxy)

No one saw anything
Everything is such that (no one saw it)
∀y *no one saw* y
∀y *no one is such that (he or she saw* y*)*
∀y (∀x: x *is a person*) ¬ x *saw* y

∀y (∀x: Px) ¬ Sxy
∀y ∀x (Px → ¬ Sxy)

[P: λx (x *is a person*); S: λxy (x *saw* y)]

The first of these sentences is perhaps slightly ambiguous (with an outside chance that it would be interpreted in the way indicated by the second analysis) but the second is pretty clearly unambiguous. It should be added that this is in part a consequence of the choice of verb and tense; the sentence *No one will eat anything* could perhaps be understood in the way indicated by the first analysis—that is, with *no one* as the main quantifier phrase—and *No one will eat just anything* has that as its most natural interpretation.

A second pair of examples involves the other sort of negative generalization.

Only experts recognized every name on the list
Only experts are such that (they recognized every name on the list)
(∀x: ¬ x *is an expert*) ¬ x *recognized every name on the list*
(∀x: ¬ x *is an expert*) ¬ *every name on the list is such that (*x *recognized it)*
(∀x: ¬ Ex) ¬ (∀y: y *is a name on the list*) x *recognized* y
(∀x: ¬ Ex) ¬ (∀y: y *is a name* ∧ y *is on the list*) Rxy

$$(\forall x{:}\ \neg Ex)\ \neg\ (\forall y{:}\ Ny \land Oyl)\ Rxy$$
$$\forall x\ (\neg Ex \rightarrow \neg\ \forall y\ ((Ny \land Oyl) \rightarrow Rxy))$$

Only experts recognized any names on the list
Every name on the list is such that (only experts recognized it)
(∀y: y *is a name on the list*) *only experts recognized* y
(∀y: y *is a name on the list*) *only experts are such that (they recognized* y*)*
(∀y: y *is a name* ∧ y *is on the list*) (∀x: ¬ x *is an expert*) ¬ x *recognized* y

$$(\forall y{:}\ Ny \land Oyl)\ (\forall x{:}\ \neg Ex)\ \neg\ Rxy$$
$$\forall y\ ((Ny \land Oyl) \rightarrow \forall x\ (\neg Ex \rightarrow \neg\ Rxy))$$

[E: λx (x *is an expert*); N: λx (x *is a name*); O: λxy (x *is on* y); R: λxy (x *recognized* y); l: *the list*]

Again, though there may be some hint of ambiguity, the interpretations represented by these analyses are by far the most likely ones. However, restating the second sentence as *Only experts recognized any name on the list* might increase the chance that it would be understood as equivalent with the first.

Another example shows that the use of *any* occurs not only with negative generalizations but also in the restricting predicates of affirmative generalizations.

Everything that is relevant to everything is worth knowing
Everything that is relevant to everything is such that (it is worth knowing)
(∀x: x *is relevant to everything*) x *is worth knowing*
(∀x: *everything is such that (*x *is relevant it)*) x *is worth knowing*
(∀x: ∀y x *is relevant to* y) Wx

$$(\forall x{:}\ \forall y\ Rxy)\ Wx$$
$$\forall x\ (\forall y\ Rxy \rightarrow Wx)$$

Everything that is relevant to anything is worth knowing
Everything is such that (everything that is relevant to it is worth knowing)
∀y *everything that is relevant to* y *is worth knowing*
∀y *everything that is relevant to* y *is such that (it is worth knowing)*
∀y (∀x: x *is relevant to* y) x *is worth knowing*

$$\forall y\ (\forall x{:}\ Rxy)\ Wx$$
$$\forall y\ \forall x\ (Rxy \rightarrow Wx)$$

[R: λxy (x *is relevant to* y); W: λx (x *is worth knowing*)]

Notice that we could reverse the order of ∀y and ∀x in the statement of the second analysis with unrestricted quantifiers. That would trace the difference between it and the corresponding way of writing the first analysis to the location of ∀y in relation to the parentheses. The difference in meaning between the these two sentences should make it clear that the placement of parentheses is as important in the case of quantifiers as it is in the case of connectives.

