

7.4.xa. Exercise answers

1. a. *Every picture pleased everyone*

Every picture is such that (it pleased everyone)

$(\forall x: x \text{ is a picture})$ x pleased everyone

$(\forall x: Cx)$ everyone is such that (x pleased him or her)

$(\forall x: Cx) (\forall y: y \text{ is a person})$ x pleased y

$(\forall x: Cx) (\forall y: Py) Lxy$

$\forall x (Cx \rightarrow \forall y (Py \rightarrow Lxy))$

[C: λx (x is a picture); L: λxy (x pleased y); P: λx (x is a person)]

- b. *No picture pleased everyone*

No picture is such that (it pleased everyone)

$(\forall x: x \text{ is a picture}) \neg x$ pleased everyone

$(\forall x: Cx) \neg$ everyone is such that (x pleased him or her)

$(\forall x: Cx) \neg (\forall y: y \text{ is a person})$ x pleased y

$(\forall x: Cx) \neg (\forall y: Py) Lxy$

$\forall x (Cx \rightarrow \neg \forall y (Py \rightarrow Lxy))$

[C: λx (x is a picture); L: λxy (x pleased y); P: λx (x is a person)]

- c. *No picture pleased anyone.*

Everyone is such that (no picture pleased him or her)

$(\forall x: x \text{ is a person})$ no picture pleased x

$(\forall x: Px)$ no picture is such that (it pleased x)

$(\forall x: Px) (\forall y: y \text{ is a picture}) \neg y$ pleased x

$(\forall x: Px) (\forall y: Cy) \neg Lyx$

$\forall x (Px \rightarrow \forall y (Cy \rightarrow \neg Lyx))$

[C: λx (x is a picture); L: λxy (x pleased y); P: λx (x is a person)]

Notice that we are forced here to change from *anyone* to *everyone* when using subject-predicate expansion because the result of retaining *anyone* would be awkward at best. In general, although it is not impossible for *anyone* to serve as the subject of a sentence (see **f** below), it is best to avoid using it as the subject of sentence in expanded form.

- d. *Each provision of the law affected every sector of the economy.*

Each provision of the law is such that (it affected every sector of the economy)

$(\forall x: \underline{x \text{ is a provision of the law}}) x \text{ affected every sector of the economy}$

$(\forall x: Px) \text{ every sector of the economy is such that } (x \text{ affected it})$

$(\forall x: Px) (\forall y: \underline{y \text{ is a sector of the economy}}) x \text{ affected } y$

$(\forall x: Px) (\forall y: Sy) Axy$

$\forall x (Px \rightarrow \forall y (Sy \rightarrow Axy))$

[A: $\lambda xy (x \text{ affected } y)$; P: $\lambda xy (x \text{ is a provision of } y)$; S: $\lambda xy (x \text{ is a sector of } y)$; e: *the economy*; l: *the law*]

- e. *No picture pleased anyone except photographers.*

All people except photographers are such that (no picture pleased them)

[or: *Everyone who is not a photographer is such that (no picture pleased him or her)*]

$(\forall x: x \text{ is a person} \wedge \neg x \text{ is a photographer}) \text{ no picture pleased } x$

$(\forall x: Px \wedge \neg Hx) \text{ no picture is such that (it pleased } x)$

$(\forall x: Px \wedge \neg Hx) (\forall y: y \text{ is a picture}) \neg y \text{ pleased } x$

$(\forall x: Px \wedge \neg Hx) (\forall y: Cy) \neg Lyx$

$\forall x ((Px \wedge \neg Hx) \rightarrow \forall y (Cy \rightarrow \neg Lyx))$

[C: $\lambda x (x \text{ is a picture})$; H: $\lambda x (x \text{ is a photographer})$; L: $\lambda xy (x \text{ pleased } y)$; P: $\lambda x (x \text{ is a person})$]

The phrase *all people* is used in the first restatement so that it agrees in number with *except photographers*. It has the disadvantage that *no picture pleased them* might be misunderstood to say that no picture pleased them all. (That would be a misunderstanding because *them* used in the context *them all* would need a subject not already containing *all*—something like *people other than photographers*—as its antecedent.) The alternative using *everyone who isn't a photographer* instead of *all people except photographers* is designed to avoid this misunderstanding. In general, it is best to choose a singular subject when using subject-predicate expansion.

- f. *Anyone who likes all mammals likes all horses*

Everyone who likes all mammals is such that (he or she likes all horses)

$(\forall x: x \text{ is a person} \wedge x \text{ likes all mammals}) x \text{ likes all horses}$

$(\forall x: Px \wedge \text{every mammal is such that } (x \text{ likes it})) \text{ every horse is such that } (x \text{ likes it})$

$(\forall x: Px \wedge (\forall y: y \text{ is a mammal}) x \text{ likes } y) (\forall z: z \text{ is a horse})$
 $x \text{ likes } z$

$(\forall x: Px \wedge (\forall y: My) Lxy) (\forall z: Hz) Lxz$

$\forall x ((Px \wedge \forall y (My \rightarrow Lxy)) \rightarrow \forall z (Hz \rightarrow Lxz))$

[H: $\lambda x (x \text{ is a horse})$; L: $\lambda xy (x \text{ likes } y)$; M: $\lambda x (x \text{ is a mammal})$; P: $\lambda x (x \text{ is a person})$]

- g. *The law stimulated only sectors of the economy that were affected by all the law's provisions*

Only sectors of the economy that were affected by all the law's provisions are such that (the law stimulated them)

$(\forall x: \neg x \text{ is a sector of the economy that was affected by all the law's provisions}) \neg \text{the law stimulated } x$

$(\forall x: \neg (x \text{ is a sector of the economy} \wedge x \text{ was affected by all the law's provisions})) \neg Tlx$

$(\forall x: \neg (Sxe \wedge \text{every provision of the law is such that } (x \text{ was affected by it}))) \neg Tlx$

$(\forall x: \neg (Sxe \wedge (\forall y: y \text{ is a provision of the law}) x \text{ was affected by } y)) \neg Tlx$

$(\forall x: \neg (Sxe \wedge (\forall y: Pyl) Fxy)) \neg Tlx$

$\forall x (\neg (Sxe \wedge \forall y (Pyl \rightarrow Fxy)) \rightarrow \neg Tlx)$

[F: $\lambda xy (x \text{ was affected by } y)$; P: $\lambda xy (x \text{ is a provision of } y)$; S: $\lambda xy (x \text{ is a sector of } y)$; T: $\lambda xy (x \text{ stimulated } y)$; e: *the economy*; l: *the law*]

- h. *No one who doesn't like all mammals likes any badger. Every badger is such that (no one who doesn't like all mammals likes it)*

$(\forall x: x \text{ is badger}) \text{ no one who doesn't like all mammals likes } x$

$(\forall x: Bx) \text{ no one who doesn't like all mammals is such that (he or she likes } x)$

$(\forall x: Bx) (\forall y: y \text{ is a person who doesn't like all mammals}) \neg y \text{ likes } x$

$(\forall x: Bx) (\forall y: y \text{ is a person} \wedge y \text{ doesn't like all mammals}) \neg Lyx$

$(\forall x: Bx) (\forall y: Py \wedge \neg y \text{ likes all mammals}) \neg Lyx$

$(\forall x: Bx) (\forall y: Py \wedge \neg \text{every mammal is such that } (y \text{ likes$

it) $\neg Lyx$

$(\forall x: Bx) (\forall y: Py \wedge \neg (\forall z: z \text{ is a mammal}) y \text{ likes } z) \neg Lyx$

$(\forall x: Bx) (\forall y: Py \wedge \neg (\forall z: Mz) Lyz) \neg Lyx$

$\forall x (Bx \rightarrow \forall y ((Py \wedge \neg \forall z (Mz \rightarrow Lyz)) \rightarrow \neg Lyx))$

[B: $\lambda x (x \text{ is a badger})$; L: $\lambda xy (x \text{ likes } y)$; M: $\lambda x (x \text{ is a mammal})$; P: $\lambda x (x \text{ is a person})$]

i. *Everyone saw everything that anyone saw.*

Everyone is such that (everyone saw everything that he or she saw)

$(\forall x: x \text{ is a person})$ *everyone saw everything that x saw*

$(\forall x: Px)$ *everyone is such that (he or she saw everything that x saw)*

$(\forall x: Px) (\forall y: y \text{ is a person})$ *y saw everything that x saw*

$(\forall x: Px) (\forall y: Py)$ *everything that x saw is such that (y saw it)*

$(\forall x: Px) (\forall y: Py) (\forall z: z \text{ is a thing that x saw})$ *y saw z*