The moral to be drawn from the last three pairs of examples is to watch for cases where there are several quantifier phrases indicating generalization and one of them uses the word *any* or uses the word *every* in such a way that replacing it by *any* would change the meaning. As a rule, in cases where *any* and *every* contrast with one another, the word *any* indicates that the quantifier phrase has wider relative scope than some other operation (either a connective or a quantifier) and should be analyzed before this other operation while the word *every* indicates narrower scope than this other operation. There are many possibilities for “other operation” mentioned here. Negation and negative generalization are probably the most common, but we have seen examples also of a contrast between *any* and *every* occurring in the antecedents of conditionals and in the restrictions of affirmative generalizations. When the other operation is one that we do not capture in our analyses, we will be able to identify the generalization only in the sentence in which it has wide scope. For example, we can analyze *It might affect anyone* by way of *Everyone is such that (it might affect him or her)* but we cannot analyze *It might affect everyone* without seeing it as the result of applying an operation marked by the modal auxiliary *might* to a generalization.

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7.4.s. Summary

Although our way of analyzing multiple generalizations forces us to assign differences in relative scope to the quantifier phrases, these differences do not always affect the propositions expressed. One example of this is a sentence containing two affirmative direct quantifier phrases. We can analyze these in either order, and the result of either analysis can be thought of as a generalization concerning pairs of values. Such generalizations are sometimes restricted to pairs whose members stand in a certain relation. In this case, we may leave the quantifier with widest scope unrestricted, using the relation to restrict the quantifier with narrower scope.

In many other cases, the scope assigned to quantifier phrases makes a difference. This is usually true in cases where there are negative generalizations. Subject-predicate expansion can be used to see which quantifier phrase should be given widest scope, but there are other signs. For example, *any* can be used in contrast to *every* to indicate that an affirmative generalization has wider scope than a negative generalization. It also can be used to show that one quantifier phrase that appears in the class indicator of another nevertheless has wider scope. Uses of *every* that contrast with *any* have the opposite significance.

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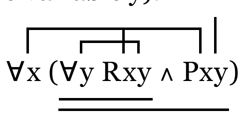
7.4.x. Exercise questions

1. Analyze the following in as much detail as possible:

- **a.** *Every picture pleased everyone*.
- **b.** *No picture pleased everyone*.
- **c.** *No picture pleased anyone*.
- **d.** *Each provision of the law affected every sector of the economy*.
- **e.** *No picture pleased anyone except photographers*.
- **f.** *Anyone who likes all mammals likes all horses*.
- **g.** *The law stimulated only sectors of the economy that were affected by all its provisions*.
- **h.** *No one who doesn't like all mammals likes any badger*.
- **i.** *Everyone saw everything that anyone saw*.
- **j.** *No one saw anything that anyone liked*.
- **k.** *No one who anyone could recall spoke to everyone*.
- **l.** *No one who everyone could recall spoke to anyone*.
- **m.** *Of the pictures anyone saw, no candid ones pleased everyone in them*.
- **n.** *No law will affect only sectors of the economy that figure in all its provisions*.

2. In the logical forms below, indicate the scope of connectives and quantifiers and the patterns of binding of variables as in the example below (where a vertical line is used to mark a free occurrence of the variable y).



- **a.** $\forall x\, Fx \to \forall y\, Gy$
- **b.** $\forall x\, (Fx \to \forall y\, Gy)$
- **c.** $\forall y\, (\forall x\, Fx \to Gy)$
- **d.** $\forall y\, \forall x\, Fx \to Gy$
- **e.** $(\forall x{:}\ \forall y\, Rxy)\, Fx$
- **f.** $\forall y\, (\forall x{:}\ Rxy)\, Fx$
- **g.** $(\forall x{:}\ Rxy)\, \forall y\, Fx$
- **h.** $(\forall x{:}\ \forall y\, Rxy)\, Pxy$

3. Synthesize idiomatic English sentences that express the propositions associated with the following logical forms using the intensional interpretation below. The way quantifiers are most naturally stated in English can depend on what other quantifiers in the sentence, so you may need to back up and revise the way you put one quantifier into English in order to state another.