$(\forall x: Px) (\forall y: Py) (\forall z: x \text{ saw } z)$ *Syz*

$(\forall x: Px) (\forall y: Py) (\forall z: Sxz)$ *Syz*

$\forall x (Px \rightarrow \forall y (Py \rightarrow \forall z (Sxz \rightarrow Syz)))$

[P: $\lambda x (x \text{ is a person})$; S: $\lambda xy (x \text{ saw } y)$]

j. *No one saw anything that anyone liked.*

Everyone is such that (no one saw anything that he or she liked)

$(\forall x: x \text{ is a person})$ *no one saw anything x liked*

$(\forall x: Px)$ *everything x liked is such that (no one saw it)*

$(\forall x: Px) (\forall y: y \text{ is a thing x liked})$ *no one saw y*

$(\forall x: Px) (\forall y: x \text{ liked } y)$ *no one is such that (he or she saw y)*

$(\forall x: Px) (\forall y: Lxy) (\forall z: z \text{ is a person}) \neg z \text{ saw } y$

$(\forall x: Px) (\forall y: Lxy) (\forall z: Pz) \neg Szy$

$\forall x (Px \rightarrow \forall y (Lxy \rightarrow \forall z (Pz \rightarrow \neg Szy)))$

[L: $\lambda xy (x \text{ liked } y)$; P: $\lambda x (x \text{ is a person})$; S: $\lambda xy (x \text{ saw } y)$]

The quantifier phrases could have been analyzed in a different order to yield an equivalent interpretation but that would have forced us to change one or both of the two *any*s to *some*.

- k. *No one who anyone could recall spoke to everyone. Everyone is such that (no one who he or she could recall spoke to everyone)*
- $(\forall x: x \text{ is a person})$ *no one who x could recall spoke to everyone*
- $(\forall x: Px)$ *no one who x could recall is such that (he or she spoke to everyone)*
- $(\forall x: Px) (\forall y: y \text{ is a person who x could recall}) \rightarrow y$ *spoke to everyone*
- $(\forall x: Px) (\forall y: y \text{ is a person} \wedge x \text{ could recall } y) \rightarrow$ *everyone is such that (y spoke to him or her)*
- $(\forall x: Px) (\forall y: Py \wedge Rxy) \rightarrow (\forall z: z \text{ is a person}) y$ *spoke to z*
- $(\forall x: Px) (\forall y: Py \wedge Rxy) \rightarrow (\forall z: Pz) Syz$
- $\forall x (Px \rightarrow \forall y ((Py \wedge Rxy) \rightarrow \neg \forall z (Pz \rightarrow Syz)))$
- [P: $\lambda x (x \text{ is a person})$; R: $\lambda xy (x \text{ could recall } y)$; S: $\lambda xy (x \text{ spoke to } y)$]
- l. *No one who everyone could recall spoke to anyone everyone is such that (no one who everyone could recall spoke to him or her)*
- $(\forall x: x \text{ is a person})$ *no one who everyone could recall spoke to x*
- $(\forall x: Px)$ *no one who everyone could recall is such that (he or she spoke to x)*
- $(\forall x: Px) (\forall y: y \text{ is a person who everyone could recall}) \rightarrow y$ *spoke to x*
- $(\forall x: Px) (\forall y: y \text{ is a person} \wedge \text{everyone could recall } y)$
 $\neg Syx$
- $(\forall x: Px) (\forall y: Py \wedge \text{everyone is such that (he or she could recall } y)) \rightarrow Syx$
- $(\forall x: Px) (\forall y: Py \wedge (\forall z: z \text{ is a person}) z \text{ could recall } y)$
 $\neg Syx$
- $(\forall x: Px) (\forall y: Py \wedge (\forall z: Pz) Rzy) \rightarrow Syx$
- $\forall x (Px \rightarrow \forall y ((Py \wedge \forall z (Pz \rightarrow Rzy)) \rightarrow \neg Syx))$
- [P: $\lambda x (x \text{ is a person})$; R: $\lambda xy (x \text{ could recall } y)$; S: $\lambda xy (x \text{ spoke to } y)$]
- m. *Of the pictures anyone saw, no candid ones pleased everyone in them*

Everyone is such that (of the pictures he or she saw, no candid ones pleased everyone in them)

$(\forall x: x \text{ is a person})$ of the pictures x saw, no candid ones pleased everyone in them

$(\forall x: Px)$ of the pictures x saw, no candid one is such that (it pleased everyone in it)

$(\forall x: Px) (\forall y: y \text{ is a picture } x \text{ saw} \wedge y \text{ is candid}) \rightarrow y$ pleased everyone in y

$(\forall x: Px) (\forall y: (y \text{ is a picture} \wedge x \text{ saw } y) \wedge y \text{ is candid}) \rightarrow$ everyone in y is such that (y pleased him or her)

$(\forall x: Px) (\forall y: (Cy \wedge Sxy) \wedge Dy) \rightarrow (\forall z: z \text{ is a person in } y) y$ pleased z

$(\forall x: Px) (\forall y: (Cy \wedge Sxy) \wedge Dy) \rightarrow (\forall z: z \text{ is a person} \wedge z \text{ is in } y) Lyz$

$(\forall x: Px) (\forall y: (Cy \wedge Sxy) \wedge Dy) \rightarrow (\forall z: Pz \wedge Nzy) Lyz$

$\forall x (Px \rightarrow \forall y (((Cy \wedge Sxy) \wedge Dy) \rightarrow \neg \forall z ((Pz \wedge Nzy) \rightarrow Lyz)))$

[C: $\lambda x (x \text{ is a picture})$; D: $\lambda x (x \text{ is candid})$; L: $\lambda xy (x$ pleased $y)$; P: $\lambda x (x \text{ is a person})$; S: $\lambda x (x \text{ saw } y)$]

- n. *No law will affect only sectors of the economy that figure in all its provisions*

No law is such that (it will affect only sectors of the economy that figure in all its provisions)

$(\forall x: x \text{ is a law}) \rightarrow x$ will affect only sectors of the economy that figure in all x 's provisions)

$(\forall x: Lx) \rightarrow$ only sectors of the economy that figure in all x 's provisions are such that (x will affect them)

$(\forall x: Lx) \rightarrow (\forall y: \neg y \text{ is a sector of the economy that figures in all } x$'s provisions) $\rightarrow x$ will affect y

$(\forall x: Lx) \rightarrow (\forall y: \neg (y \text{ is a sector of the economy} \wedge y \text{ figures in all } x$'s provisions)) $\rightarrow Axy$

$(\forall x: Lx) \rightarrow (\forall y: \neg (Sye \wedge \text{all } x$'s provisions are such that (y figures in them))) $\rightarrow Axy$

$(\forall x: Lx) \rightarrow (\forall y: \neg (Sye \wedge (\forall z: z \text{ is a provision of } x) y \text{ figures in } z)) \rightarrow Axy$

$(\forall x: Lx) \rightarrow (\forall y: \neg (Sye \wedge (\forall z: Pzx) Fyz)) \rightarrow Axy$

$\forall x (Lx \rightarrow \neg \forall y (\neg (Sye \wedge \forall z (Pzx \rightarrow Fyz)) \rightarrow \neg Axy))$

[A: $\lambda xy (x \text{ affects } y)$; F: $\lambda xy (x \text{ figures in } y)$; L: $\lambda x (x \text{ is a$

law); P: λxy (*x is a provision of y*); S: λxy (*x is a sector of y*); e: *the economy*]

or (and perhaps better): $(\forall x: Lx) \rightarrow (\forall y: Sye \wedge \neg (\forall z: Pzx) Fyz) \rightarrow Axy$ —this is the result of taking *sectors of the economy* to indicate bounds so that the formula

x will affect only sectors of the economy that figure in all x's provisions

would be expanded to

among sectors of the economy, only those that figure in all x's provisions are such that (x will affect them)