[B: λxy (x *has bitten* y); D: λxy (x *despises* y); M: λx (x *is a mosquito*); P: λx (x *is a person*); S: λxy (x *is smaller than* y)]

- **a.** $(\forall x{:}\ Mx)\, (\forall y{:}\ Py)\, Dxy$
- **b.** $(\forall x{:}\ Px)\, \neg\, (\forall y{:}\ My)\, Dxy$
- **c.** $(\forall x{:}\ Mx)\, (\forall y{:}\ Py)\, \neg\, Dyx$
- **d.** $(\forall x{:}\ Px)\, (\forall y{:}\ My \land Byx)\, \neg\, Dxy$
- **e.** $(\forall x{:}\ Px \land (\forall y{:}\ My)\, Dxy)\, (\forall z{:}\ Mz)\, \neg\, Bzx$
- **f.** $\forall x\, (\forall y{:}\ Sxy)\, \neg\, Syx$

7.4.xa. Exercise answers

1. **a.** *Every picture pleased everyone*
Every picture is such that (it pleased everyone)
(∀x: x *is a picture*) x *pleased everyone*
(∀x: Cx) *everyone is such that (*x *pleased him or her)*
(∀x: Cx) (∀y: y *is a person*) x *pleased* y

$$(\forall x\text{: } Cx)\ (\forall y\text{: } Py)\ Lxy$$
$$\forall x\ (Cx \rightarrow \forall y\ (Py \rightarrow Lxy))$$

[C: λx (x *is a picture*); L: λxy (x *pleased* y); P: λx (x *is a person*)]

b. *No picture pleased everyone*
No picture is such that (it pleased everyone)
(∀x: x *is a picture*) ¬ x *pleased everyone*
(∀x: Cx) ¬ *everyone is such that (*x *pleased him or her)*
(∀x: Cx) ¬ (∀y: y *is a person*) x *pleased* y

$$(\forall x\text{: } Cx)\ \neg\ (\forall y\text{: } Py)\ Lxy$$
$$\forall x\ (Cx \rightarrow \neg\ \forall y\ (Py \rightarrow Lxy))$$

[C: λx (x *is a picture*); L: λxy (x *pleased* y); P: λx (x *is a person*)]

c. *No picture pleased anyone*.
Everyone is such that (no picture pleased him or her)
(∀x: x *is a person*) *no picture pleased* x
(∀x: Px) *no picture is such that (it pleased* x*)*
(∀x: Px) (∀y: y *is a picture*) ¬ y *pleased* x

$$(\forall x\text{: } Px)\ (\forall y\text{: } Cy)\ \neg\ Lyx$$
$$\forall x\ (Px \rightarrow \forall y\ (Cy \rightarrow \neg\ Lyx))$$

[C: λx (x *is a picture*); L: λxy (x *pleased* y); P: λx (x *is a person*)]

Notice that we are forced here to change from *anyone* to *everyone* when using subject-predicate expansion because the result of retaining *anyone* would be awkward at best. In general, although it is not impossible for *anyone* to serve as the subject of a sentence (see **f** below), it is best to avoid using it as the subject of sentence in expanded form.

d. *Each provision of the law affected every sector of the economy.*
Each provision of the law is such that (it affected every sector of the economy)
(∀x: x *is a provision of the law*) x *affected every sector of the economy*
(∀x: Pxl) *every sector of the economy is such that (*x *affected it)*
(∀x: Pxl) (∀y: y *is a sector of the economy*) x *affected* y

$$(\forall x\text{: } Pxl)\ (\forall y\text{: } Sye)\ Axy$$
$$\forall x\ (Pxl \rightarrow \forall y\ (Sye \rightarrow Axy))$$

[A: λxy (x *affected* y); P: λxy (x *is a provision of* y); S: λxy (x *is a sector of* y); e: *the economy*; l: *the law*]

e. *No picture pleased anyone except photographers*.
All people except photographers are such that (no picture pleased them)
[*or: Everyone who is not a photographer is such that (no picture pleased him or her)*]
(∀x: x *is a person* ∧ ¬ x *is a photographer*) *no picture pleased* x
(∀x: Px ∧ ¬ Hx) *no picture is such that (it pleased* x*)*
(∀x: Px ∧ ¬ Hx) (∀y: y *is a picture*) ¬ y *pleased* x

$$(\forall x\text{: } Px \wedge \neg\ Hx)\ (\forall y\text{: } Cy)\ \neg\ Lyx$$
$$\forall x\ ((Px \wedge \neg\ Hx) \rightarrow \forall y\ (Cy \rightarrow \neg\ Lyx))$$

[C: λx (x *is a picture*); H: λx (x *is a photographer*); L: λxy (x *pleased* y); P: λx (x *is a person*)]

The phrase *all people* is used in the first restatement so that it agrees in number with *except photographers*. It has the disadvantage that *no picture pleased them* might be misunderstood to say that no picture pleased them all. (That would be a misunderstanding because *them* used in the context *them all* would need a subject not already containing *all*—something like *people other than photographers*—as its antecedent.) The alternative using *everyone who isn't a photographer* instead of *all people except photographers* is designed to avoid this misunderstanding. In general, it is best to choose a singular subject when using subject-predicate expansion.

f. *Anyone who likes all mammals likes all horses*
Everyone who likes all mammals is such that (he or she likes all horses)
(∀x: x *is a person* ∧ x *likes all mammals*) x *likes all horses*
(∀x: Px ∧ *every mammal is such that (*x *likes it)*) *every horse is such that (*x *likes it)*

(∀x: Px ∧ (∀y: y *is a mammal*) x *likes* y) (∀z: z *is a horse*) x *likes* z

$$(\forall x\colon Px \wedge (\forall y\colon My)\ Lxy)\ (\forall z\colon Hz)\ Lxz$$
$$\forall x\, ((Px \wedge \forall y\, (My \rightarrow Lxy)) \rightarrow \forall z\, (Hz \rightarrow Lxz))$$

[H: λx (x *is a horse*); L: λxy (x *likes* y); M: λx (x *is a mammal*); P: λx (x *is a person*)]

g. *The law stimulated only sectors of the economy that were affected by all the law's provisions*

Only sectors of the economy that were affected by all the law's provisions are such that (the law stimulated them)

(∀x: ¬ x *is a sector of the economy that was affected by all the law's provisions*) ¬ *the law stimulated* x

(∀x: ¬ (x *is a sector of the economy* ∧ x *was affected by all the law's provisions*)) ¬ Tlx

(∀x: ¬ (Sxe ∧ *every provision of the law is such that (*x *was affected by it)*)) ¬ Tlx

(∀x: ¬ (Sxe ∧ (∀y: y *is a provision of the law*) x *was affected by* y)) ¬ Tlx

$$(\forall x\colon \neg\, (Sxe \wedge (\forall y\colon Pyl)\ Fxy)\ \neg\, Tlx$$
$$\forall x\, (\neg\, (Sxe \wedge \forall y\, (Pyl \rightarrow Fxy)) \rightarrow \neg\, Tlx)$$

[F: λxy (x *was affected by* y); P: λxy (x *is a provision of* y); S: λxy (x *is a sector of* y); T: λxy (x *stimulated* y); e: *the economy*; l: *the law*]

h. *No one who doesn't like all mammals likes any badger.*

Every badger is such that (no one who doesn't like all mammals likes it)

(∀x: x *is badger*) *no one who doesn't like all mammals likes* x

(∀x: Bx) *no one who doesn't like all mammals is such that (he or she likes* x*)*