2. a.
$$\frac{\frac{\frac{}{\forall x Fx} \rightarrow \frac{\frac{}{\forall y Gy}}{\forall y Gy}}{\forall y Gy}}{\forall y Gy}}{\forall y Gy}$$
 b.
$$\frac{\frac{\frac{}{\forall x (Fx \rightarrow \forall y Gy)} \rightarrow \frac{\frac{}{\forall y Gy}}{\forall y Gy}}{\forall y Gy}}{\forall y Gy}}{\forall y Gy}$$
- c.
$$\frac{\frac{\frac{}{\forall y (\forall x Fx \rightarrow Gy)}}{\forall y (\forall x Fx \rightarrow Gy)}}{\forall y (\forall x Fx \rightarrow Gy)}}{\forall y (\forall x Fx \rightarrow Gy)}$$
 d.
$$\frac{\frac{\frac{}{\forall y \forall x Fx \rightarrow Gy}}{\forall y \forall x Fx \rightarrow Gy}}{\forall y \forall x Fx \rightarrow Gy}}{\forall y \forall x Fx \rightarrow Gy}$$
- e.
$$\frac{\frac{\frac{}{(\forall x: \forall y Rxy) Fx}}{(\forall x: \forall y Rxy) Fx}}{(\forall x: \forall y Rxy) Fx}}{(\forall x: \forall y Rxy) Fx}}$$
 f.
$$\frac{\frac{\frac{}{\forall y (\forall x: Rxy) Fx}}{\forall y (\forall x: Rxy) Fx}}{\forall y (\forall x: Rxy) Fx}}{\forall y (\forall x: Rxy) Fx}}$$
- g.
$$\frac{\frac{\frac{}{(\forall x: Rxy) \forall y Fx}}{(\forall x: Rxy) \forall y Fx}}{(\forall x: Rxy) \forall y Fx}}{(\forall x: Rxy) \forall y Fx}}$$
 h.
$$\frac{\frac{\frac{}{(\forall x: \forall y Rxy) Pxy}}{(\forall x: \forall y Rxy) Pxy}}{(\forall x: \forall y Rxy) Pxy}}{(\forall x: \forall y Rxy) Pxy}}$$

3. a. $(\forall x: x \text{ is a mosquito}) (\forall y: y \text{ is a person}) x \text{ despises } y$
($\forall x: x \text{ is a mosquito}$) every person is such that (x despises him or her)
($\forall x: x \text{ is a mosquito}$) x despises every person
Every mosquito is such that (it despises every person)
Every mosquito despises every person or: Every mosquito despises all people
- b. $(\forall x: x \text{ is a person}) \neg (\forall y: y \text{ is a mosquito}) x \text{ despises } y$
($\forall x: x \text{ is a person}$) \neg every mosquito is such that (x despises it)
($\forall x: x \text{ is a person}$) \neg x despises every mosquito
No one is such that (he or she despises every mosquito)
No one despises every mosquito
- c. $(\forall x: x \text{ is a mosquito}) (\forall y: y \text{ is a person}) \neg y \text{ despises } x$
($\forall x: x \text{ is a mosquito}$) no person is such that (he or she despises x)
($\forall x: x \text{ is a mosquito}$) no one despises x
Every mosquito is such that (no one despises it)

No one despises any mosquito or: No one despises a mosquito

- d. $(\forall x: x \text{ is a person}) (\forall y: y \text{ is a mosquito} \wedge y \text{ has bitten } x)$
 $\neg x \text{ despises } y$
 $(\forall x: x \text{ is a person}) (\forall y: y \text{ is a mosquito that has bitten } x)$
 $\neg x \text{ despises } y$
 $(\forall x: x \text{ is a person})$ no mosquito that has bitten x is such that (x despises it)
 $(\forall x: x \text{ is a person})$ x despises no mosquito that has bitten x

Every person is such that (he or she despises no mosquito that has bitten him or her)

A person despises no mosquito that has bitten him or her

The sentence *No one despises any mosquito that has bitten him or her* is equivalent, and more natural, but its closest analysis would take a slightly different form.

- e. $(\forall x: x \text{ is a person} \wedge (\forall y: y \text{ is a mosquito})$ x despises y)
 $(\forall z: z \text{ is a mosquito}) \neg z$ has bitten x
 $(\forall x: x \text{ is a person} \wedge$ every mosquito is such that (x despises it)) no mosquito is such that (it has bitten x)
 $(\forall x: x \text{ is a person} \wedge x$ despises every mosquito) no mosquito has bitten x
 $(\forall x: x \text{ is a person who despises every mosquito})$ no mosquito has bitten x

Every person who despises every mosquito is such that (no mosquito has bitten him or her)

No mosquito has bitten anyone who despises every mosquito or: No mosquito has bitten anyone who despises mosquitoes

- f. $\forall x (\forall y: x \text{ is smaller than } y) \neg y$ is smaller than x
 $\forall x$ nothing that x is smaller than is such that (it is smaller than x)
 $\forall x$ nothing that x is smaller than is smaller than x
Everything is such that (nothing that it is smaller than is smaller than it)

Nothing that anything is smaller than is smaller than it