(∀x: Bx) (∀y: y *is a person who doesn't like all mammals*) ¬ y *likes* x

(∀x: Bx) (∀y: y *is a person* ∧ y *doesn't like all mammals*) ¬ Lyx

(∀x: Bx) (∀y: Py ∧ ¬ y *likes all mammals*) ¬ Lyx

(∀x: Bx) (∀y: Py ∧ ¬ *every mammal is such that (*y *likes it)*) ¬ Lyx

(∀x: Bx) (∀y: Py ∧ ¬ (∀z: z *is a mammal*) y *likes* z) ¬ Lyx

$$(\forall x\colon Bx)\ (\forall y\colon Py \wedge \neg\, (\forall z\colon Mz)\ Lyz)\ \neg\, Lyx$$
$$\forall x\, (Bx \rightarrow \forall y\, ((Py \wedge \neg\, \forall z\, (Mz \rightarrow Lyz)) \rightarrow \neg\, Lyx))$$

[B: λx (x *is a badger*); L: λxy (x *likes* y); M: λx (x *is a mammal*); P: λx (x *is a person*)]

i. *Everyone saw everything that anyone saw.*

Everyone is such that (everyone saw everything that he or she saw)

(∀x: x *is a person*) *everyone saw everything that* x *saw*

(∀x: Px) *everyone is such that (he or she saw everything that* x *saw)*

(∀x: Px) (∀y: y *is a person*) y *saw everything that* x *saw*

(∀x: Px) (∀y: Py) *everything that* x *saw is such that (*y *saw it)*

(∀x: Px) (∀y: Py) (∀z: z *is a thing that* x *saw*) y *saw* z

(∀x: Px) (∀y: Py) (∀z: x *saw* z) Syz

$$(\forall x\colon Px)\ (\forall y\colon Py)\ (\forall z\colon Sxz)\ Syz$$
$$\forall x\, (Px \rightarrow \forall y\, (Py \rightarrow \forall z\, (Sxz \rightarrow Syz)))$$

[P: λx (x *is a person*); S: λxy (x *saw* y)]

j. *No one saw anything that anyone liked.*

Everyone is such that (no one saw anything that he or she liked)

(∀x: x *is a person*) *no one saw anything* x *liked*

(∀x: Px) *everything* x *liked is such that (no one saw it)*

(∀x: Px) (∀y: y *is a thing* x *liked*) *no one saw* y

(∀x: Px) (∀y: x *liked* y) *no one is such that (he or she saw* y*)*

(∀x: Px) (∀y: Lxy) (∀z: z *is a person*) ¬ z *saw* y

$$(\forall x\colon Px)\ (\forall y\colon Lxy)\ (\forall z\colon Pz)\ \neg\, Szy$$
$$\forall x\, (Px \rightarrow \forall y\, (Lxy \rightarrow \forall z\, (Pz \rightarrow \neg\, Szy)))$$

[L: λxy (x *liked* y); P: λx (x *is a person*); S: λxy (x *saw* y)]

The quantifier phrases could have been analyzed in a different order to yield an equivalent interpretation but that would have forced us to change one or both of the two *any*s to *some*.

k. *No one who anyone could recall spoke to everyone.*

Everyone is such that (no one who he or she could recall spoke to everyone)

(∀x: x *is a person*) *no one who* x *could recall spoke to everyone*

(∀x: Px) *no one who* x *could recall is such that (he or she spoke to everyone)*

(∀x: Px) (∀y: y *is a person who* x *could recall*) ¬ y *spoke to everyone*

(∀x: Px) (∀y: y *is a person* ∧ x *could recall* y) ¬ *everyone is such that (*y *spoke to him or her)*

(∀x: Px) (∀y: Py ∧ Rxy) ¬ (∀z: z *is a person*) y *spoke to* z

$$(\forall x{:}\ Px)\ (\forall y{:}\ Py \wedge Rxy)\ \neg\ (\forall z{:}\ Pz)\ Syz$$
$$\forall x\ (Px \rightarrow \forall y\ ((Py \wedge Rxy) \rightarrow \neg\ \forall z\ (Pz \rightarrow Syz)))$$

[P: λx (x *is a person*); R: λxy (x *could recall* y); S: λxy (x *spoke to* y)]

l. *No one who everyone could recall spoke to anyone*

everyone is such that (no one who everyone could recall spoke to him or her)

(∀x: x *is a person*) *no one who everyone could recall spoke to* x

(∀x: Px) *no one who everyone could recall is such that (he or she spoke to* x*)*

(∀x: Px) (∀y: y *is a person who everyone could recall*) ¬ y *spoke to* x

(∀x: Px) (∀y: y *is a person* ∧ *everyone could recall* y) ¬ Syx

(∀x: Px) (∀y: Py ∧ *everyone is such that (he or she could recall* y*)*) ¬ Syx

(∀x: Px) (∀y: Py ∧ (∀z: z *is a person*) z *could recall* y) ¬ Syx

$$(\forall x{:}\ Px)\ (\forall y{:}\ Py \wedge (\forall z{:}\ Pz)\ Rzy)\ \neg\ Syx$$
$$\forall x\ (Px \rightarrow \forall y\ ((Py \wedge \forall z\ (Pz \rightarrow Rzy)) \rightarrow \neg\ Syx))$$

[P: λx (x *is a person*); R: λxy (x *could recall* y); S: λxy (x *spoke to* y)]

m. *Of the pictures anyone saw, no candid ones pleased everyone in them*

Everyone is such that (of the pictures he or she saw, no candid ones pleased everyone in them)

(∀x: x *is a person*) *of the pictures* x *saw, no candid ones pleased everyone in them*

(∀x: Px) *of the pictures* x *saw, no candid one is such that (it pleased everyone in it)*

(∀x: Px) (∀y: y *is a picture* x *saw* ∧ y *is candid*) ¬ y *pleased everyone in* y

(∀x: Px) (∀y: (y *is a picture* ∧ x *saw* y) ∧ y *is candid*) ¬ *everyone in* y *is such that (*y *pleased him or her)*

(∀x: Px) (∀y: (Cy ∧ Sxy) ∧ Dy) ¬ (∀z: z *is a person in* y) y *pleased* z

(∀x: Px) (∀y: (Cy ∧ Sxy) ∧ Dy) ¬ (∀z: z *is a person* ∧ z *is in* y) Lyz

$$(\forall x{:}\ Px)\ (\forall y{:}\ (Cy \wedge Sxy) \wedge Dy)\ \neg\ (\forall z{:}\ Pz \wedge Nzy)\ Lyz$$
$$\forall x\ (Px \rightarrow \forall y\ (((Cy \wedge Sxy) \wedge Dy) \rightarrow \neg\ \forall z\ ((Pz \wedge Nzy) \rightarrow Lyz)))$$

[C: λx (x *is a picture*); D: λx (x *is candid*); L: λxy (x *pleased* y); P: λx (x *is a person*); S: λx (x *saw* y)]

n. *No law will affect only sectors of the economy that figure in all its provisions*

No law is such that (it will affect only sectors of the economy that figure in all its provisions)

(∀x: x *is a law*) ¬ x *will affect only sectors of the economy that figure in all* x*'s provisions*)

(∀x: Lx) ¬ *only sectors of the economy that figure in all* x*'s provisions are such that (*x *will affect them)*)

(∀x: Lx) ¬ (∀y: ¬ y *is a sector of the economy that figures in all* x*'s provisions*) ¬ x *will affect* y

(∀x: Lx) ¬ (∀y: ¬ (y *is a sector of the economy* ∧ y *figures in all* x*'s provisions*)) ¬ Axy

(∀x: Lx) ¬ (∀y: ¬ (Sye ∧ *all* x*'s provisions are such that (*y *figures in them)*)) ¬ Axy

(∀x: Lx) ¬ (∀y: ¬ (Sye ∧ (∀z: z *is a provision of* x) y *figures in* z)) ¬ Axy

$$(\forall x{:}\ Lx)\ \neg\ (\forall y{:}\ \neg\ (Sye \wedge (\forall z{:}\ Pzx)\ Fyz))\ \neg\ Axy$$
$$\forall x\ (Lx \rightarrow \neg\ \forall y\ (\neg\ (Sye \wedge \forall z\ (Pzx \rightarrow Fyz)) \rightarrow \neg\ Axy))$$

[A: λxy (x *affects* y); F: λxy (x *figures in* y); L: λx (x *is a* *law*); P: λxy (x *is a provision of* y); S: λxy (x *is a sector of* y); e: *the economy*]

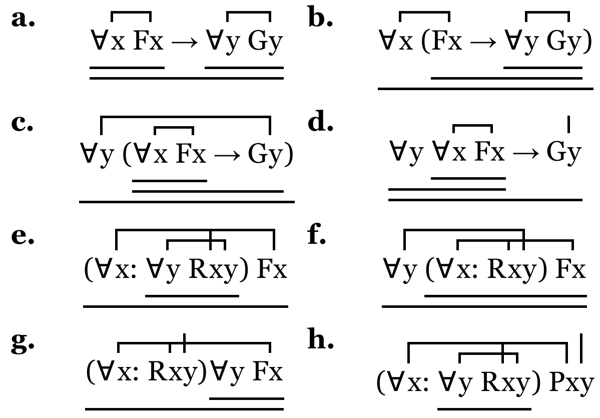
or (and perhaps better): (∀x: Lx) ¬ (∀y: Sye ∧ ¬ (∀z: Pzx) Fyz) ¬ Axy —this is the result of taking *sectors of the economy* to indicate bounds so that the formula

x *will affect only sectors of the economy that figure in all* x*'s provisions*

would be expanded to

among sectors of the economy, only those that figure in all x*'s provisions are such that (*x *will affect them)*

2.



3. a. (∀x: x *is a mosquito*) (∀y: y *is a person*) x *despises* y

(∀x: x *is a mosquito*) *every person is such that* (x *despises him or her*)

(∀x: x *is a mosquito*) x *despises every person*

Every mosquito is such that (it despises every person)

Every mosquito despises every person or: Every mosquito despises all people

b. (∀x: x *is a person*) ¬ (∀y: y *is a mosquito*) x *despises* y

(∀x: x *is a person*) ¬ *every mosquito is such that (*x *despises it)*

(∀x: x *is a person*) ¬ x *despises every mosquito*

No one is such that (he or she despises every mosquito)

No one despises every mosquito

c. (∀x: x *is a mosquito*) (∀y: y *is a person*) ¬ y *despises* x

(∀x: x *is a mosquito*) *no person is such that (he or she despises* x*)*

(∀x: x *is a mosquito*) *no one despises* x

Every mosquito is such that (no one despises it)

No one despises any mosquito or: No one despises a mosquito

d. (∀x: x *is a person*) (∀y: y *is a mosquito* ∧ y *has bitten* x) ¬ x *despises* y

(∀x: x *is a person*) (∀y: y *is a mosquito that has bitten* x) ¬ x *despises* y

(∀x: x *is a person*) *no mosquito that has bitten* x *is such that (*x *despises it)*

(∀x: x *is a person*) x *despises no mosquito that has bitten* x

Every person is such that (he or she despises no mosquito that has bitten him or her)

A person despises no mosquito that has bitten him or her

The sentence *No one despises any mosquito that has bitten him or her* is equivalent, and more natural, but its closest analysis would take a slightly different form.

e. (∀x: x *is a person* ∧ (∀y: y *is a mosquito*) x *despises* y) (∀z: z *is a mosquito*) ¬ z *has bitten* x

(∀x: x *is a person* ∧ *every mosquito is such that (*x *despises it)*) *no mosquito is such that (it has bitten* x*)*

(∀x: x *is a person* ∧ x *despises every mosquito*) *no mosquito has bitten* x

(∀x: x *is a person who despises every mosquito*) *no mosquito has bitten* x

Every person who despises every mosquito is such that (no mosquito has bitten him or her)

No mosquito has bitten anyone who despises every mosquito or: No mosquito has bitten anyone who despises mosquitoes

f. ∀x (∀y: x *is smaller than* y) ¬ y *is smaller than* x

∀x *nothing that* x *is smaller than is such that (it is smaller than* x*)*

∀x *nothing that* x *is smaller than is smaller than* x

Everything is such that (nothing that it is smaller than is smaller than it)

Nothing that anything is smaller than is smaller than it